UPDATING OF AIRCRAFT STRUCTURE DYNAMIC MODEL TO GROUND VIBRATION TEST RESULTS

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Abstract

This paper deals with an aircraft structure analytical dynamic model updating to ground vibration tests (GVT) results. Paper describes possible approaches of flutter analysis as regards to the incorporation of GVT results - the direct usage of GVT results and analytical model updating. Then the theoretical background of the model updating methods including Bayesian parameter estimation and more generic optimization using powerful nonlinear gradient-based methods are given. Finally, the methodology of an aircraft structure dynamic model updating is described. The practical application is documented on the EV-55M turboprop utility aircraft model updating. Selected results are shown, evaluated and the conclusions are formulated.

1 Introduction

Analytical models of aircraft structures, that are used for aeroelastic analyses must be validated by means of experimental results. Aeroelastic calculations, in particular flutter analyses, have ultimate character and reliability of flutter results are directly dependent on reliability of input data. However, the data, in particular stiffness characteristics, based on the theoretical virtual model are not enough reliable. Furthermore, airworthiness regulations include this demand. One of the most important experiments during the aircraft development is the ground vibration test (*GVT*). It measures the modal characteristics of the aircraft, which are the main input data for the flutter analyses.

With regard to *GVT*, there are two approaches, how to incorporate results into the flutter analysis: 1) direct usage of *GVT* results and 2) updating of a structural model to *GVT* results.

The former approach is used mainly for the general aviation aircraft certified according UL, *LSA*, *VLA* or *FAR / CS 22* specifications (e.g., [1]). The advantage is a direct relation to the real aircraft structure; however, flutter analyses are more or less limited to the tested structure. Although there are the possibilities of parameter variation (e.g., [2]), the large parametric studies or major parameter changes or modifications of the structure are not applicable.

The latter approach is used mainly for the utility or commuter aircraft certified according *FAR / CS 23* or *FAR / CS 25* specifications, where the aircraft development is longer and the certification procedure is more complex (e.g., [3]). The analytical model, that is usually based on *FE* is updated in order to match the *GVT* modal characteristics as closely as possible. Obviously, there must be some difference between updated model and the test results. On the other side, the updated model allows to make parametrical studies and to include further modifications of the structural parameters.

The methods of the model updating and application to the aircraft structure is the subject of the presented paper.

2 Theoretical Background

Updating of an analytical model structural parameters to *GVT* results is basically problem of the structural optimization. The basic optimization problem can be expressed as seeking the optimal combination of parameters aimed to minimize the specified target function, respecting the constraining functions. The procedure, how to match the modal characteristics measured by *GVT* may be based on the following approaches ([4], [5]):

1) Model updating by Bayesian least squares method

The objective function is the distance between actual modal responses and target (*GVT*) modal responses. The solution is iterative, based on the matrix equation of the form:

$$\{P_u\} = \{P_0\} + [G]\{-\Delta R\} \qquad (1)$$

where $\{P_\mathrm{u}\}$ is vector of design parameters after updating; $\{P_0\}$ is vector of design parameters before updating; $\{\Delta R\}$ is response change vector and $[G]$ is "gain matrix". Considering the relation of number of design parameters and responses, $[G]$ is computed according Bayesian estimation theory as:

$$[G] = \left([W_P] + [S][W_R][S]^T\right)^{-1} [W_R][S]^T \qquad (2)$$

or

$$[G] = [W_P]^{-1} [S]^T \left([W_R]^{-1} + [S][W_P]^{-1}[S]^T\right)^{-1} \qquad (3)$$

where $[W_\mathrm{P}]$ and $[W_\mathrm{R}]$ are diagonal weight matrices for design parameters and design responses respectively; $[S]$ is a sensitivity matrix representing rates of response changes with respect to change in parameters:

$$[S] = \left[\frac{\partial R_i}{\partial P_j}\right] \qquad (4)$$

Design responses are represented by the measured modal parameters (natural frequencies and mode shapes), design parameters are represented by the structural parameters of the model, in particular by stiffness characteristics.

There is also possibility to link parameters each other either by a linear or nonlinear relation.

Despite the limitations in the theory of this approach, it is usually more direct and cost effective for the presented type of the model updating.

2) Model optimization by gradient-based methods

Model optimization is more general approach. Optimization algorithms belong to the family of methods called “gradient-based”, because the optimization algorithms determine the gradients of the objective function and constraints to determine a direction of searching for the optimum in the design space. The direction of searching is based on the sensitivity analysis. The optimization algorithms then proceed in that direction as far as they can go. They then investigate, whether it is at the optimum point and if not, the process is repeated until no more improvement of the objective function is possible without violating some constraint.

