DYNAMIC RESPONSE OF AN AIRPLANE ELASTIC STRUCTURE IN TRANSONIC FLOW

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Abstract
Two computational methods are presented for determination dynamic response and loads of an elastic airplane in transonic flow under controls deflection and wind gust. One of these approaches is based on integration of the nonlinear Euler equations with the use of the Godunov finite-difference method. In the second approach time-harmonic solutions of the linearized Euler equations in transonic viscous flow are used. The paper describes shortly the applied mathematical models and procedure for research of a structure dynamic response. The comparison of the results of computations based on the developed approaches and on linear aerodynamics is presented. The results of dynamic response, aeroservoelasticity and flutter analysis have been shown for the middle range airplane with the high aspect ratio wing.

1 Introduction
Modern airplanes are elastic structures that easily respond on variation of aerodynamic forces, for this reason it is very important to estimate correctly dynamic loads on aircraft design. In dynamic aeroelasticity analysis two main components are required: aircraft structural model and unsteady aerodynamic model. Nowadays in practice the linear approximation of both models is mainly used. As for nonlinear aerodynamics in transonic flight regime, a development and application of advanced, convenient and accurate computational methods is necessary till now.

For example, buzz-aileron vibration, one degree of freedom phenomenon of aeroelasticity including shock oscillations, is one of problems, which could not be solved by the linear aerodynamic methods. In such case it is necessary to determine solutions of nonlinear aerodynamic equations in order to simulate a behavior of aeroelastic structure in flow with mixed subsonic-supersonic zones and with moving shock waves. Sometimes in practice such phenomena have appeared in wind tunnel or flight test; it led to necessity of an airplane modification and, as a consequence, to structural weight increase. It will be possible to decrease of an airplane weight and its development cost in the case of more accurate analysis of dynamic response in transonic regime and essential reduction of a number of expensive WT tests on dynamically-scaled models and/or flight tests. The determination of unsteady aerodynamic loads generated by the structure vibration is the most essential part in flutter, dynamic response and aeroservoelasticity analysis.

It is known that in transonic flight local supersonic zones and shock waves have appeared on the surface of an elastic wing of the modern passenger airplane. The definite relation between the wing deflection, flow separation and shock displacement has been formed in dependence on flow parameters [1]. Results of the WT test [2] had shown that moving shock directly influences on flow parameters on trailing edge and on level of disturbance, achieving shock and forcing shock to shift up and down on flow. Such dynamic structure-flow interaction can stimulate appearance of nonlinear flutter.

In set of papers [1-4] it was shown that shock wave oscillation with definite frequency may arise because of the flow separation or because of the wing deformation (vibration)
with the same frequency. In WT test [4] the bending-torsion flutter (reduction of aerodynamic damping) was received; it has been accompanied by the shock wave oscillation on the wing surface with definite frequency and amplitude. According to the experimental results, it was determined that the shock frequency depends mainly on speed of the disturbance propagation up and down on flow and, on shock mean position on the wing surface.

According to aeroelasticity analysis [5-7] in the range of the transonic dip aeroelastic instability often has arisen as a type of single degree of freedom (SDOF) flutter or limit cycle oscillation (LCO). Experimental results of the wing NLR 7301 in WT (DLR) test have also demonstrated SDOF torsion flutter in transonic flight regime [8-10]. Limit cycle oscillations are the result of nonlinear aeroelastic interaction between structure dynamic responses and unsteady aerodynamic forces. Therefore, one of the important features of the structural dynamic response in transonic flow is the interaction of the shock wave displacement with the elastic oscillation of the structure. This peculiarity depends on flow regime (Mach number, angle of attack, amplitude of oscillation, flow viscosity and separation) and may be the reason for nonlinear damping and flutter and, also, for complicated dependence of dynamic loads on flow parameters.

