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NONLINEARITIES AND UNCERTAINTIES OF AEROELASTICITY CHARACTERISTICS IN AIRCRAFT DESIGN AND CERTIFICATION

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Abstract

Some aspects of structural design and certification of aircraft related to the nonlinearities and uncertainties of aeroelasticity characteristics are considered. The influence of quasi-steady flight deformation of the high aspect ratio wing, some structural and aerodynamic nonlinearities and uncertainties on aeroelasticity characteristics have been investigated. The analysis and discussion of mentioned aeroelasticity aspects are applied to specific examples of airplanes for the various stages of development.

1 Introduction

During a practical problems solution of ensuring of airplane safety according to aeroelastic stability requirements linear approaches are usually used. However the development of aeronautical engineering demands to investigate some problems in nonlinear statement. Nonlinearities are closely related to uncertainties in structural parameters and flow conditions.

Uncertainties and nonlinearities, which influence on determination of aeroelastic characteristics, may be conditionally divided into geometrical, structural and aerodynamic ones [1-4]. Their sources can include joint units between structural parts, inaccuracy in aerodynamic forces prediction, instability and dispersion in material properties, different fuel loading cases, scatter of inertial and stiffness characteristics of external suspended loads, etc. The large group of nonlinearities and uncertainties is related to characteristics of control drives and control system [4].

One of the modern trends of the aviation development is enlarging the aspect ratio to improve aerodynamic quality and reduce specific fuel consumption.

For advanced long-distance passenger and transport airplanes with the composite wing it is supposed to enlarge the aspect ratio from AR=8-10 up to AR=11-12 and more, and for some unmanned vehicles (UAV) structures with aspect ratio 30 and more are considered. Flight vehicles with high aspect ratio are acted with high static deformations in different flight conditions. In this regard a question arises about possible influence of the quasi-steady flight deformations on aeroelasticity characteristics, including deformations due to a maneuver or wind gust. Should these deformations be considered in modern aircraft design and certification (for example, on flutter safety)? Note that nowadays the Airworthiness Requirements prescribed to ensure flutter safety margin for horizontal undisturbed flight. However, further clarification of the Airworthiness Requirements and certification procedure may be required for the airplanes with very high aspect ratio. Possible influence of high static deformations on aeroelasticity characteristics is actual modern problem, and many investigations were performed in recent years [5-6]. One of the determinations of the methodology of high deformations modeling with nonlinear aerodynamic model for performance the joint nonlinear analysis of aeroelasticity characteristics and results of it's using for wing with high aspect ratio are presented in the paper [5]. Project of unmanned vehicle with AR=32, including its aeroelasticity characteristics and influence on it elastic deformations due to flight loads is investigated in the paper [6]. One of the interesting results, it should be noticed, is a founded significant influence of the loading due to rising attack angle and connected with this angle deformations on the flutter speed and frequency. The flutter speed noticeably is reduced due to wing attack angle rising.

In this paper a quite simple methods of the calculations researches of a quasi-steady deformations influence on modal and flutter characteristics of the wing with high aspect ratio in dependence on a maneuver or wing gust parameters is used. This method is based on a linearization of a problem by creation a number of calculation models with deformed structure.

Linear analysis results can noticeable change due to accounting of structural nonlinearities which usually displays in attachment points between aggregates; especially it is important in the case of the mounting of control surfaces and engines. Such nonlinearity can be interpreted as joint stiffness uncertainty quantification. Usually in the aeroelasticity problems two types of structural nonlinearities are interesting.

The first type is related to free-play in control linkage. The methodology of nonlinear flutter and limit cycle oscillations (LCO) investigation is presented for the case of free-play presence in a control linkage, as well as the safety of structure ensuring in this case [7, 8]. A lot of researches of LCO connected with structural nonlinearities of control surfaces were performed. It shows that usually the speed of LCO appearing is less than the linear flutter speed for nominal structure and due to these factors the aeroservoelasticity problem is complicated.

The second type of nonlinearity is related to nonlinear dependence of the flight parameters (thrust, vertical and horizontal load factors) stiffness of the statically indeterminate attachment engine/pylon/wing on the flight parameters (thrust, vertical and horizontal load factors). In the paper typical experimental nonlinearities are considered for such type attachment in dependence on flight parameters. The analysis of the flutter boundaries and the possibility of flutter margin decrease are presented for this case.

For advanced long-distance airplanes the flight regime at high subsonic Mach numbers is a specific one. The nonlinearities related to transonic flow nature are very important for aeroelasticity safety ensuring. A lot of investigations are devoted to LCO appearance, shock waves movement and its connection with elastic deformations [9, 10]. The influence of transonic nonlinearities and flow viscosity on flutter characteristics and control surfaces with respect to problems of the aircrafts certification is considered.