Design variables (parameters) and responses may include any kind of quantity like mass, strain, stress, modal, flutter etc. It is also possible to define combined composite parameters or responses formed using other ones. Parameters and responses can be constrained as:

$$\begin{aligned} P_i^I &\le P_i \le P_i^U \\ R_i^I &\le R_i \le R_i^U \end{aligned} \qquad (5)$$

Both linear and nonlinear linkage between parameters is also often used:

$$\begin{aligned} P_j &= C_0 + \sum_i C_i P_i \\ P_j &= f\left(\{P\}, \{C\}\right) \end{aligned} \qquad (6)$$

The objective function or constraints may include combination of any kind of responses.

The example of the typical objective function used for the model updating purpose is shown in eqn.(7).

$$OBJ = \min\Big(\{\varepsilon_f\}^T [W_f] \{\varepsilon_f\} + \{W_{MAC}\}^T \{1 - [MAC]\} + \{\varepsilon_P\}^T [W_P] \{\varepsilon_P\} \Big) \tag{7}$$

This objective function includes the weighted squared minimization of the relative error in modal frequencies (*f*):

$$\{\varepsilon_f\} = \left\{ \frac{f_a - f_e}{f_e} \right\} \tag{8}$$

where subscript (*a*) denotes for analytical (*FE*) data while subscript (*e*) denotes for experimental (*GVT*) data. The weighting factor is represented by the diagonal matrix $[W_{\mathrm{f}}]$ reflecting the confidence in the test data.

In the similar way, eqn.(7) includes the relative error in mode shapes expressed by means of the correlation criterion *MAC* (Modal Assurance Criterion):

$$MAC(\Psi_a, \Psi_e) = \frac{|\,(\{\Psi_a\}^T \{\Psi_e\})\,|^2}{((\{\Psi_a\}^T \{\Psi_a\})\,(\{\Psi_e\}^T \{\Psi_e\}))} \tag{9}$$

The weighting factor is here represented by the diagonal matrix $[W_{\mathrm{MAC}}]$.

The objective function of eqn.(7) includes also the demand to minimize the changes of design parameters, especially those ones, which are considered as reliable. Therefore the eqn.(7) includes weighted squared minimization of the relative error in design parameters. Otherwise, the optimization process would make the change of design parameters with no matter regarding the magnitude or sign. It might lead to a model, which may not be representative to the physics of an aircraft. Relative error in design parameters is expressed as:

$$\{\varepsilon_P\} = \left\{ \frac{P_u - P_0}{P_0} \right\} \tag{10}$$

The weighting factor for design parameters is represented similarly by the diagonal matrix $[W_{\mathrm{p}}]$.

This approach is more general and thus, it is more complicated and costly for standard problems. It may be useful for the specific problems, which include complicated composite objective or constraints.

3 Methodology of Aircraft Structure Model Updating

Model updating of an aircraft structure is very complex problem. In the most cases, updating is performed in a several steps. The key issue is the appropriate selection of design parameters and responses. It requires good knowledge regarding the updated structure, possible error sources and the appropriate strategy of updating.

Described methodology is formed for a twin turboprop utility aircraft. It employs the former approach using the Bayesian method. We expect the dynamic beam FE model. Stiffness characteristics of such a model are included via mass-less beam elements, inertia characteristics are included using concentrated mass elements. There are also usually used spring elements for the specific connections (attachment of engine, actuation of control surfaces, connection of structural parts). Model usually includes half-span and either symmetric or antisymmetric boundary condition. A structural FE model is shown in the fig.1.

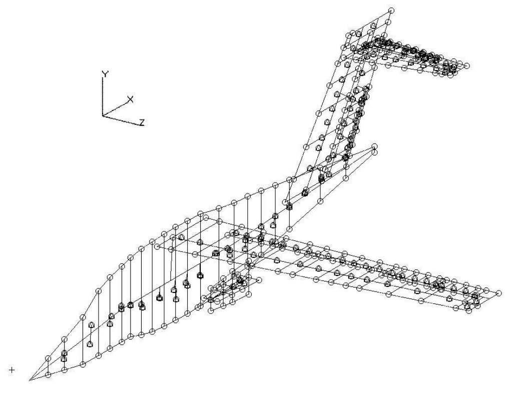

Fig.1. FE model of a turboprop commuter

The grid of experimental measured points is adjusted and reduced to those ones, that are used for updating. The reduced grid (fig.2) includes points on the main structural parts and engine, while the full grid (fig.3) includes in addition points on control surfaces.