In the paper the results of research of the mentioned above phenomena with the use of two approaches for unsteady aerodynamic forces computation in transonic flow are presented. One of these approaches is based on integration of the nonlinear Euler equations by using the Godunov finite-difference method [11]. In the second approach time-harmonic solutions of the linearised Euler equations are used in viscous transonic flow [12]. In both methods the structural displacements are determined on the basis of equations in modal coordinates, which have been created in the ARGON and/or NASTRAN systems. Computational results were obtained on mathematical model of the middle range passenger airplane with transonic cruise flight regime at Mach number M=0.82. The model was developed in the frame of the ISTC Project #4035, and the results of various researches were presented in [13].

2 Computation methods

As was pointed above, the main features of transonic aeroelasticity problems are nonlinearities with respect to vibration amplitude and loads (angle of attack), and increased influence of viscosity. In principle, all of these phenomena may be studied on unified computational model of aerodynamic forces on the basis of the Navier-Stokes equations [14]. Nevertheless, for the practical applications the separate investigation of the considered aspects is more preferable with the use of different realizations of transonic aerodynamics on the basis of the Euler equations. The brief description of the approaches, which are complementary of each other, for the solution of the transonic aeroelasticity problems, is presented below. With the use of the developed algorithms the different aeroelasticity disciplines may be solved in transonic flight regime.

2.1 Nonlinearity with respect to amplitude of oscillation

The iterative method (TRAN-n) has been developed for computation of dynamic response and flutter in transonic flow in frequency and time domain with the use of the finite-difference Godunov algorithm for the nonlinear Euler equations for the ideal gas. As an initial approximation of the frequency and deformation shape the results of linear flutter problem or experimental data are used. After that the flow near the wing oscillating with specified flutter frequency, mode (with the given amplitude of oscillation) and angle of attack is analyzed using nonlinear transonic theory. Then the Fourier components of main frequency are extracted from the obtained dynamic pressure and new aerodynamic matrices are computed. The equations of vibration in the flow are solved anew. The algorithm in more details is presented in [11]. The method comprises two algorithms: computation of the linear flutter and
study of transonic flow near an elastic wing that oscillates with the specified frequency and mode (amplitude) [11]. Therefore it is important to note that the main feature of the TRAN-n method is the possibility to investigate in transonic flow the dependence of level of dynamic response and flutter on values of both vibration amplitude and angle of attack. In the time domain TRAN-n is used for determination of dynamic response and parameters of nonlinear transonic limit cycle oscillations. The method of coupled numerical integration of equations, which describe lifting surface deformations and transonic aerodynamic flow, is used.

2.2 Influence of viscosity and loads

The TRAN-v method [13] has been developed for analysis of transonic flow over an elastic airplane of complex aerodynamic configuration with taking into consideration the viscous effects on the wings including thin separation zones. The finite-difference solution of the linearized small disturbance unsteady Euler equations is conducted for each mode and each reduced frequency. On the basis of linear aeroelasticity analysis the flutter parameters are determined in advance, and a set of natural modes, which participate in dynamic response and flutter evolution, have been chosen (15-25 modes for symmetrical or antisymmetrical cases for complete airplane). A set of reduced frequencies, allowing to determine an unsteady aerodynamic forces with sufficient accuracy in considered range of frequencies and speeds, is also determined on the basis of the linear flutter analysis (in general 5-7 values). Natural mode shapes are input to transonic solver as the nodal displacements of the DLM aerodynamic grid. Modal shape is determined by the displacements of four corners of each panel for the lifting surfaces (wing, horizontal tail, vertical tail), and by the displacements of the nodes of central line for bodies (fuselage, nacelle) as beam deformations. Finite-difference solution of linearized unsteady Euler equations is performed for each mode and each reduced frequency. The obtained pressure distribution is transformed to the same grid where modal shapes were specified; then aeroelasticity analysis is carried out with the use of the same methods and computational procedure as for linear aerodynamics. The following data are obtained as a result:

- the distribution of real and imaginary parts of non-dimensional pressure difference in panel nodes (for lifting surfaces),
- the pressure distribution for body is represented as two complex (vertical and horizontal) components.

