NOSES OF MINIMAL WAVE DRAG IN SUPERSONIC FLOW

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Abstract

Optimization of aerodynamic forms of axisymmetrical noses was carried out. Optimization was realized in the class of bodies with a front face and power-law geneatrix. The radius of the front face and the power-law exponent are used as parameters of optimization. Optimization was performed with the help of the method of cyclical coordinate-wise descent. The nose parts of the minimal wave drag have been obtained for lengthening λ = 2 -8 at the range Mach number M = 1.5-4. The obtained nose parts have been compared with the well-known optimal bodies on values of wave deag and geometrical parameters.

Introduction

The first results of the search for axisymmetrical noses that have minimal wave drag were obtained within the limit of simplified flow models. The Karman ogive was established on the basis of the linear slender body theory. The blunted nose and equality to zero of the derivative of the geneatrix near the bottom are among the peculiar features of this body. The Newton model was used in the process of solving of this task. This model is based on the following assumption: the pressure coefficient depends on the angle between the vector of the free stream and the normal to the element of the surface. It is another simplified model. Within the framework of the Newton model, important peculiar features of optimal noses were established. It was shown that with fixed dimensions, the noses of minimal wave drag have a front face.

At the present time optimization methods allow establishing optimal nose parts within the framework of Euler equations [1, 2]. In particular, an analytical solution has been obtained that provides for bodies that considerably exceed the Newton noses. It was shown that the values of wave drag close to the minimal are obtained in the class of bodies that have a front butt-end and a power-mode geneatrix with an exponent of power equal to two-thirds.

The search of aerodynamic shapes close to optimal, designed in the analytical form, is continued in the present study, which contains results of numeric optimization of blunted powermode bodies on the basis of two parameters: the diameter of the butt-end and the exponent of power.

Statement of the problem

The target function is the coefficient of drag $c_x(r', n)$ = min. In case of nonviscous task the target function is the coefficient of wave drag, in case of viscous task – the coefficient of total drag. The exponent of power n and the relative dimension of the butt-end $r' = r_т/R$ served as parameters of optimization. The geneatrix of the body was represented with the power dependence, as follows:

$$r = R\left[\frac{x}{L} + \left(\frac{r_т}{R}\right)^{1/n}\left(1 - \frac{x}{R}\right)\right]^n$$

where L is the length of the body, R is the radius of the bottom, $r_т$ is the radius of the front butt-end, and n is the exponent of power.

Lengthening of the body was fixed and calculated in accordance with the following formula λ=L/2R.

The base area was the typical dimension on calculation of the drag. Noses obtained as a result of the analytical solution were taken as the initial form [2].

Optimization was performed using the method of cyclical coordinate-wise incline. The essence of this method is in the following. It is sequential minimization of the objective function *f(x)*, first of all, along the direction of the first basis vector, then the second basis vector, etc. On termination of minimization, there is a rerun along the direction of the last basis vector. Minimization on each of the parameters was performed on the basis of the method of parabolas [3].

In case of nonviscous task the flow near the noses was calculated within the Euler model. Two calculation areas were determined. The common boundary of the areas belonged to the cross-section, which was drawn off at a distance equal to the diameter of the front butt-end. In the proximity of the butt-end, calculation of the flow was performed with the help of the method of establishing with time [1]. Dimension of the butt-end was taken as a unit of length. This helped simplify calculation in the first area. The supersonic area of flow calculation was affected with the help of the March method in the axial coordinate [4]. In the first section, flow parameters were determined on the basis of the solution for the first calculation domain. The noses for lengthening λ = 2 -8 at the range of Mach number M = 1.5-4 have been obtained.

In case of viscous task the flow near the noses was calculated within the Navier-Stoks model. Computational complexes Fluent and Antares were used. Fore providing numerical research two-dimensional grid was created. It was used axisymmetrisal flow condition. The size of numerical grid was about 150000 cells. Minimal size of cell was corresponded to Y^{+}=30. The length of noses (L=1m.) was fixed. Research was provided for Mach number M=3 at the range of lengthening λ = 2 -8. The Reynolds number was equal Re_L = 25000000.

Optimization method testing

Testing of the represented method of optimization was held. Nose was considered with the lengthening λ = 2 streamlined on M = 2. Two cases have been discussed. As for the first case, the truncated cone was taken as a basic body. The radius of the front face *r'* was 1% from the maximal radius *R*, and the power law generatrix *n*, respectively, was equal to 1. As for the second case, the body obtained upon analytical solution of the problem was taken as the basis. Comparison of values *r'*, *n*, and c_x, obtained as a result of optimization for the first and second cases, showed that they coincide.