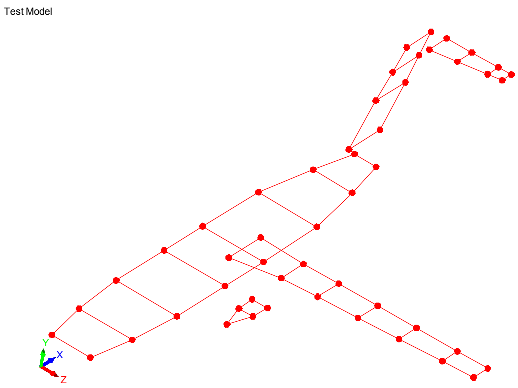

Fig.2. Grid of experimental points (reduced) used for updating

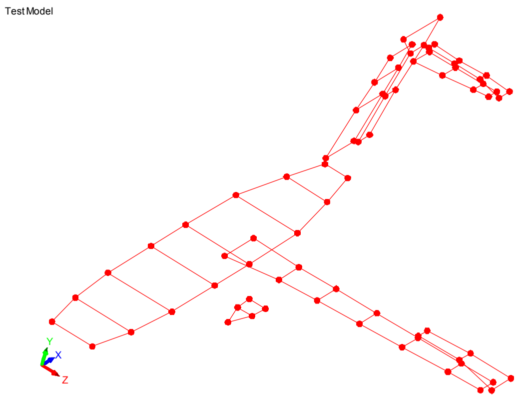

Fig.3. Grid of experimental points (full) used for updating

Pairing of the measured points with the nodes of *FE* model is performed according the topology. For this purpose, coincident nodes to measured points are added to *FE* model.

3.1 Preparatory Activities

The main updating uses usually stiffness characteristics as design parameters. The main reason is, that stiffness data, that are based on design drawings or a virtual prototype are usually less accurate and reliable compare to inertia data.

Thus, the inertia and other characteristics, that will not be used as design parameters must be sufficiently accurate and reliable. These parameters are therefore verified according available experimental data (typically mass measurements) before the main updating.

First, adjustment of the engine attachment is performed. The engine attachment is usually modeled via spring elements. Nodes connected by the spring element must be stationed at the node point of the appropriate engine vibration mode. Note, that stations of node points of symmetric and antisymmetric modes are usually different. Next step is updating of inertia characteristics. Mass distribution obtained from the virtual prototype is ordinarily considered as enough reliable. Inertia characteristics (mass, mass moment of inertia) of control surfaces and tabs are updated according the weighting. Finally, the total inertia characteristics (mass, center of gravity, mass moments of inertia) are updated according the prototype weighting, that is performed just before the *GVT*. Stiffness parameters of tab actuation systems, which are modeled by means of the rotational spring elements, are updated according the static stiffness measurements. The reason is, that modal measurement of tab flapping modes is not reliable due to the high natural frequencies and complicated identification of tab flapping modes.

3.2 Main Phase

After described preparatory steps the model is prepared for the main updating phase. Updating includes the modes of the main structural parts (wing, fuselage, horizontal and vertical tail), engines and control surfaces. Symmetric and antisymmetric model are updated separately; therefore, we obtain different values of design parameters for both models. Applicable design parameters are: 1) stiffness characteristics (vertical bending, in-plane bending, torsion) of beam elements modeling main structural parts and controls and 2) stiffness parameters of spring elements modeling engine attachments, control actuations and structural part connections. Beam element characteristics can be specified as global or local design parameters. Changes of global parameters are the same within the specified group (e.g., wing, fuselage etc.), whereas changes of local parameters may vary for each element.

Applicable design responses for updating are 1) modal frequencies and 2) mode shapes. Mode shape responses are included by means of the *MAC* criterion. Selection of the modal deformations directly as design responses is not recommended.

Pairing of analytical and experimental modes can be performed automatically, based on the *MAC* values. However, considering the character of aircraft structures, it is recommended to use manual pairing according the visual comparison of the mode shapes. For this purpose, special graphic format showing the node lines and modal deformations of particular structural parts (see fig.4) is useful.

