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OBLIQUE MODE ENHANCEMENT BY LONGITUDINAL WALL OSCILLATION IN 2D CHANNEL FLOW

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Abstract

The effects of longitudinal wall oscillation on the small disturbances developing in the two dimensional channel flow is investigated by the Floquet theory. The base flow consists of the 2D plane Poiseuille flow and the Stokes layer. A time-dependent Orr-Sommer (OS) equation expanded by the collocation points is used for the Floquet analysis. The parameters governing the present model are the frequency and amplitude of the wall oscillation, and Reynolds number. For the 2D Tollmein-Schlichting (TS) mode, the wall oscillation shows stabilizing or destabilizing effect depending on the parameters. The Floquet analysis also elucidated that the oblique mode can be more unstable than the TS mode. These results suggest that the oblique mode can appear earlier than TS mode contrary to the Squire's Theorem.

1 Introduction

Drag reduction is one of the most important issues of the air transport system. The intensity of surface friction drag strongly depends on the flow condition of the boundary layer around airfoil. In order to suppress the friction drag, the ideas are roughly divided into two types; namely the passive control and the active one. As an example of the former, the airfoil-surface optimization technology can be given[1,2]. Although this approach shows some good results for subsonic airfoil, it is very hard for transonic one. For the latter, the boundary layer blowing or suction is given as an example[3,4]. The problem of this approach is that the input total energy generally exceeds the net gain. However Jung. et al[5] showed the drag reduction by spanwise wall oscillation on the 2D channel flow. Then Quadrio and Ricco[6] numerically demonstrate the drag reduction of 44.7% which corresponds to the net gain of 7.3%. These fundamental studies shows the possibility of engineering feasibility.

On the other hand, the above studies could not explain sufficiently the mechanism of the drag reduction. Thus author tried to investigate it using the model flow which is constructed by the 2D channel flow with longitudinal wall oscillation[7]. This system has a great advantage from the analytical viewpoint because the flow can be explained as an exact solution of the governing equation. In this study author showed by the Floquet analysis that the amplification rate of TS disturbance wave can be suppressed. This result suggests that the laminar-turbulence transition might be delay. This study also found that the amplification rate of oblique TS mode larger than the 2D TS mode in some cases. This finding implies massive potential in the flow control technology.

Thus the present study focuses on the behavior of oblique wave developing in 2D channel flow with longitudinal wall oscillation. Since this system has a time periodicity, the Floquet theory might suit to examine the characteristics of the stability. To do this, the collocation method is used to build the eigenequation from the linearized disturbance equation. In the section 2, the model flow based on the plane Poseuille flow is defined. Then the governing equation and the procedure of the Floquet analysis are explained in Section 3. Numerical results for 2D TS and the oblique TS mode are given in the Section 4, and the conclusion is given in the Section 5.

2 Model flow

The model flow examined in the present study is shown in Fig.1. A pair of infinite plates are arranged in parallel at distance of $2h$. The maximum velocity of the base flow is of U_{max} and the nonslip condition above the walls is adopted. The two walls are oscillated in phase with the amplitude of U_w, and the frequency of Ω. The Reynolds number R is defined as $R \equiv U_{max}\, h / \nu$, here ν is the kinematic viscosity. Then the parameters which control this model flow are U_w , Ω and R. The Cartesian coordinate system (x,y,z) is defined as x in the flow direction, y in the perpendicular to the walls, and z in the spanwise direction.

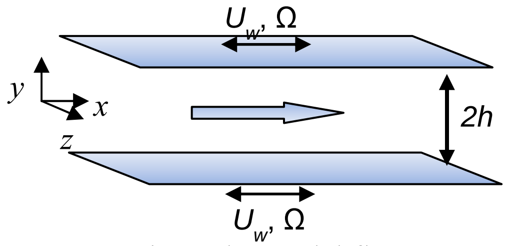


Fig.1 The model flow.

