MODELING AND OPTIMIZING OF BONDED PATCH REPAIRS FOR POLYMER COMPOSITE STRUCTURES

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Abstract

The long-term usage of composite structures challenges to develop new tools to support effective and reliable restore procedures for composites. The present paper describes an analytical method of stress distribution modeling based on inclusion theory that is used to create new software aimed to solve maintenance problems that will appear during service life of composite structures.

Stress distribution is formulated in accordance with constitutive equations of mathematical theory of elasticity and can be applied for various patch repair configurations.

1 Introduction

Maintenance of modern civil aircraft airworthiness is the important task for existing airlines. The significant part of load-carrying structure of currently producing airplanes is made of composite materials – fiber-reinforced plastics with thermoset resins as a matrix. Patch repairs of aircraft composite structures damaged during the airplane service life need to provide adequate models, methods and software tools in order to increase efficiency of applying repairs, decrease expenses for composite airplane maintenance and reduction of labour intensity of pre- and post-flight service.

The repair manuals describing the technology of evaluation and patching the damage of structures are usually developed by a company-producer of aircraft but one of the most frequent damages (partial depth nicks or scratches) can be assessed and neutralized by in-line technical personnel at the airplane location. Thus, the most common and frequent repairs could be excluded from manuals in order to transfer the possibility of defining the repair patch to airline directly.

To realize this ability the novel simple but reliable software containing solutions of basic equations of theory of elasticity and fracture mechanics should be created.

Stress distribution along the damaged skin panel and composite patch is the core value of developing repair software.

2 Stress Distribution Model

2.1 Repair Configuration

The repaired part to be analyzed is flat supported skin panel made of tape fiber reinforced polymer composite with rigidly bonded composite patch. Patch is made of the same material as a skin panel. Linear dimensions of skin are much more than patch dimensions so the repaired skin in this particular case can be represented as infinite. Skin panel is supposed damaged with line crack initially oriented normally to zero direction of skin panel layup. Patch layup is organized to bridge the supposed crack with zero direction normal to crack face. Adhesive between skin and patch is omitted during stress calculation. Load transfer path in adhesive is assumed much less than patch dimension.

Skin panel is subjected to a tension stress $\sigma_{x\infty}$ along the zero layup direction. No bending effects are accounted at patch location: this idealization may take place in case of partial depth damage or stiffener supported skin.

Repair configuration is shown on Fig. 1.

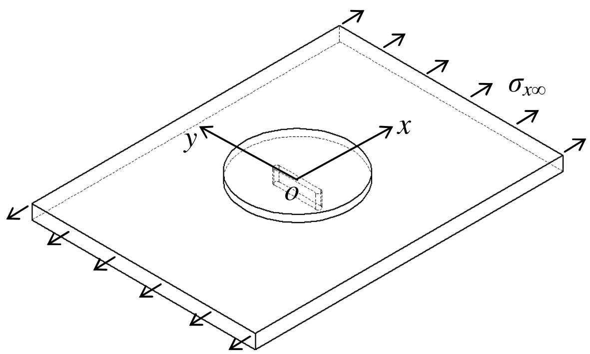

Fig. 1. Patch-to-skin Repair Configuration

Prior to define conditions affected to opening the existing crack, it is reasonable to research the influence of patch application to stress distribution along the skin under the patch. Prospective stress σ_0 applied to the faces of crack can be determined via investigation of repair configuration where patch is bonded to uncracked skin first [1].

2.2 Effective Elastic Constants of Composite Layup

Skin panel and repair patch are assumed consisting of several unidirectional composite plies. Mechanical properties of each unidirectional ply are described with Young's moduli along and normal to fiber direction E_1 and E_2, shear modulus G_{12} and Poisson's ratios ν_{12} and ν_{21}, where indices 1 and 2 are related to axis along fiber: 1 – along fiber direction, 2 – normal to fiber direction.