With regard to the fact, that the aircraft structures represent very complicated dynamic system, the updating process is divided into several steps. First, the modes of main structural parts and engines are updated by means of the beam global stiffness parameters. Second, the modes of the main structural parts and engines are updated using local parameters. Finally, the modes of the main structural parts, engines and controls are updated by means of the local parameters. Note that the points on controls are excluded from the *MAC* calculation during the first and second step.

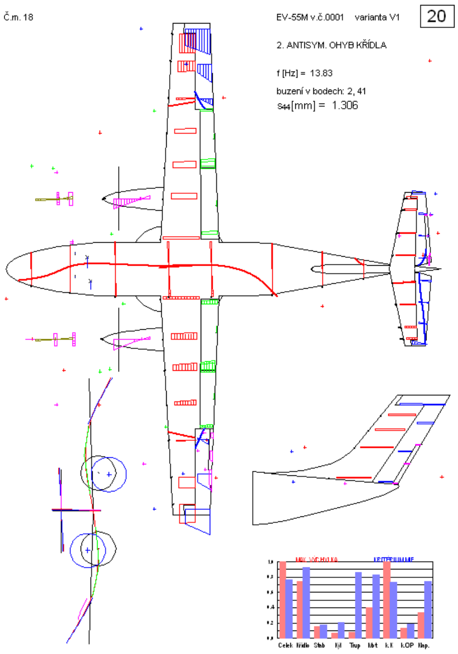

Fig.4. Mode shape visualization (example: 2nd wing antisymmetric bending)

Complete *GVT* measurement of a turboprop commuter aircraft is ordinarily performed for a single (baseline) mass configuration (e.g., with no fuel loading). Selected modes are then measured also for the optional mass configurations (e.g., maximal fuel loading). Model updating is performed considering the data of baseline mass configuration. As the last optional step, it is possible to make the multi-model updating, that includes *FE* models and experimental modes coming from multiple mass configurations.

Updating should not include unnecessary modes. In general, more modes make more restrictions to the design space and consequently, updating gives worse results. During the follow-on flutter analyses, it is also possible to make further updating considering the subset of major modes, contributing to a specific flutter instability. It may increase the accuracy of flutter analyses targeted to some specific flutter issue. It may help provided the flutter speed is approaching the certification velocity margin.

4 Application to EV-55M Aircraft Structure

Model updating procedure is demonstrated on *EV-55M* aircraft. *EV-55M* is twin turboprop utility aircraft for *9 - 13* passengers with a total length of *14.35 m*, the wingspan of *16.10 m* and a maximal take-off weight of *4600 kg*. The aircraft was developed by Evektor, Kunovice company. The aircraft prototype *GVT* was accomplished at the *VZLU* in *2012*; the test arrangement is shown in fig.5. As the example, updating of the symmetric model is demonstrated here.

Fig.5. Arrangement of EV-55M aircraft GVT at VZLU

Set of experimental modes included *10* modes of the main structural parts and engines and further three modes of controls (see tab.1).

Tab.1. Experimental modes (symmetric)

#	title	f_0 [Hz]
1	1st wing bending	5.953
2	Elevator flapping (fixed stick)	6.501
3	1st engines vertical vibrations	8.639
4	Aileron flapping (fixed pedals)	10.22
5	1st engines lateral vibrations	10.60
6	1st fuselage vertical bending	12.50
7	1st wing in-plane bending	16.15
8	2nd wing bending	18.54
9	1st horizontal tail bending	23.03
10	2nd engines vertical vibrations	23.69
11	1st wing torsion	38.28
12	Elevator torsion	52.49
13	1st horizontal tail torsion	93.66

FE model included 25 modes within the same frequency range. Comparison of the initial and final pairing of modes is shown in fig.6 and fig.7. Pair numbers correspond to the experimental mode number in tab.1. Fig.6 demonstrates relative error in natural frequencies. The final errors are less than *1.5%*. This is pretty good result. Ordinarily, the errors up to *5%* are considered as good, up to *10%* as acceptable. Note that high initial frequency errors in the flapping modes are caused by the fact, that the initial stiffness parameters of actuation were set guessingly.