2 Deflection of quasi-steady deformations on aeroelasticity characteristics of the aircraft with increased flexibility wing

At the beginning of the chapter on simplified methodic examples it is shown that geometry changing due to elastic deformation has main influence on aeroelasticity characteristics for loaded wing. Essential deflection of deformations on aeroelasticity characteristics of the wing with high aspect ratio is demonstrated. Then estimations for advanced long-distance airplane are performed.

2.1 Test model for estimation of the role of geometry and stressed state changing

Both of changing geometric form of the loaded wing and stressed state in structure influence on oscillations frequencies and shapes. Methodic calculations of the beam wing structure with high aspect ratio were performed for three cases to determinate the each factor influence.

In the first case wing is modeling as a console-attached straight beam. Wing box with rectangular cross-section, with the length 16 m, width 1.5 m and height 0.6 m is considered. Eigen-frequency of the first mode is equal 2 Hz. The finite element model calculations were performed at the system MSC.Nastran with the use of the nonlinear static calculations block with modal analysis of the deformed pre-stressed structure [11,12]. The changing of eigen-frequencies of the structure in dependence on its end deflection was investigated. The deflection was caused by external load which is modeling aerodynamic forces. Constant pressure, which acts on the lower surface of the beam, was used as a load. Pressure value corresponds to the end beam deflection in the nonlinear calculation from 0 to 30 % from the beam length.

In the second case the curved beam without loading and with deflection determined in the first case was set. Form of the beam corresponded to the loaded beam form, but inherent stresses were absence.

A bending beam loaded in the opposite direction is considered in the third case. The loading pressure unbends the beam in this case and the straight beam with internal stresses corresponding to deformation under load in the first case

Analysis of load and strain influence on modal characteristics obtained in the first calculation has shown that oscillations modes of vertical bending practically don’t change when deflection increases. Due to deflection increase the rotation appears in the horizontal bending oscillations modes. Bending oscillations frequencies decrease (on 2%-4% in a case of maximum deflection), but torsion oscillations frequencies increase (on 4%-9%).

The comparative analysis of these three cases shows that the geometry change, i.e. deflection, is the main influence on eigenvalues of the loaded beam. The influence of the internal stressed state due to the load on eigenvalues change is quite small. Typical example is presented on the fig.1.

Special comparative calculations showed that stressed state influence become appreciable only due to approach to boundary of stability loss of the skin (due to deformations increase more than 90-95% from the critical value).This important conclusion allows to use in the first approximation linear calculative models taking into account the geometry change in structure during the flight.

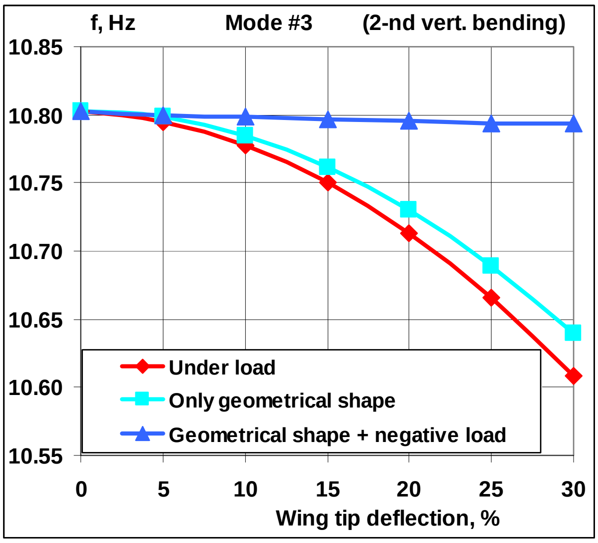

Fig.1. Eigen frequency dependence on the beam tip deflection, 3-d mode

For the analysis of the geometry change on the aeroelasticity characteristics the set of calculative models, which are corresponding to different wing deflections (i.e. to the level of the loading which is determined by the maneuver load factor or wind gust value) are created. For practical analysis automatic procedure is created to preparing these calculative models in the system ARGON [13].

2.2 UAV with AR=30

To reveal main mechanisms of deformation influence on aeroelasticity characteristics a flight vehicle with large flight deformations was considered, namely the calculation model of UAV "Helios Prototype", which was studied within the frame of program of creation of high altitude and long endurance aircraft “HALE” [5].

The first level model of ARGON system was applied. The doublet-lattice method was used for aerodynamic forces calculations. Mass-elastic properties of structure were described based on Ritz polynomial method.

This UAV is a flying wing with AR=30. Fuselage and pylons were joined to the wing. Wing end parts were deflected from horizontal plane by 10° (Fig.2). The wing was divided along the span into three parts, which were considered as separate elastic surfaces (ES).