It can be considered that the present system consists of the plane Poiseuille flow and the Stokes layer. Since these two flow are the exact solutions of the linearlized equation which is derived from the Navier-Stokes equation with 2D parallel flow approximation, the model flow can be described as the superposition of these two exact solution given by the following equation, and is shown in Fig.2.

$$U(y,t) = 1 - y^2 + U_w \,\mathrm{Re}[\frac{\cosh(ky)}{\cosh(k)}]\exp(i\Omega t) \qquad (1)$$

Here $k \equiv \sqrt{\Omega / 2\nu}$, i denotes imaginary unit. Using above velocity profile, the amplification rate of the small disturbances is estimated by the linear stability analysis with the Floquet theory.

3 Eigenvalue equation and Floquet analysis

In common linear stability analysis, the famous Orr-Sommerfeld (OS) equation is used and the growth rate is estimated as magnitude of the eigenvalues. This OS equation is obtained by assuming the disturbance as the plane modal wave which the eigenfunction doesn't have the time dependence. In the present study, however, the assumption as the plane modal wave is not suitable because the base flow is variable in time. Thus the form of small disturbance $\boldsymbol{u}'$ is assumed as the follows,

$$\boldsymbol{u}'(x,y,z,t) = \hat{\boldsymbol{u}}(y,t)\exp[i(\alpha x + \gamma z)]\,, \qquad (2)$$

here α and γ are the wavenumber in x and z direction. Substituting above equation for the linearized disturbance equation, so called time-dependent OS equation is derived.

$$[(\frac{\partial}{\partial t} + i\alpha U(y,t))(D^2 - \alpha^2 - \gamma^2) - i\alpha D^2 U(y,t)]\hat{v}(y,t)$$
$$= \frac{1}{R}(D^2 - \alpha^2 - \gamma^2)^2 \hat{v}(y,t), \quad \left(here\, D \equiv \frac{\partial}{\partial t}\right) \qquad (3)$$

Appling the Chebyshev spectral collocation method in y direction, a matrix equation is obtained.

$$\frac{d}{dt}F(t) = G(t)F(t)\,. \qquad (4)$$

The collocation point y_j is

$$y_j = \cos\frac{\pi j}{N+1}, \quad (j = 0,1,2,\cdots,N). \qquad (4)$$

When F(t) is rewrote as,

$$Q = \frac{1}{T}\ln F\,, \qquad (4)$$

the behavior of the system is described by the eigenvalues μ of the matrix Q. When μ is positive, the system is unstable.

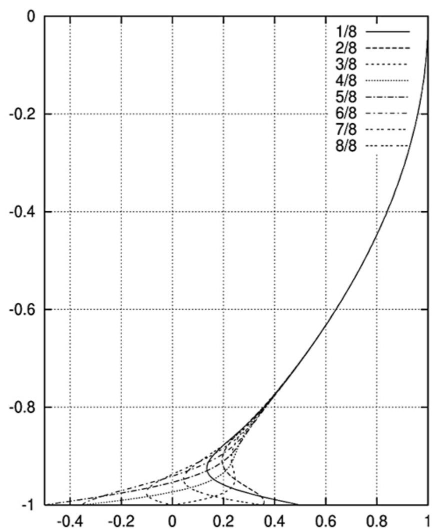


Fig.2 Velocity profiles at several instants.

4 Results

4.1 Tollmein-Schlichting mode

Orszag [8] numerically investigated the stability of the small disturbances developing in the 2D channel flow, and showed the critical Reynolds number of 5,722 for disturbance of (α, γ) = (1.0, 0.0). To compare this results, the mode is fixed as (1.0, 0.0) in this subsection. Then For the remaining parameters of (U_w, Ω), the several cases of the Floquet analysis are examined and are shown in Fig.3. The case of (U_w, Ω) = (0, 0) corresponds to the original channel flow investigated by Orszag. It can be seen that the sign of the floquet exponent changes from negative to positive around R = 6,000 with the increase of R. It is confirmed that this critical value of R is of 5,722. Turning the attention to the other cases, the floquet exponents generally decrease affected by the wall oscillation. For this reason, the critical Reynolds numbers are larger than the case of the original channel flow. This means that the laminar-turbulent transition might be delayed. Since the Floquet exponent for the case of (U_w, Ω) = (0.4, 0.2) is larger than that of the original one, there is some possibility that the oblique TS wave can dominate the flow field.