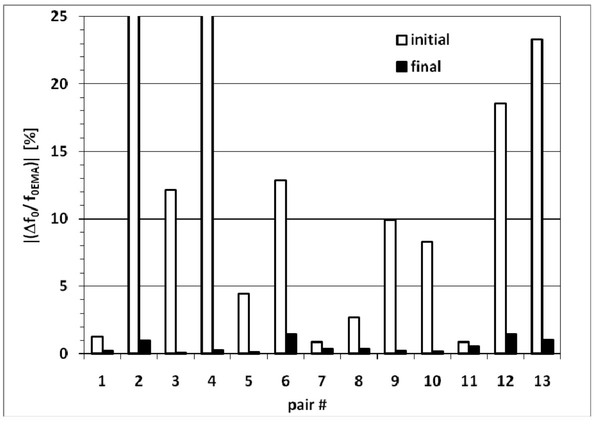

Fig.6. Initial and final pairing of modes (frequency relative error)

Fig.7 shows a comparison of the initial and final state in terms of *MAC* values. The results are also good, all *MAC* values got increased or remained the same. Apart from *MAC* values, agreement in mode shapes must be evaluated by means of visual comparison as well.

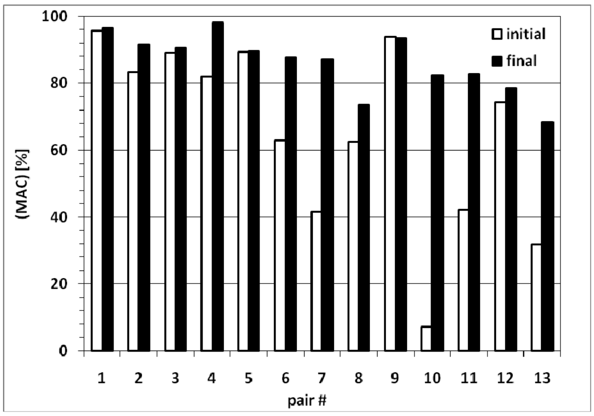

Fig.7. Initial and final pairing of modes (MAC values)

Fig. 8 and fig.9 demonstrate the changes of design parameters. Both figures show stiffness distribution in the spanwise direction expressed as the cross-section inertia. Fig.8 shows the wing vertical bending stiffness. There are two drops in the spanwise distribution. First one is in the root area. It is caused by the flexibility of the wing and fuselage connection. The second one is in the engine area. It is caused by influence of the engine vibration mode. Fig.9 shows the horizontal tail vertical bending stiffness. It includes the similar drop in the root area caused by the flexibility of horizontal tail and vertical tail connection.

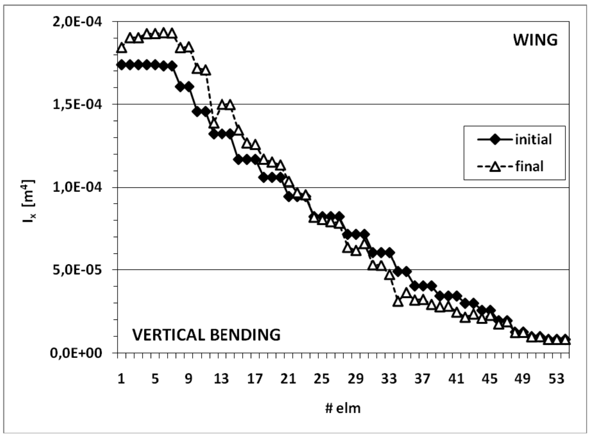

Fig.8. Wing vertical bending stiffness spanwise distribution

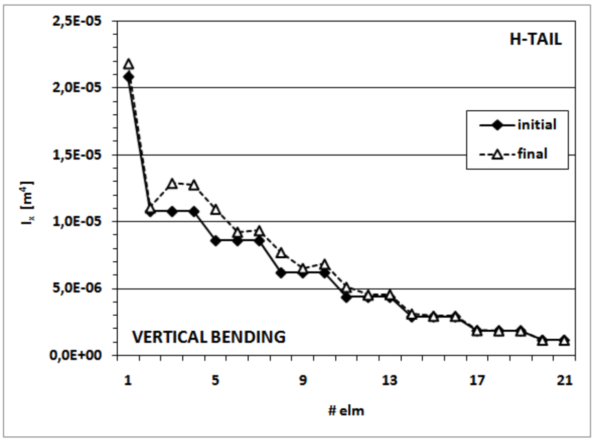

Fig.9. Horizontal tail vertical bending stiffness spanwise distribution

5 Conclusion

Presented paper describes the aircraft structure dynamic model updating in order to match the results of the *GVT*. Paper describes the theoretical background and the methodology, which is demonstrated on the example of *EV-55M* utility aircraft. The results of the symmetric model are presented as an example. Updated modal parameters got much closer to the target *GVT* data. Updated model is prepared to the final phase of flutter calculations.

6 Acknowledgement

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