Effective layup elastic constants can be found based on composite plate theory for the known number of plies and their angle orientation φ with principal axes of the composite plate [2] (layup is consider symmetric):

$$E_x = B_{11} - \frac{B_{12}^2}{B_{22}},\qquad E_y = B_{22} - \frac{B_{12}^2}{B_{11}}, \tag{1}$$

$$G_{xy} = B_{33},\qquad \nu_{xy} = \frac{B_{12}}{B_{11}}, \nu_{yx} = \frac{B_{12}}{B_{22}}. \tag{2}$$

Stiffness coefficients B_{ij} of layup are calculated as follows:

$$B_{ij} = \sum_{k=1}^{n} \bar{h}_k\, b_{ij}^k,\qquad \bar{h}_k = \frac{h_k}{h}, \tag{2}$$

h_k – thickness of k ply, h – thickness of layup, n – number of plies in layup and stiffness components b_{ij}^k of each ply:

$$\begin{aligned} b_{11}^k &= \bar{E}_1^k c_k^4 + \bar{E}_2^k s_k^4 + 2E_{12}^k c_k^4 s_k^4,\\ b_{12}^k &= \bar{E}_1^k \nu_{12}^k + \left(\bar{E}_1^k + \bar{E}_2^k - 2E_{12}^k\right) c_k^2 s_k^2,\\ b_{22}^k &= \bar{E}_1^k s_k^4 + \bar{E}_2^k c_k^4 + 2E_{12}^k c_k^4 s_k^4,\\ b_{33}^k &= \left(\bar{E}_1^k + \bar{E}_2^k - 2\bar{E}_1^k \nu_{12}^k\right) c_k^2 s_k^2 + G_{12}^k (c_k^2 - s_k^2)^2, \end{aligned} \tag{3}$$

where:

$$\bar{E}_{1,\ 2}^k = \frac{E_{1,\ 2}^k}{1 - \nu_{12}^k \nu_{21}^k},\qquad E_{12}^k = \bar{E}_1^k \nu_{12}^k + 2G_{12}^k,\qquad c_k = \cos(\varphi_k), s_k = \sin(\varphi_k). \tag{4}$$

Defined effective elastic constants E_x, E_y, G_{xy} and ν_{xy} will be used as mechanical characteristics of composite skin and patch to determine the sought stress distribution.

2.3 Inclusion Methodology

Inclusion method supposes the representation of repair area as thin plate with included repair "sandwich" – patch adhesively bonded to the part of skin under the patch [3]. Figure 2 shows the proposed division of the patch repair.

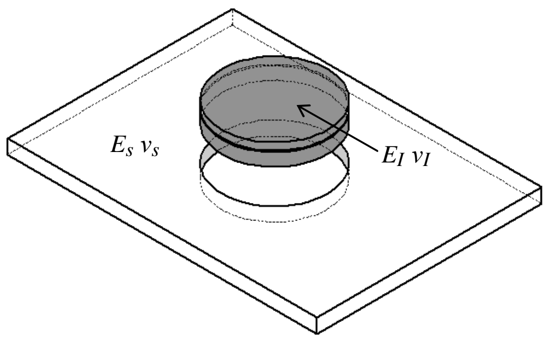

Fig. 2. Inclusion Method in the Repaired Area

This inclusion analogy allows describing differences of stress and strain states between patched area and surrounding skin area. Mechanical properties for the skin and inclusion can be defined separately and then used in constitutive static equations of the inhomogeneous plate subjected to tension.

Stiffness matrix of the inclusion can be calculated using formula for composed plate consisting of layers with different materials from static equation of forces' equality applied to skin and inclusion:

$$[C^I] = [C^s] + \frac{h_p}{h_s}[C^p], \tag{5}$$

with skin stiffness matrix:

$$[C^s] = \begin{bmatrix} B_{11}^s & B_{12}^s \\ B_{12}^s & B_{22}^s \end{bmatrix}, \tag{6}$$

and patch stiffness matrix:

$$[C^p] = \begin{bmatrix} B_{11}^p & B_{12}^p \\ B_{12}^p & B_{22}^p \end{bmatrix}. \tag{7}$$

h_s and h_p are thickness of skin and patch portion of inclusion, respectively.

2.4 Stress Distribution in Repair Stack-up

To calculate stresses inside inclusion area and their split to skin and patch partition solution from mathematical theory of elasticity is used. Stress and strain states are considered plain.

The stresses and strains inside the circular inclusion are assumed uniform. Because of the traction continuity across the interface, the tractions at the boundary between the inclusion and the surrounding skin plate along the *x* and *y* direction are related to the constant stresses *p* and *q* as depicted in Figure 3 (*p* and *q* represent the *x* and *y* components of the stresses inside the inclusion, respectively).

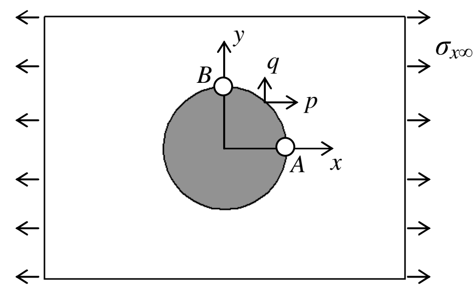

Fig. 3. Stress Distribution at Inclusion Location

The unknown stresses *p* and *q* can be found from the displacement continuity conditions along the interface between the inclusion and the surround skin.