Distribution of elastic displacements of wing along the span were obtained via static loads calculation at eight values of angle of attack from α=4.9° to α=7.2°. It is supposed that maneuver or discrete gust impact may cause changing of angle of attack. Wing tip deflections from 2% to 30% (relative to semi-span of wing) at nominal vehicle speed V=12.2 m/sec correspond to that values of angle of attack. Obtained forms of deformed wing were piecewise-linear approximated by the indicated above three ES. Thus, eight additional computational models (for eight angles of attack) were obtained. They were used for estimation of effect of angle of attack in flight, (that is effect of relevant static wing deformations) on aeroelasticity characteristics. UAV's angle of attack in level flight α=5.8° corresponds to 12.7% of deflection.

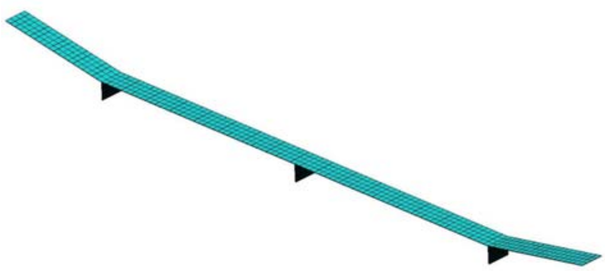

Fig.2. UAV's computational model

Under consideration UAV structure has low oscillation frequencies (the frequency of the first mode of wing bending is 0.19Hz). Static deformations have a considerable effect on oscillation forms (up to 30-40%) that leads to appreciable change of flutter characteristics.

Two flutter forms with frequencies about 1 Hz and 1.6 Hz appear for an unstrained structure in symmetrical case. Interaction of several, at least of three oscillation tones causes the both flutter forms. A flutter form with frequency about 0.8 Hz occurs in an anti-symmetrical case.

Main mechanism of static deflection effect on flutter consists in interrelation increasing of horizontal bending oscillation of wing with torsional oscillations.

Starting from angle of arrack α=5.7° (steady horizontal flight), critical speed of the 1st form of symmetrical and anti-symmetrical flutter has almost linearly decreasing, approximately by 10%-15% when increasing the angle of attack by 1° or when increasing bending by 10% of semi-span of wing that is almost the same (Fig. 3). Critical flutter speed of the 2nd form grows up when increasing deflection. The results are close to findings obtained for other objects/vehicles with high aspect ratio (see, for example [6]).

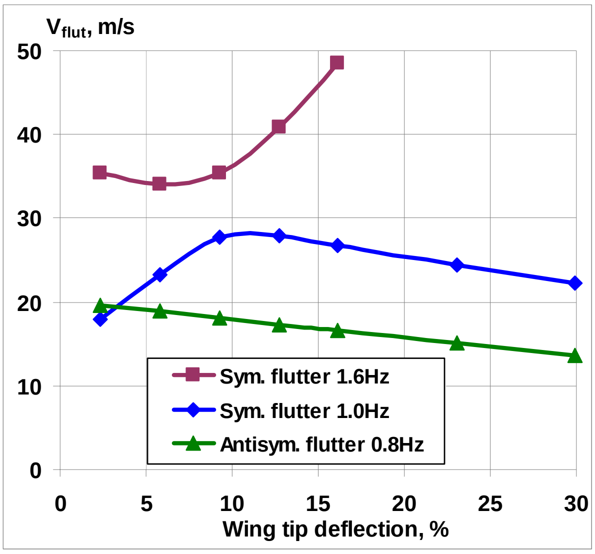

Fig.3. Flutter speed dependence on deflection of wing tip (angle of attack) for UAV with high aspect ratio AR=30

2.3 Middle-range aircraft

As shown above when analyzing aeroelasticity characteristics of unmanned flight vehicle (UAV) with high aspect ratio it is necessary to take into account quasi-steady wing deformations in flight. What about modern airliners?

Let consider a typical example of middle-range aircraft (MRA) c with composite wing of high aspect ratio AR=11.5 (Fig.4). Deflections for various values of load factor obtained by calculation in ARGON system at maximum flight speed are shown in the Fig.5. Ratio of deflection equals 4.4% corresponds to horizontal flight and deflection equals 10% corresponds to maneuver with load factor equals 2.5.

Oscillation frequencies change is not big for this type of aircraft and at maximum load factor *n*=2.5 does not exceed 2%-5%.

Flutter analysis shows that two flutter forms of wing may arise. The first flutter form with frequency 3 Hz connected with the 1st mode of vertical bending and pitch oscillations of engine. The second form with frequency 5 Hz connected with the 2nd mode of vertical bending and wing tip torsion.