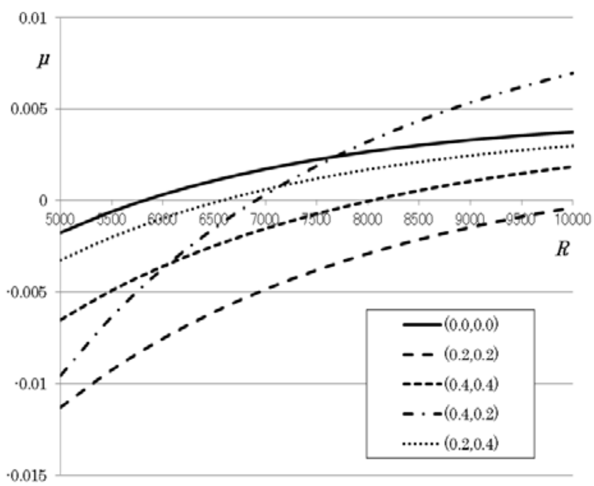


Fig.3 Variation of the Floquet exponents with the Reynolds number for the cases of (U_w, Ω) = (0.0, 0.0), (0.2, 0.2), (0.4, 0.4), (0.4, 0.2), (0.2, 0.4), and (α, γ) = (1.0, 0.0).

Depending on the parameters of U_w and Ω, the critical Reynolds number is changes. Thus, the critical Reynolds numbers for various sets of the parameters are examined and the result is plotted in Fig.4. In this figure, the original channel flow corresponds to the origin. It can be seen that when the wall oscillation is added even just little bit, the system is stabilized. The system is strongly stabilized around (U_w, Ω) = (0.15, 0.15). On the other hand, when U_w exceeds 0.25, the effect of the wall oscillation changes to destabilize feature around Ω=0.2. Near the U_w axis, there is the stable area even if the U_w is increased. Although it is difficult to explain this feature, the assumption of the superposed model flow doesn't appropriate because the thickness of the Stokes layer increases when Ω decreases. In the present model flow, the discussion has to be in a restriction that the wall motion doesn't affect the another side.

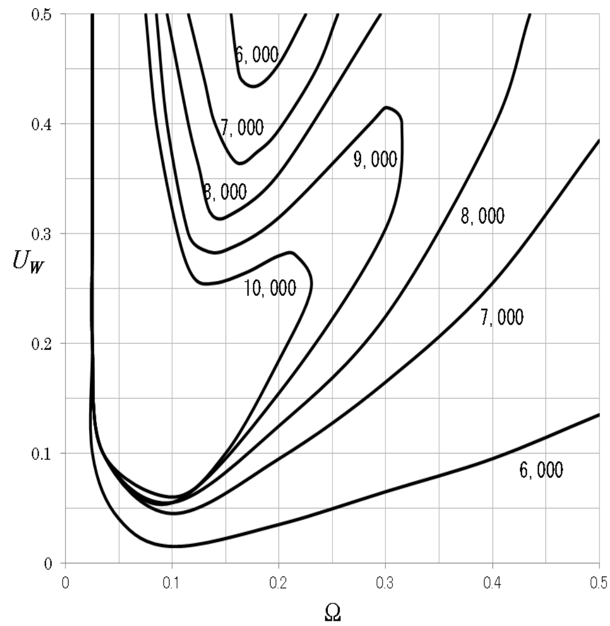


Fig.4 Neutral curves of the Floquet exponents for R=6,000, 7,000, 8,000, 9,000, 10,000 in U_w–Ω plan for (α, γ) = (1.0, 0.0).