Since the strains in the inclusion are constant and independent of *x* and *y*, the displacement continuity suggests that the *y* strain $\varepsilon_{y,A}^I$ is continuous at point A, and the *x* strain $\varepsilon_{x,B}^I$ is continuous at point B. This condition is formulated as follows:

$$\begin{aligned} \varepsilon_{x,B}^I &= \varepsilon_{x,B}^s, \\ \varepsilon_{y,A}^I &= \varepsilon_{y,A}^s. \end{aligned} \tag{8}$$

Strains inside inclusion can be expressed by:

$$\begin{bmatrix} \varepsilon_x^I \\ \varepsilon_y^I \end{bmatrix} = [C^I]^{-1} \begin{bmatrix} p \\ q \end{bmatrix}. \tag{9}$$

Stresses *p* and *q* can be found based on methods of mathematical theory of elasticity and functions of complex variables theory [4]. Resulting expressions for *p* and *q*:

$$\begin{aligned} p &= \sigma_{x\infty}\frac{\beta^I + \delta^I}{2}, \\ q &= \sigma_{x\infty}\frac{\beta^I - \delta^I}{2}. \end{aligned} \tag{10}$$

Here,

$$\beta^I = \frac{\mu^I(\chi^s + 1)}{2\mu^I + \mu^s(\chi^I - 1)}, \tag{11}$$

$$\delta^I = \frac{\mu^I(\chi^s + 1)}{\mu^s + \mu^I \chi^s};$$

$$\mu^{I,s} = \frac{1}{2}\frac{E_{I,s}}{\nu_{I,s} + 1}, \ \chi^{I,s} = \frac{3 - \nu_{I,s}}{1 + \nu_{I,s}}.$$

Thus, the stresses in the plate can be defined as:

$$\begin{bmatrix}\sigma_x^s \\ \sigma_y^s\end{bmatrix} = [C^s][C^I]^{-1}\begin{bmatrix}p \\ q\end{bmatrix}. \qquad (12)$$

Stresses in skin underneath the patch are the quantity of intent interest because of their affect to the prospective crack-opening stress:

$$\sigma_x^s = \sigma_0. \qquad (13)$$

For the case of fully supported one-sided repair (no out-of-plane bending), this prospective stress is uniform through the plate thickness.