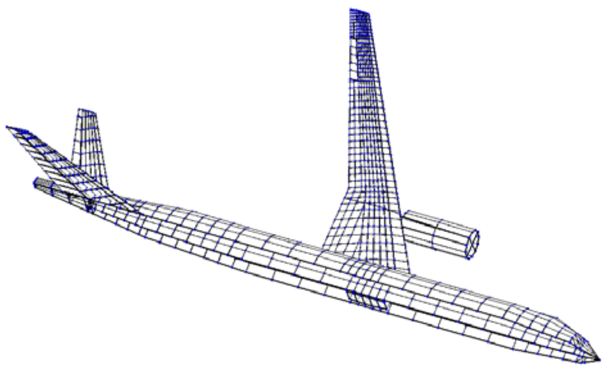

Fig.4. Structural model of MRA

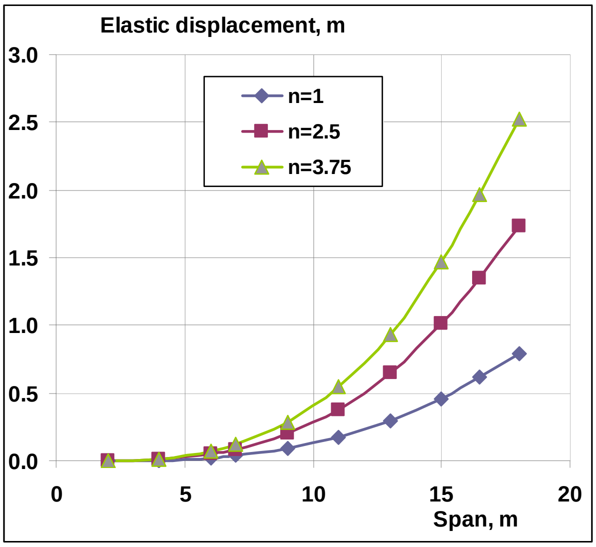

Fig.5. Deflections of MRA wing for various load factors, $V=V_D$

Deflection of quasi-steady deformations on flutter speed is shown in Fig.6. One can see that speed of the flutter first form sharply grows under deflection increase. Speed of the second flutter form decreases by 2% at the deflection range from 0% to 10%, and then starts to increase.

At this stage a conclusion can be drawn that for the type of under consideration airplane static deformations do not lead to appearance of new dangerous flutter forms and a decrease of the flutter speed due to static deformations is within the accuracy of the calculations. Therefore today at certification of passenger airplanes under flutter safety requirements these effects may leave out.

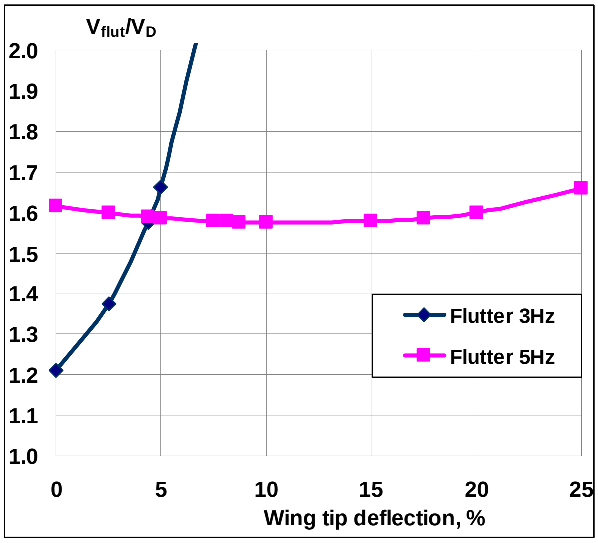

Fig.6. Dependence of flutter velocity on deflection of wing tip of MRA

3 Structural nonlinearities and uncertainties

Arrangement of the engines under wing on pylons is very widely used configuration for modern civil airplane. Nonlinearities and uncertainties in stiffness values of engine statically indeterminable suspension are very significant parameters because of their influence on limit flutter modes characteristics and dynamic loads. The example of such nonlinearities and uncertainties of structural type is considered below.

3.1 Stiffness testing of “wing-pylon-engine” system

Special method and equipment for ground stiffness test of natural structure of A/C are developed and used in TsAGI [14] for experimental determination of nonlinear stiffness for engine supporting on pylon under wing.

The method of investigation allows deciding the following tasks:

- creation of math. model for statically undetermined pylon attachment structure,
- experimental investigation of nonlinear attachments stiffness taking into account engine thrust force and maneuver loads,
- investigation of nonlinear attachment stiffness influence on flutter

characteristics, limit cycle oscillations and dynamic loads,

- decreasing of uncertainties for analysis and expertise of flutter safety problem and dynamic loading during certification of A/C.

The scheme fragment of pylon stiffness test for regional A/C is presented on fig.7 as example. Experimental data (fig.8) show approximately 1.5÷2 times difference of vertical stiffness (flexibility) values for small and large static forces (or amplitude of elastic vibration).