Features of the target function can be investigated based on analysis of the level lines. Level lines and descent trajectory of these parameters of the problem are presented in Fig. 1.

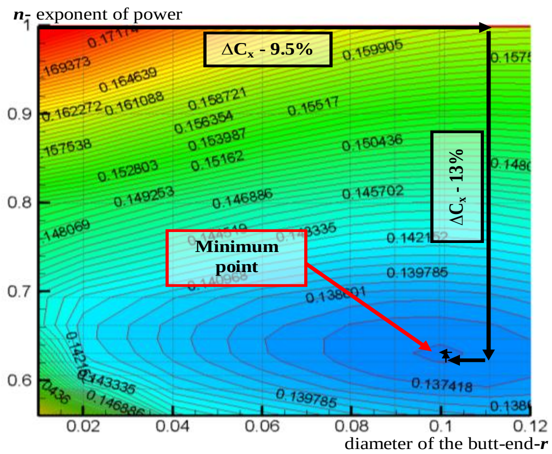

Fig. 1: Level lines and direction of descent for each parameter (M = 2; λ = 2)

It is seen that at the first iteration after the descent of the radius of the butt-end, the drag of the body decreased by 9.5%, and after the descent of the degree even at 14%, and has a value close to optimal. After the second and third iterations, the drag decreased by ~ 0.5%.

Analysis of level curves showed that the minimum is the unique and level lines in the vicinity of the minimum point have the elliptical shape. Thus, the objective function is well approximated by a quadratic dependence. These features of the target function are the cause of good convergence of the optimization method.

Results of optimization

Optimization in case of Euler model was held for lengthening λ = 1-8 at the range Mach number M = 1.5-4. The nose parts obtained in this study will hereafter be called the blunted power-law nose parts, or of two-parametric bodies.

The comparison of the nose parts with the well-known bodies on the values of the wave drag and geometrical parameters has been completed [6]. For comparison the body taken with an exponent, which is found in the direct optimization using the Euler model [5]. In the future, we will call them power bodies. The second example is the nose parts, the shape of which is found in the exact formulation of the problem [1]. These bodies will be called optimal.

Comparison of the nose parts showed that the power nose parts are the worst. Using the values of the wave drag, they lag behind the found nose parts by > 5.5%. Comparison of two-parametric body and nose parts derived from the exact formulation showed the two parameters of the body inferior to the best by no more than 2.5% (Fig. 2). At the same time with increasing Mach number, the difference in drag is significantly reduced.

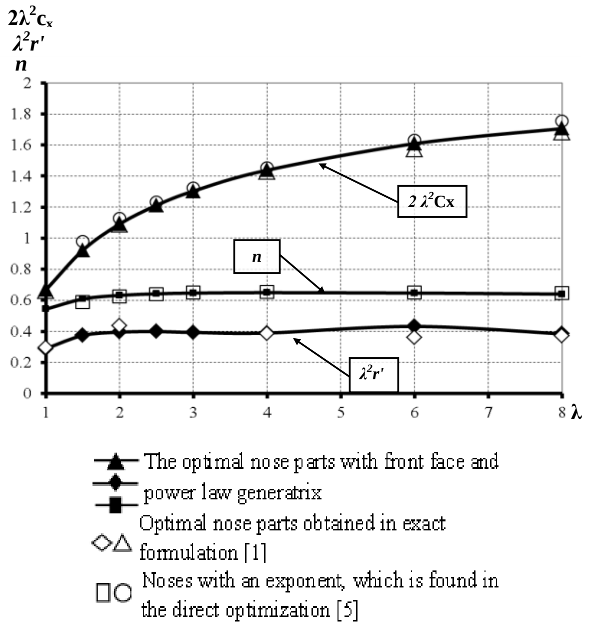

Fig. 2: Dependence of the radius of the butt-end, the exponent of power and coefficient of wave drag on the lengthening (M = 2).

Comparison of blunted power-law bodies and bodies which obtained in exact formulation, by the geometrical parameters showed, that the maximum difference between diameters of the front butt-ends amounts to 20% at λ = 6. Quite a big difference in the diameter of the butt-end due to the fact that, at large lengthenings, its size is many times smaller than the maximum diameter of the body and therefore has no significant effect on the drag of the body. Analysis of the level lines showed that, near the optimum, even with small lengthening, the size of the end face had a slight effect on drag. Thus, for M = 2 for body lengthening λ = 2 deviation from the optimum diameter of the butt-end of 10% leads to a change in drag of < 0.01%.