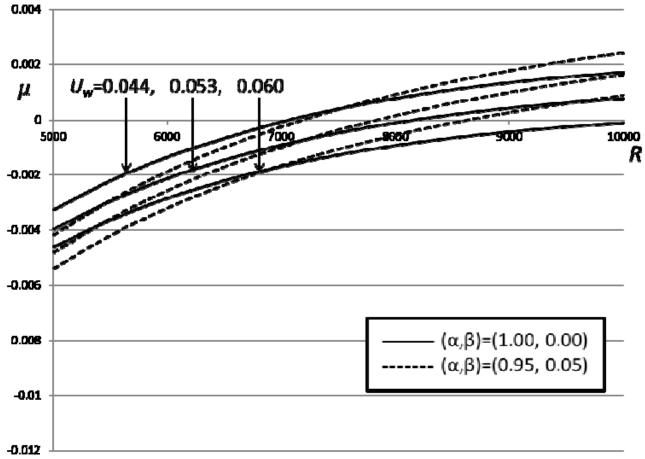
It is mentioned that there is a possibility which the flow might be controlled by the oblique TS modes in some cases. However, the 2D TS modes generally appeared earlier than the oblique TS modes, and rapidly develop due to the nonlinear growth. If the oblique TS mode could be dominant, it is only the vicinity of the neutral curves. Thus, the subsequence section focuses on the stability of the oblique TS mode around the neutral curves.

4.2 Oblique TS mode

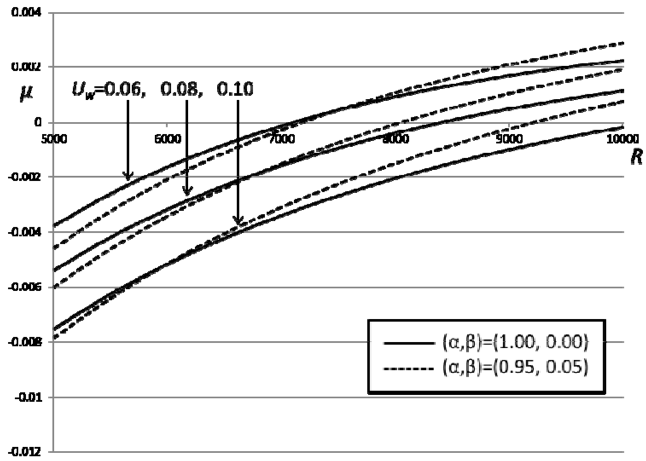
The variation of the Floquet exponents versus Reynolds number for Ω = 0.1, 0.15, 0.2 are shown in Fig.5, respectively. the solid lines in these figures correspond to the 2D TS mode of (α , γ) = (1.0, 0.0), and the dotted lines correspond to the oblique TS mode of (0.95, 0.05). For each Ω, a few cases of U_w are shown and one of them is chosen so that U_w doesn't exceed the neutral curve at R=10,000. Thus the lines corresponding to the most stable case in each figures reach the horizontal axis at R=10,000. In such condition, it can be expect that the oblique TS mode appears earlier than the 2D TS mode. In other cases, of course, the appearance of the oblique TS mode can be expected when dotted lines exceed the horizontal axis.

As compared with these figures, some features are revealed.

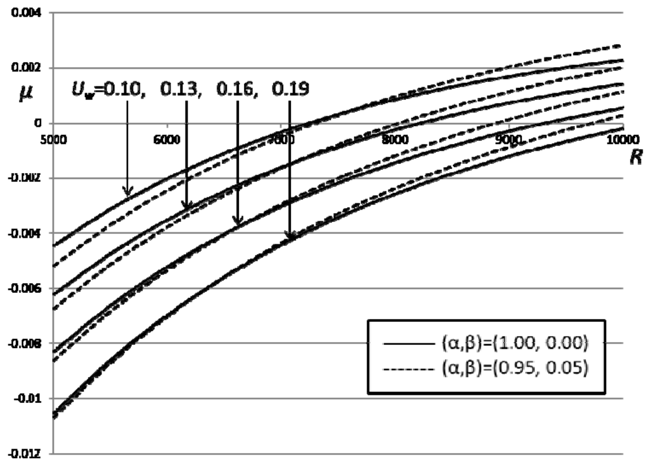
- 2D TS modes are more unstable than the oblique TS mode at small R region. On the other hand, the oblique TS modes become unstable than 2D TS modes at large R region. These are able to say for all the cases of Ω.
- At small R region, the Floquet exponents decrease when Ω is increased. However, it seems that at large R region the growth of the Floquet exponents doesn't change much.
- At large R region, the gap of the Floquet exponents between 2D TS and the oblique TS modes is narrowed when Ω is increased.
- Finally the overtaking 2D TS mode at large R region disappears when Ω exceeds about 0.3.