For aeroelasticity analysis of dynamic response and flutter the modal generalized aerodynamic forces are determined on the basis of computed pressure distributions.

3 Results of investigations

In the paper the computational results are presented for middle range passenger airplane with transonic cruise speed at Mach number M=0.82. The main features of the airplane structure behavior with the presence of such aerodynamic nonlinearities as flow - shock wave interaction and viscosity effect are analyzed. The airplane of traditional configuration with high aspect ratio wing AR=12.5 and two engines under the wing is considered. There are supercritical aerodynamic airfoils with the root thickness 15.8%, 11% - on the kink and 9% - on the wing tip.

Preliminary analysis with the use of linear aerodynamics has shown that aeroelasticity characteristics of the airplane are in general limits. The aileron effectiveness decreases with the increase of dynamic pressure and Mach number. The boundary of the aileron reversal is close to the extreme regimes on dynamic pressure and Mach number.

Flutter analysis shows that two flutter forms take place for symmetrical motion. The first form is connected by the engine pitch, the wing bending and torsion of the wing root (Flutter 5Hz). Flutter dynamic pressure margin of the form is on the limit. The second form is related with bending and torsion of the wing tip (Flutter 8Hz); in this case the flutter margin is large. The main interest is focused on the first flutter form and on the dependences of flutter characteristics on flow parameters. The same
dependencies are obtained on levels of dynamic responses in the range of the lowest frequencies of elastic modes. For this reason it is very important to analyze different characteristics carrying out studies of an interaction between the pressure distribution and the deformation of the elastic structure.

3.1 Investigation of influence of nonlinearity with respect to amplitude of oscillation

Initially, the dependence of flutter characteristics on vibration amplitude was considered. Then the influence of amplitude on aerodynamic damping and, finally, on dynamic response was analyzed. The level of oscillations was determined by amplitude of stream-wise angle of the wing tip vibrations $\alpha_0$ at the flutter frequency.

3.1.1 Flutter

The results of flutter computation using nonlinear Euler equations for determination of aerodynamic coefficients in transonic flow are presented in Fig.1. The flutter boundaries, which were determined for two different angles of the wing twist, are shown in the figure. In fact, the dynamic pressure on the wing was determined at forced vibrations with flutter frequency 5Hz and flutter form with given amplitude. For comparison, the result of the linear flutter analysis is also shown in Fig.1.

The existence of shock waves has led to the considerable change of the elastic structure behavior in flow, namely, to the occurrence of the transonic dip (Fig.1). For example, Fig. 3 demonstrates the stationary pressure distributions and shocks locations on the wing upper and lower surfaces in the dip range of Mach number for the wing section for nondimensional span $z^*$=0.1 at zero angle of attack.

![Fig.1. Flutter boundary versus Mach number](attachment:image1.png)

In the range of transonic dip the influence of vibration amplitude on flutter dynamic pressure is essential (Fig. 2). For example, at Mach number $M$=0.84 with the dynamic pressure increase the abrupt jump of amplitude takes place, then amplitude of oscillation increases continuously as Fig. 2 shows. The presented results indicate that the limit cycle oscillations (LCO) may exist at dynamic pressure less than linear flutter critical dynamic pressure. Such response of the structure is undesirable.

![Fig.2. Correlation between dynamic pressure and amplitude of flutter oscillation with frequency 5Hz at $M=0.84$](attachment:image2.png)

The real and imaginary parts of dynamic pressure on upper and lower surfaces of the wing section $z^*$=0.1 at flutter "5Hz" with the given amplitude 0.5 degree are presented in Fig.4 for different Mach numbers: $M=0.82$ и 0.84.