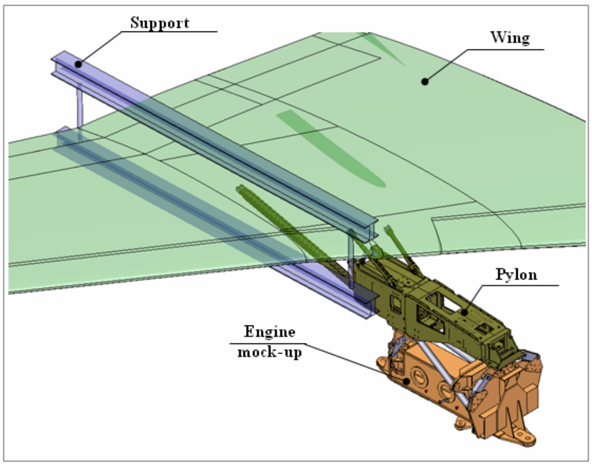

Fig.7. Scheme of pylon stiffness test

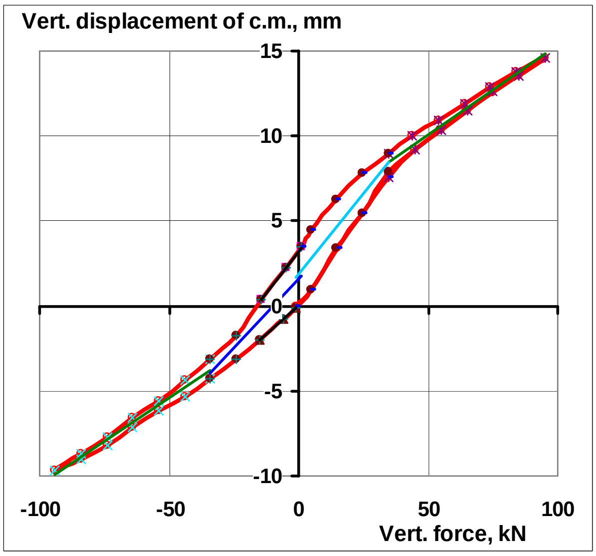

Fig.8. Vertical flexibility of "wing-pylon-engine" system

3.2 Uncertainties in flutter characteristics of aircraft with engines on pylons under wing

The nonlinearity mentioned above may be interpreted as expansion of uncertainty in stiffness of engine attachment. Wide parametric investigations are necessary for providing flutter clearance and safety of A/C taking in account of uncertainties.

Typical dependence of critical flutter speed from engine attachment pitch stiffness for considered A/C is presented on fig. 9 as example. Two main flutter modes are proper for such A/C configuration as a rule. The first flutter type is characterized by engines pitch oscillations (it significantly depends on engine attachment stiffness) and the second one – by wing tip elastic deformations (small effect from stiffness of engine pylon and attachment). Flutter boundary for our example is presented on fig. 9. The curve has distinct minimum of flutter speed.

There are two ways to enhance A/C required flutter margins taking during design: 1) to realize more high stiffness of pylon and attachment (structure weight will increase); 2) to make pylon and attachment with smaller stiffness (the dynamic loads and adequate additional weight also may increase). Different designers in practice choose any way during multidisciplinary optimization.

It is enough difficult to provide guarantee of large flutter margins on design stage taking into consideration possible nonlinearities and uncertainties. For considered A/C, for example, the calculated flutter speed margin for pitch engine mode (fig. 9) is very close to required one for certification (dashed horizontal line) and depends from nonlinearities (dashed vertical lines). Further increase of pylon/attachment stiffness is not effective because of significant weight penalty and negative structure behavior in failure case – "engine blade failure". So, nonlinearities may effect on expertise conclusion under certification procedure.

Unfortunately modern ground vibration tests of natural A/C conducted, as a rule, without imitation of statically flight loads and engine thrust force not fully removed the uncertainties in shapes and eigen frequencies of elastic modes and therefore flutter speed. Limits of shakers forces, their quantity and positions, amplitudes of vibrations during GVT also may be source of difference (uncertainties) in comparison with real structure vibration behavior.

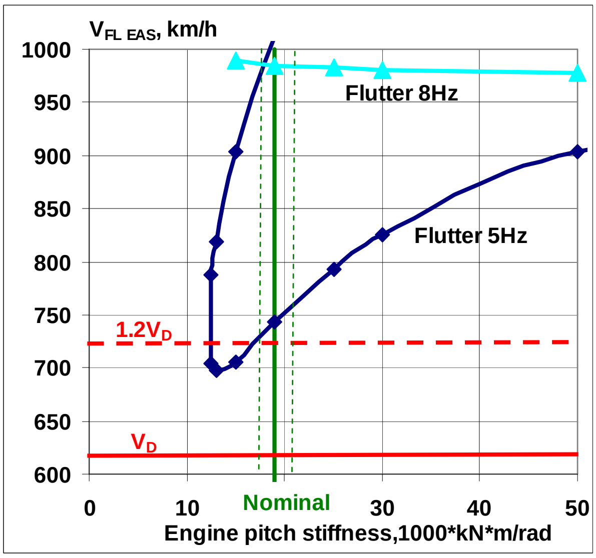

Fig.9. Flutter speed versus engine pitch stiffness

In our case of low damping engine pitch type flutter, for example, relationship between torsion amplitudes of wing root part and engine pitch vibration (shapes parameters) has the high effect on calculated flutter boundary under practically the same vibration frequencies. From the other side during different GVT regimes difference in shapes relationship was noted for base elastic modes.