(a) Ω = 0.10



(b) Ω = 0.15



(c) Ω = 0.20

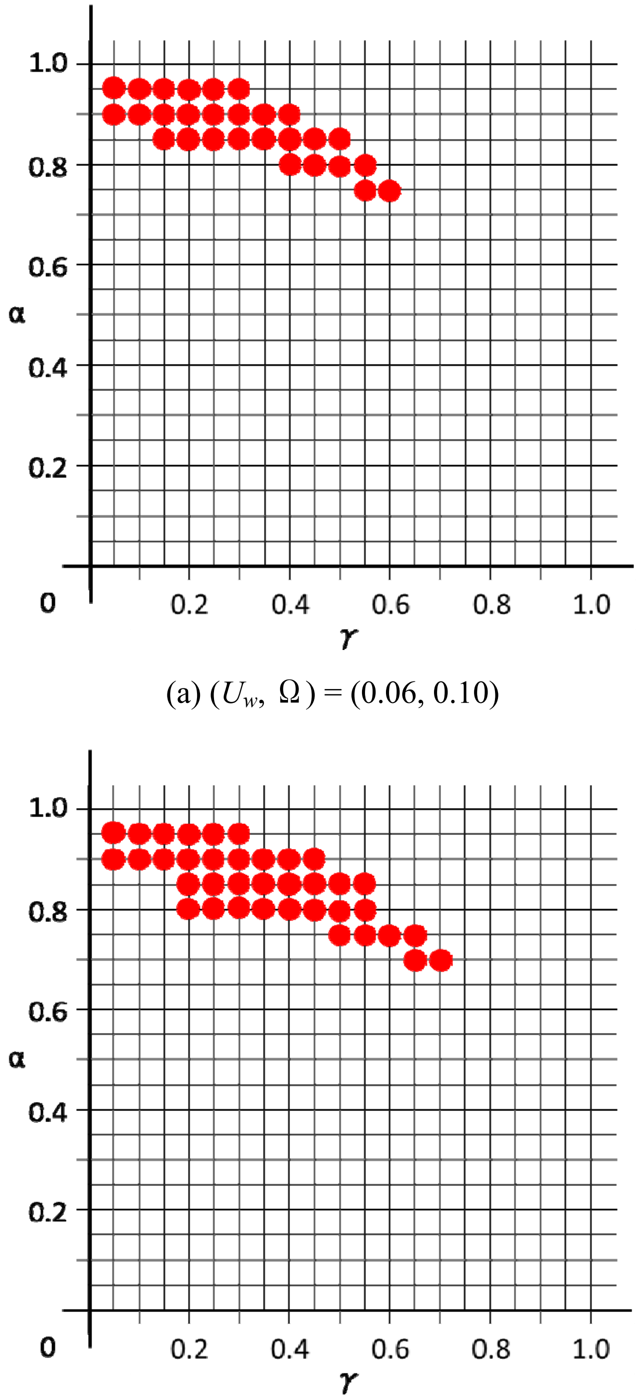
Fig.5 Variation of the Floquet exponents for the case of (a) Ω = 0.10, (b) 0.15, and (c) 0.20. The solid lines correspond to 2D TS mode and the dotted lines correspond to the oblique TS mode.