It can be seen that at $M=0.82$ the shock wave on lower wing surface pays the main role in flutter evolution; whereas at higher Mach number ($M=0.84$) displacements of both upper and lower shock waves have led to flutter at less dynamic pressure.
The presented computational results of transonic flutter, which depend on shock waves motion on the wing surfaces, demonstrate high flutter sensitivity to variation of Mach number. Obviously, strength and amplitude of the shock wave at flutter oscillations depends on flow parameters. Fig. 5 shows the real and imaginary
parts of dynamic pressure difference of the studied flutter form (given amplitude 0.5 degree, frequency 5Hz) for different Mach numbers.

3.1.2 Damping

The existence of the shock wave has changed the flutter boundary even if it is very small shock displacement. At enough big values of shock displacement the nonlinear dependence between aerodynamic forces and structural deformation has led to appearance of LCO. Phenomenon of transonic LCO to same extend reminds flutter because it occurs with definite frequency under condition of mixed subsonic/supersonic flow over lifting surfaces. In order to understand some features of the transonic LCO it is helpful to analyze behavior of aerodynamic damping (nondimensional derivative of aerodynamic moment coefficient with respect to oscillation rate), which was determined under forced wing oscillation with the flutter frequency at different values of amplitude. Especial attention was paid to small amplitudes of oscillation under the study of damping behavior. The forced oscillations around the wing stiffness axis were considered.

The results of the computational research are shown in Figs. 6 and 7, the frequency of the forced vibration is 5Hz, and amplitude is in the range from 0.125 up to 2 degree. Fig. 6 shows the fall of the aerodynamic damping coefficient at Mach number M>0.75 that has led to flutter at negative damping and, then the increase of damping at transition over Mach number M=0.84.

It is worth noting that the peak of negative damping coincides with the dip of the transonic flutter boundary. In Fig.7 the dependence of aerodynamic damping on frequency and amplitude of forced oscillations is presented for Mach number (M=0.84), corresponding to the minimum flutter dynamic pressure. The damping values obtained agrees with the results of flutter boundary computation at different amplitudes (Fig.1): the smaller amplitude the lower flutter dynamic pressure. On the basis of results determined it is possible to confirm that considered aeroelastic instability in transonic regime may be represented as LCO because at small amplitudes the oscillations are unstable, amplitudes increase up to the moment when the process is stabilized at bigger amplitudes due to the nonlinear forces.
3.1.3 Dynamic response

As mentioned above the mathematical model of aeroelasticity in inviscid flow takes into account the dependence of aerodynamic forces on oscillation amplitude. Therefore the dynamic response considerably depends on oscillation amplitude. To obtain the correct dynamic response in this case it is necessary to determine stationary structural response for each frequency at specified level of the external action (such as harmonic deflection of the control surface). This is very labour-consuming task. The simplified approach was used here when the linearised aerodynamic forces for specified level of oscillations are used. The level of oscillations (i.e. the level of external action) is characterized by the amplitude of stream-wise twist angle $\alpha_0$ of the wing tip near the flutter frequency as well as for the flutter analysis. Let us consider one of the important frequency response function (FRF) of an airplane – wing root bending moment due to aileron harmonic deflection $M_{bend}/\delta_{ail}$. The comparison of FRFs for two levels of wing oscillation with $\alpha_0=0.5^\circ$ and $\alpha_0=1.0^\circ$ for Mach number $M=0.775$ and equivalent airspeed $V_{EAS}=500\text{km/h}$ is presented on the Fig.9.

It can be seen that the linearization for smaller amplitude gives higher on 10% FRF's peaks. Note this is high enough level of amplitude: to force wing tip oscillations with...
amplitude $\alpha_0=1.0^\circ$ the amplitude of aileron oscillation should be equal approximately $5^\circ$ for considered flow regime.

Both these factors are absent in linear aerodynamic methods and the account of them may appreciable influence on structural dynamic response of modern airplane in transonic flight regimes.