Ratio of amplitudes of engine pitch oscillation and wing flow chord torsion in engine support cross section (Kteta) may be considered as generalized parameter for analysis of given uncertainty.

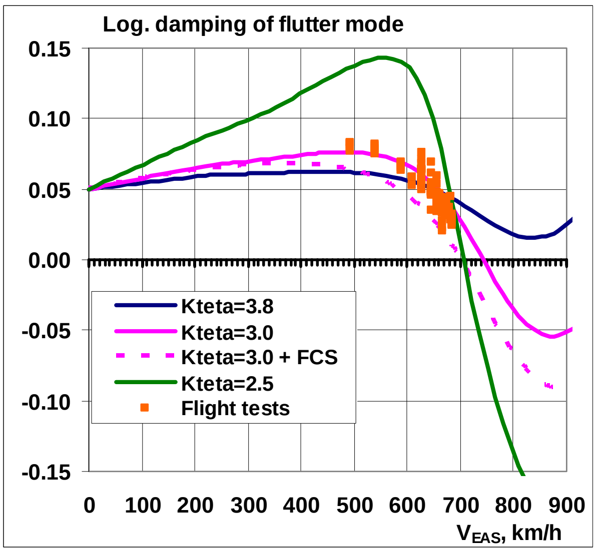

Fig.10. Damping of flutter mode for different cases

The flutter mode damping behavior for different (varied) Kteta is shown on fig.10 as example. The results show that flutter speed (point of zero damping) is very sensitive to Kteta changing for considered flutter type. It is necessary to pay attention on these dependencies under expertise and conclusion about satisfying of A/C to adequate flutter safety requirements.

3.3 Influence of control system on flutter characteristic

Uncertainty in A/C flutter boundaries analysis may amplify by possible influence of flight control automatic system. From the one point of view additional deflection of control surfaces during flutter type oscillation formed the additional vibration forces and from the another side – control system has it`s own nonlinearities and uncertainties.

Schematic diagram and parameters of simplified longitudinal channel of flight control system (FCS) is shown on fig.11 for our example. Signals of vertical acceleration (load factor) and pitch rate after filtering and amplifying enter to the actuator of elevator. The gains in channel in common are depended on Mach number and dynamic pressure. Here for estimations we used constant gain values adequate to maximum flight parameters.

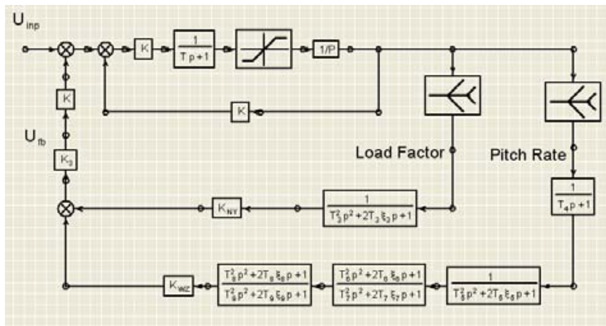

Fig.11. Simplified scheme of longitudinal channel of FCS

From the point view of influence on flutter, frequency response functions (FRF) (in longitudinal movement) near engine pitch mode frequency 5Hz and fuselage vertical first bending mode 7.5Hz are more interest in given case. Aircraft FRFs of pitch rate due to elevator oscillation for different velocities up to flutter speed at Kteta=3.0 are presented on fig.12. Results show rapid increase of the amplitude FRF peak near flutter speed and so the influence of control (elevator oscillations near flutter frequency) must change the flutter characteristic for closed loop.

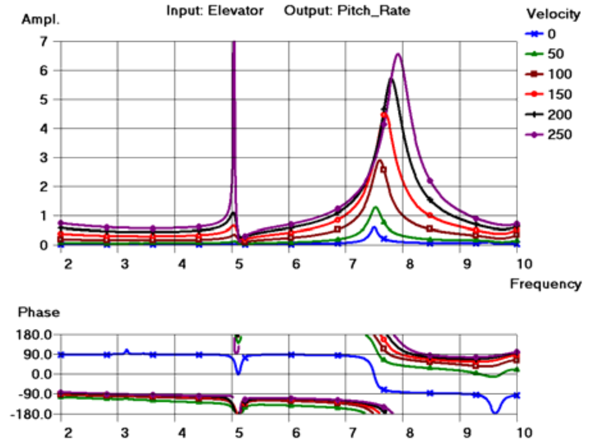

Fig.12. Pitch rate FRF under harmonically deflection of elevator (H=4km)

The analysis of open loop FRF in the form of Nyquist's diagram shows that control system has destabilizing effect (critical point of instability (+1; 0)) on flutter mode with ~5Hz frequency near critical airspeed.