For the cases of (U_w, Ω) = (0.06, 0.1), (0.1, 0.15), (0.19, 0.2), the unstable oblique TS modes are investigated and shown in Fig. 6. These three cases are in the situation that the 2D TS mode is just before the neutral points. This means that the oblique TS mode can appear if the Floquet exponent is positive. Although it is already shown in Fig.5 that the mode of (α , γ) = (0.95, 0.005) can appear in above condition, the spread of the unstable oblique TS mode is not clear. Thus in these figure, the oblique TS modes with positive Floquet exponent are marked by solid circles.

It seems that the existence of the the unstable TS mode is limited for all the cases of (U_w, Ω). There is no possibility of the appearance of the oblique TS wave of the modes with small wavenumbers. Also, the oblique TS mode along by γ axis doesn't have positive Floquet exponents. It means that the transvers wave of the small disturbance cannot appear. From these figures, the following features are perceived.

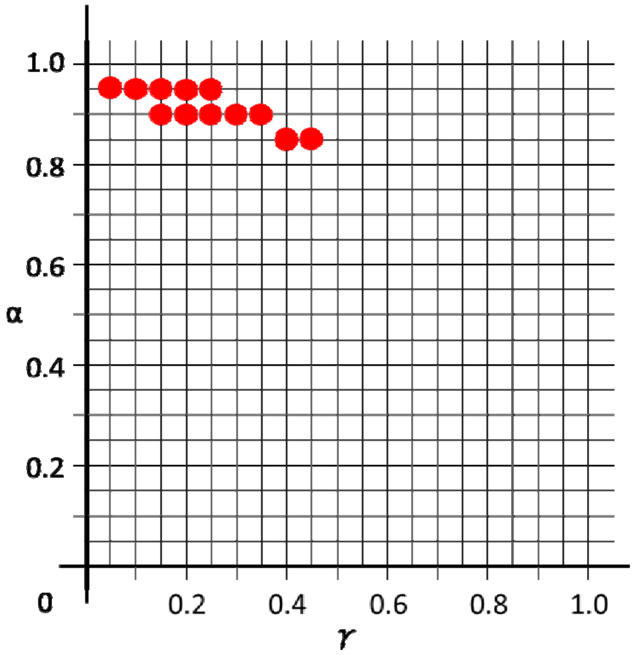
- The unstable oblique TS modes have relatively large α and small γ , namely $0.7 < \alpha < 1.0$, and $0 < \gamma < 0.7$.
- The direction of the unstable oblique TS modes is within 45 degrees from the mean flow direction.
- When the angle of the wave vector of the oblique TS modes becomes large, the gap of the Floquet exponent between these oblique TS mode and the 2D TS mode decreases.
- The region of the unstable oblique TS mode decreases with the increase of Ω. Finally, Then this region disappears.

From the comparison of the neutral curves shown in Fig. 4, it can be understood that the region of the unstable oblique TS mode disappears for large Ω for the case of R = 10,000. However, it should be emphasized that this feature lends well to other R, nevertheless the neutral curves exit.



(a) (U_w, Ω) = (0.06, 0.10)

(b) (U_w, Ω) = (0.10, 0.15)



(c) (U_w, Ω) = (0.19, 0.20)

Fig.6 Map of the unstable oblique TS modes. The oblique TS modes with positive Floquet exponent are plotted as the slid circles in α - γ plane for the case of (a) (U_w, Ω) = (0.06, 0.10), (b) (U_w, Ω) = (0.10, 0.15), and (c) (U_w, Ω) = (0.19, 0.20).

5 Conclusion

Destabilizing effects of the longitudinal wall oscillation on the oblique Tollmein-Schlichting mode developing in the two dimensional channel flow is investigated by the Floquet theorem. A time-dependent Orr-Sommerfeld equation is employed as the eigenvalue equation for the Floqeut analysis. It is cleared from the present study that the wall oscillation stabilizes the 2D TS modes for some cases of the frequency and the amplitude of the wall oscillation. Furthermore, the oblique TS mode can appear earlier than the 2D TS mode in some situations contrary to the Squire's theorem.

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