Comparison of the two-parametric and power nose part on the values of the degree exponent showed that the maximum difference is observed at low lengthenings. In this case for the two-parametric and power nose parts the exponent increases with M and tends to the theoretical value of 2/3. Note that the exponent near the minimum has a much greater effect on drag than the butt-end. To found the nose part of lengthening λ = 2 for M = 2, a deflection of the exponent of 5% from the optimum leads to an increase in drag at 0.5%.

The total results of optimization are shown in Fif 3,4 and in table 1.

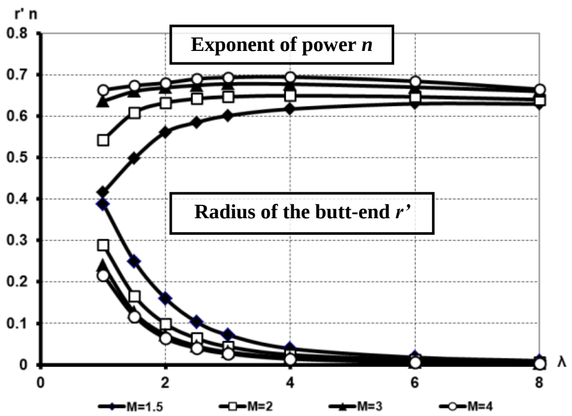

Fig. 3: Dependence of the radius of the butt-end and the exponent of power on lengthening.

The values of the relative dimension of the butt-end, of exponent of power *n* and coefficient of wave drag are shown.

Table 1

M	1.5		
λ	The relative dimension of the butt-end r’	The exponent of power *n*	Cx
1			
1.5	0.2495	0.4981	0.2125
2	0.1600	0.5612	0.1462

2.5	0.1033	0.5850	0.1052
3	0.0720	0.6010	0.0790
4	0.0391	0.6171	0.0489
6	0.0178	0.6300	0.0239
8	0.0097	0.6294	0.0139
M	2		
λ	The relative dimension of the butt-end r'	The exponent of power *n*	Cx
1	0.2895	0.5434	0.3342
1.5	0.1662	0.6089	0.2052
2	0.0988	0.6323	0.1365
2.5	0.0638	0.6420	0.0970
3	0.0437	0.6467	0.0724
4	0.0244	0.6494	0.0450
6	0.0120	0.6468	0.0224
8	0.0060	0.6400	0.0133
M	3		
λ	The relative dimension of the butt-end r'	The exponent of power *n*	Cx
1	0.2400	0.6366	0.3163
1.5	0.1263	0.6599	0.1854
2	0.0723	0.6688	0.1210
2.5	0.0455	0.6744	0.0853
3	0.0309	0.6778	0.0636
4	0.0168	0.6774	0.0396
6	0.0063	0.6700	0.0199
8	0.0035	0.6600	0.0121
M	4		
λ	The relative dimension of the butt-end r'	The exponent of power *n*	Cx
1	0.2163	0.6632	0.3027
1.5	0.1172	0.6738	0.1733
2	0.0639	0.6803	0.1117
2.5	0.0402	0.6899	0.0782
3	0.0270	0.6935	0.0582
4	0.0142	0.6945	0.0362
6	0.0058	0.6843	0.0184
8	0.0027	0.6650	0.0113

At final stage, optimization research was held in case of Navier-Stoks model. Computations were provided for lengthening λ = 1-8 at the Mach number M=3. For defining effect of viscous to the drag, numerical research was held for noses created in case of Euler model.

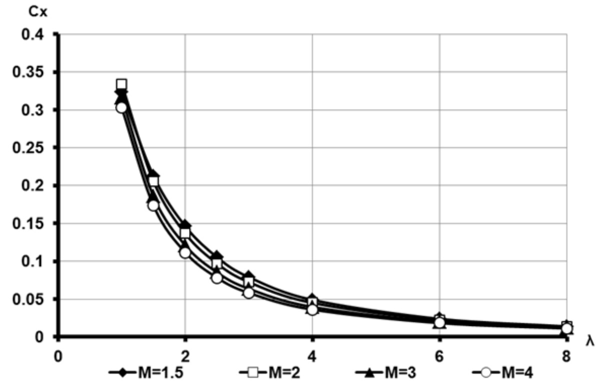

Fig. 4: Dependence jf the coefficient of wave drag on the lengthening.

Dependence on lengthening for coefficients of total, wave and friction drag are presented in Fig 5. With low values of lengthening (λ<1.5) friction have slight effect on a total drag. But with higher lengthening friction constitutes a serious part of total drag.