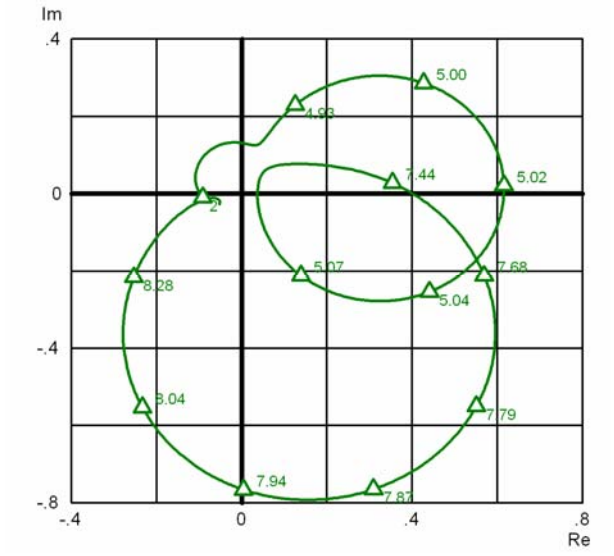

Fig.13. FRF of open loop “A/C structure –control system” (Nyquist's diagram) for V=230м/s, H=4km (V_{FL}=250m/s)

Other form of flutter speed estimation is linear analysis of closed loop system. The damping behavior of flutter mode with OFF / ON control system (fig.10) shows that activation of control system leads to decrease of flutter mode decrement approximately on 0.01÷0.02 at high speed near boundary. Meanwhile adequate flutter speed decrease may be different in dependence of damping from airspeed. In our example for variant Kteta=3.0 flutter airspeed decay equals to about 5%.

It necessary to note that even after flutter flight tests (fig. 10) the uncertainty in critical speed value remains and complicates the decision about providing of enough flutter margins during certification.

For decrease the level of uncertainty without significant weight penalty and project parameters changing the additional volume of numerical/experimental complicated investigations is necessary in several cases like as in example presented in given item.

4 Aerodynamic nonlinearities and uncertainties

Influence of transonic flow features on aeroelastic characteristic of A/C (structural flutter modes, control surfaces efficiency) from the point view of certification is considered in given section.

Nowadays in TsAGI original method for calculations of unsteady aerodynamic loads distribution in transonic flow, based on Euler equations taking into account viscosity [15], was developed and is using for practice. Current approach is intended for fast calculations both steady and unsteady (harmonically type in time domain) flow an assessment of aerodynamic characteristic for multielement aerodynamic configurations with taking into consideration viscosity on aerodynamic surfaces including no lengthy zones of flow separation. The method described may be used for calculations in areas of steady aerodynamic analysis and conceptual design, for support of wind tunnel experiment, for determination of aerodynamic derivatives for flight dynamic, aeroelasticity and aeroservoelasticity [16].

Solution of static aeroelasticity problems and flutter analysis for middle range type aircraft (MRA) are considered below as example. Mathematical model of MRA is illustrated on fig. 14.

Pressure distribution and shocks wave intensity, of course, depend significantly on flow conditions. Two typical regimes which are distinguished by load (it is characterized by lift force coefficient C_L) and viscosity (it is characterized by Reynolds number Re) are considered here. The first one (C_L=0.1, Re=3mln) is relevant for testing of aeroelastic models in transonic wind tunnel and second (C_L=0.5, Re=23mln) – for cruise flight conditions. For sensitivity estimation of every parameter the calculation for intermediate regimes (C_L=0.1, Re=23mln) and (C_L=0.5, Re=3mln) also have been conducted.

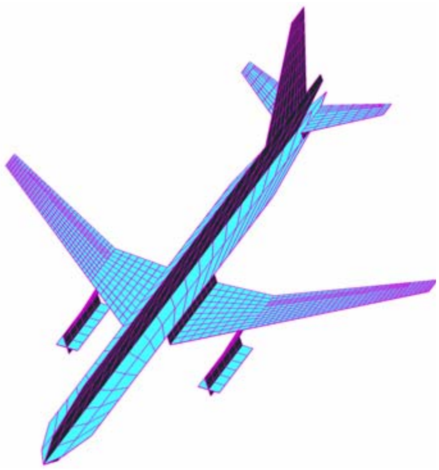

Fig.14. Scheme of MRA computational model

Main peculiarities of summarized transonic MRA aerodynamic characteristics received with influence of viscosity in comparison with linear aerodynamic are illustrated on fig.16. Lift slope coefficient C_L^α (fig.15) is significantly higher for nonlinear model than for linear calculation near of cruise regime and fast decrease under increase of Mach number more than 0.85. The deviation of C_L^α value may be equal up to ~15% for different flow parameters.