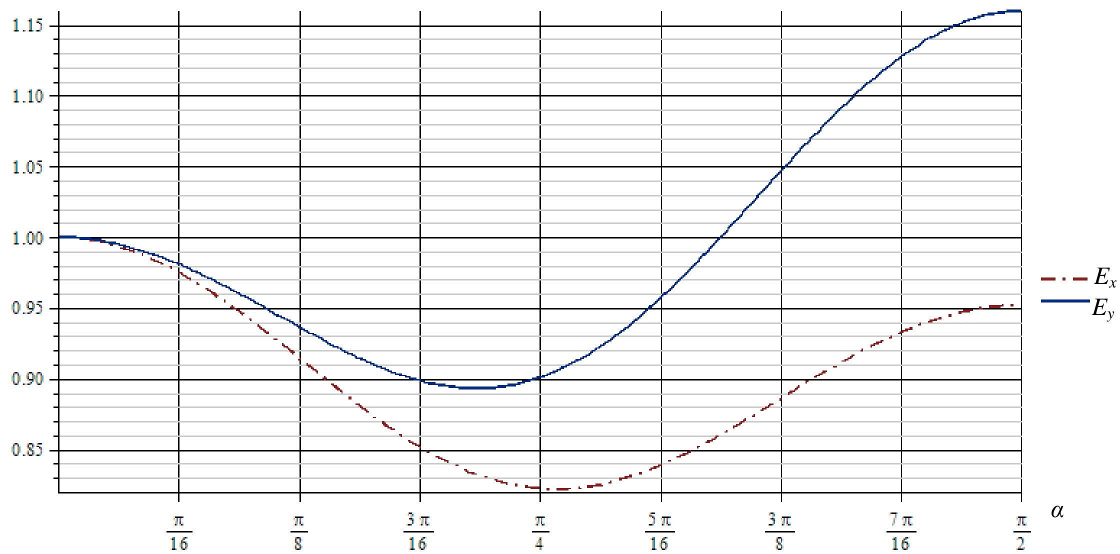

Fig. 4. Effect of Crack Orientation on Inclusion Elastic Constants

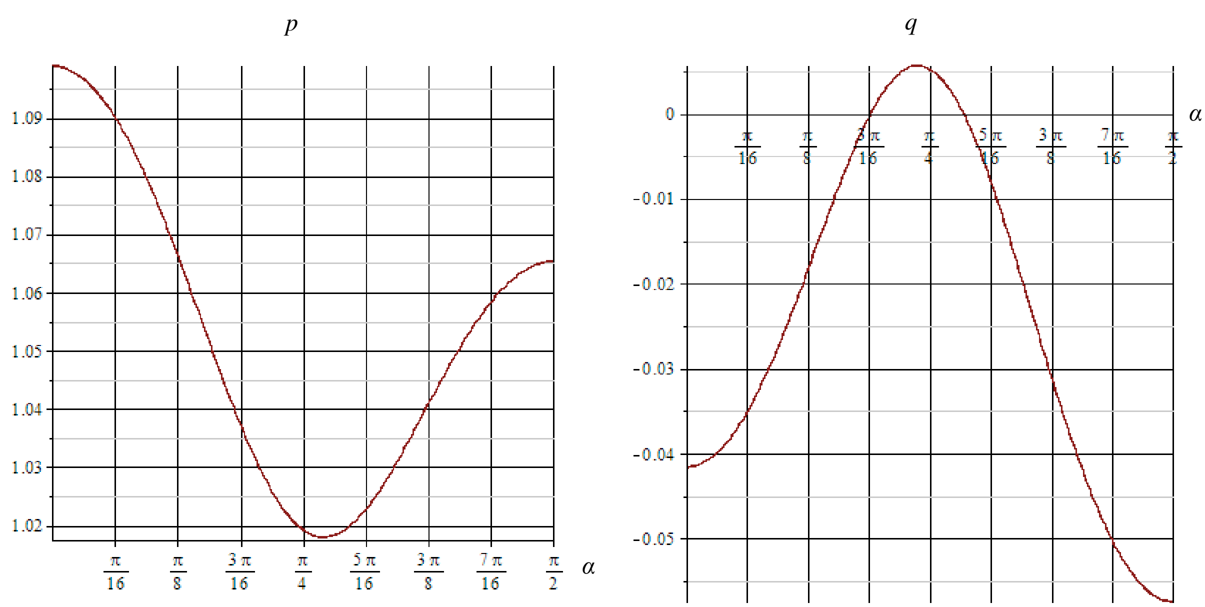

Fig. 5. Effect of Crack Orientation on Inclusion Stresses

3 Results and Conclusions

3.1 Effect of Crack Orientation

Initially supposed study is related to crack orientation that is normal to x direction of coordinate system linked to the repaired skin (refer to Fig. 1). In general case orientation (and also – location) of the crack can be arbitrary. Based on structure orthotropy the mechanical properties of the inclusion will vary depending on orientation angle α of the crack.

It is practically interesting to receive results that show variation of stress distribution within the patched skin depended on orientation of patch bridging the existing damage.

Extra comments should be added for presented results. Layup for both skin and patch is assumed symmetric. Orientation of patch is placed in relation with crack position to provide effective bridging of the damage: 0 direction of the patch is normal to crack face. Effective properties of inclusion are calculated by formulas (1) thru (4) and (5) thru (7).

Figure 4 shows relation between effective elastic constants of inclusion and orientation angle of the crack. Young's moduli are normalized to the case of 0 crack angle value. Skin is supposed as symmetric 26-ply carbon fiber laminate with $[-/+45, 0, 90, -/+45, 0, +/-45, 0, -/+45, 90]_s$ layup sequence used in real aircraft structure, patch consists of 6 carbon plies with $[0, 90, 0]_s$ layup.

Dependency of stresses inside the inclusion on the crack orientation angle is illustrated on Figure 5. Stresses are normalized to applied stress $\sigma_{x\infty}$.

Effective elastic modulus along applied stress direction has strongly marked minimum which is related to minimal stiffness of patch-skin inclusion in accordance with angle orientation of the supposed crack. Thus, crack angle orientation and angle of mutual 0-degree layup direction orientation of the patch and skin values should not be excluded from list of key parameters for the bonded repair optimization process.

3.2 Conclusive Remarks

Presented study contains the first iteration on the way to develop methodology for on-site bonded repair assessment and application and is related to a very restrictive maintenance event. Patch-adhesive optimization procedure with additional intensive analytical and numerical research need to be realized to account the important life cycle effects that are vital for structure efficiency and durability.

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