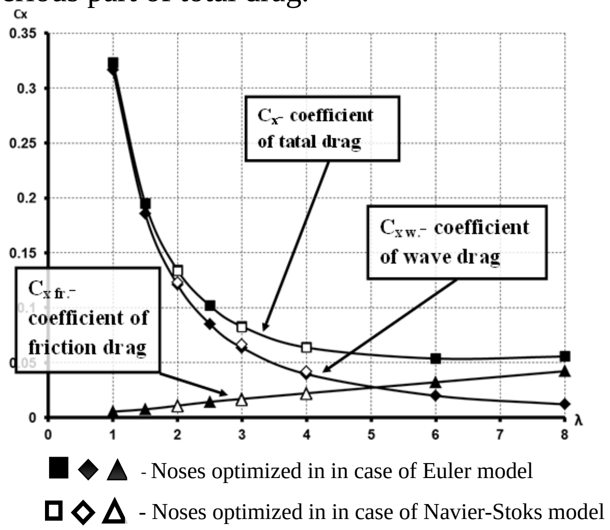

Fig. 5: Dependence of the coefficients of of total, wave and friction drag (M = 3).

In reference with the above for bodies with normal and high lengthening it's important to provide optimization in case of Navier-Stoks model. Noses with the lengthening λ = 2, 3, 4 were considered. Noses obtained in case of Euler model were taken as the initial forms. As result of optimization reducing of total drag by 0.5%-0.8% was achieved. Note that for noses which obtained in case of Navier-Stoks model, low increasing of wave drag (from 0.6% to 1%) and serious reducing of friction drag (from 3.5% to 1%) were detected. The comparison of the nose parts obtained in case of different models by the geometrical parameters is presented in Fig. 6.

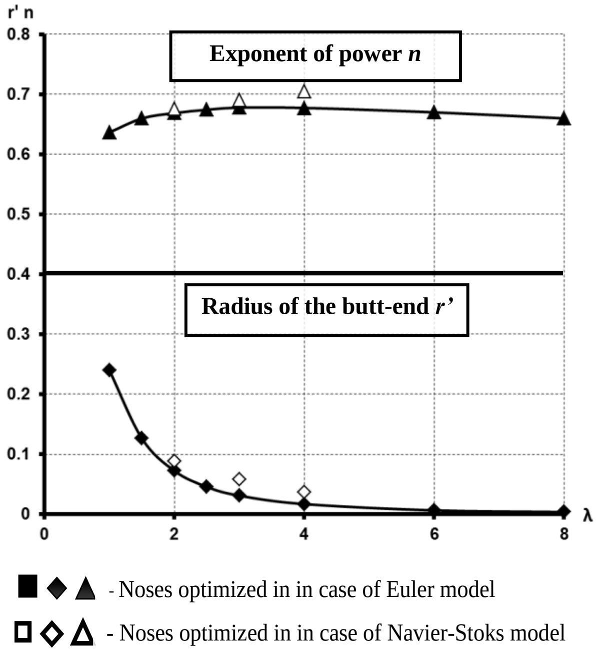

Fig. 6: Dependence of the radius of the butt-end and the exponent of power on lengthening.

For noses which obtained in case of Navier-Stoks model serious increasing of the diameter of the butt-end and some increasing of the exponent of power are detected. Such variation of the geometric parameters tends to creating bodies with smaller air-swept surface which connected with friction drag. With higher lengthening is detected changing of geneatrix to concave form. In reference with the above statement of the problem have to be changed, for example with adding other space limitations.

Conclusion

Optimization of aerodynamic forms of axisymmetrical noses was carried out. Target function is the coefficient of wave drag. The exponent and relative size of the butt-end were the parameters of optimization.

Optimization was carried out by the method of cyclic coordinate-wise descent. At each stage of the minimization, one parameter was carried out by the method of parabolas. At the stage of verification, the results were obtained in the vicinity of extreme points of the line built-level objective function. It is shown that the minimum is unique level line close to an elliptical shape, and the objective function with high accuracy can be approximated by a quadratic form.

We found the nose part of the minimum wave drag in a wide range of defining parameters. It is shown that, for an optimal bodies with increasing Mach number and lengthening, the diameter of the butt-end decreases monotonically, while the values of the exponent (for medium and large lengthenings $\lambda > 2\text{-}3$) are close to the theoretical value of 2/3. The results are compared with well-known optimal bodies, obtained by the exact formulation of the problem, and power bodies, optimized on the value of the exponent.

Optimization research of two-parametric bodies was held in case of Navier-Stoks model. For lengthening $\lambda = 2, 3, 4$ noses of minimum drag were found. It was established that for bodies with normal and high lengthening is important to provide optimization in case of Navier-Stoks model.

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