Several types of dynamic response in the frequency domain for the range of lowest modes of elastic oscillations are considered here: wing root bending moment due to harmonic actions of aileron and wind gust, as well as load factor (acceleration) at the wing tip due to harmonic action of aileron. These characteristics are important for study of dynamic load and load alleviation system.

Figs. 11-13 show the comparison of FRFs on bending moment due to aileron deflection for different Mach numbers and flow regimes. FRFs are computed for the same airspeed value $V_{EAS}=500\text{km/h}$. Two typical regimes which are distinguished by wing load and viscosity are considered here. The first one ($C_L=0.1$, $Re=3\text{mln}$) is relevant for testing of aerelastic models in transonic wind tunnel (WT) and second ($C_L=0.5$, $Re=23\text{mln}$) – for cruise flight conditions. For sensitivity estimation of every parameter the calculation for intermediate regimes ($C_L=0.1$, $Re=23\text{mln}$) and ($C_L=0.5$, $Re=3\text{mln}$) also have been conducted.

3.2 Viscosity and wing load effect

The second approach is based on the finite-difference solution of linearised unsteady Euler equations to determine unsteady time-harmonic flow. A stationary flow field is preliminary computed in the frame of viscous-inviscid interaction procedure of the boundary layer theory. Therefore there are no nonlinearities on amplitude in this case but there are two important factors for aerelasticity characteristics in transonic flow:

- The first factor is the effect of basic stationary flow field on aerodynamic derivatives. In addition to Mach number and density, the basic flow field is determined by the angle of attack, camber of airfoils and wing twist. Integral wing load is determined by the lift coefficient $C_L$ (it is close to static angle of attack around which the oscillations occur in the first approach).

- The second factor is the effect of viscosity on aerodynamic derivatives. It is characterized by Reynolds number $Re$.

Aileron effectiveness on bending moment slightly decreases when Mach number grows and the difference of responses due to oscillation amplitude arises and achieves 20% an $M=0.84$ (Fig.10).

![Fig.10. The same as on Fig.9, but $M=0.84$](image)
Comparison shows that the dynamic response for low subsonic Mach number is only slightly dependent on these two parameters (Fig.11). The results are considerably dependent on flow regimes for transonic cruise Mach number M=0.82. The response is higher on 20-25% for cruise flight regime in comparison with WT test regime (Fig.12).

When Mach number grows up to M=0.9 the response decreases for high values of $C_L$ (Fig.13). It should be also mentioned that linear aerodynamics (DLM) gives overestimated results for high Mach numbers.

About a similar effect of flow regime can also be seen for dynamic response on load factor at wing tip (Figs.14-16): the effect is small for M=0.6, the response for cruise flight regime is higher on 20-25% in comparison with WT test regime for M=0.82, and the response for high $C_L$ decreases considerable for M=0.9.
Revealed features of structural dynamic response in transonic flow may lead to significance differences in dynamic load and fatigue characteristics in comparison with the results of analysis on the basis of linear panel aerodynamics. For example, fatigue damage of wing structure increases by 2-3 times for flight regimes $M=0.82-0.84$. The influence of gust load alleviation system on fatigue damage in transonic regimes increases also, but not as much, only on 5-10%.

4 Conclusions

One of important features of the structural dynamic response in transonic flow is the interaction between shock wave motion and elastic oscillations of the structure. This peculiarity depends on flow regime (Mach number, angle of attack, flow viscosity, existence of flow separations) and it is the reason of negative damping, flutter and, also, of complicated dependence of dynamic loads on flow parameters.

The computational results presented in the paper, which were obtained on mathematical model of the middle range passenger airplane with transonic cruise flight regime, show the essential influence of transonic features on characteristics of dynamic aeroelasticity. Naturally, such peculiarities must be taken into account in design and certification of modern aircraft. Hope that our methods and software developed for research may be one more step in this important direction.

References


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