Flow parameters (Reynolds number and lift force coefficient) also have significant effect on static aeroelasticity characteristics. Comparison of structure elasticity influence on aileron lift efficiency for different flow conditions under constant Mach number M=0.82 is shown on fig.16 as example. According to results obtained, reverse critical dynamic pressure for cruise conditions (C_L=0.5, Re=23mln) may be smaller on ~20% in comparison with one calculated for typical (C_L=0.1, Re=3mln) testing regimes of aeroelastic models in transonic wind tunnel.

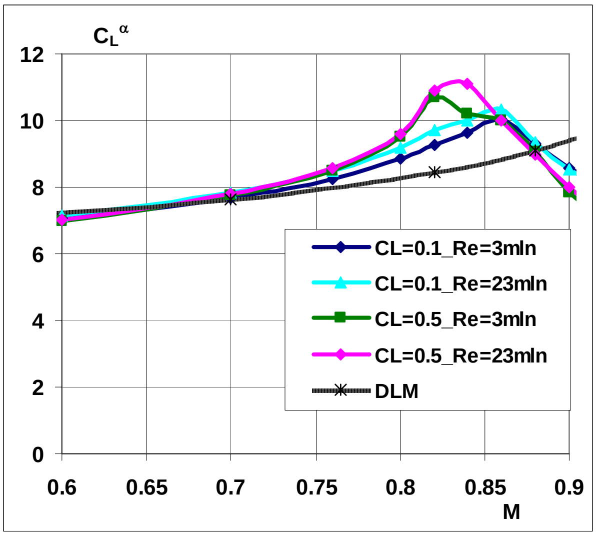

Fig.15. Influence of C_L and Re on lift slope coefficient

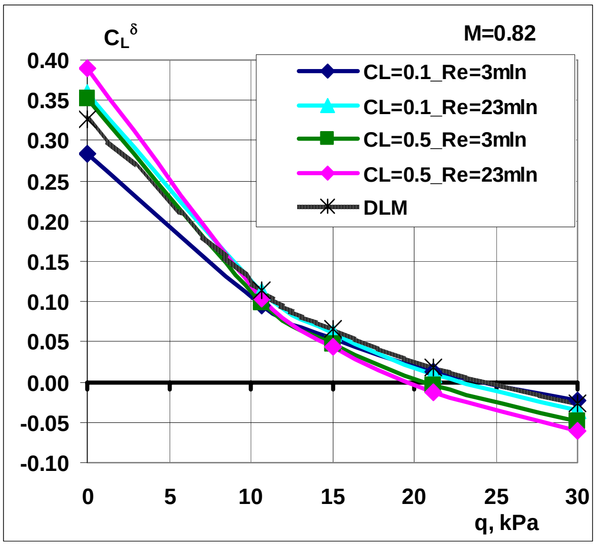

Fig.16. Influence of C_L and Re on aileron efficiency

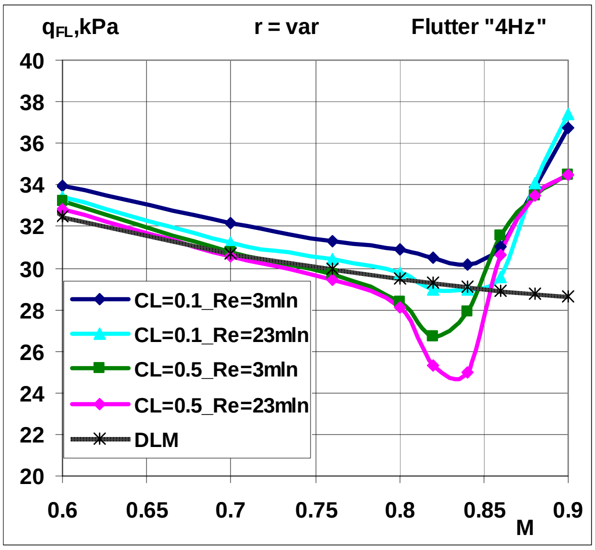

Fig.17. Influence of C_L and Re on flutter dynamic pressure

Dependence of limit flutter mode dynamic pressure q_{FL} on Mach number for different flow regimes is presented on fig.17. Calculated flutter boundary for cruise regime also appreciably below (up to 20%) in comparison with transonic wind tunnel conditions. Analysis of influence of every parameter separately (viscosity and lift slope coefficient nonlinearity) shows more input of lift force nonlinearity in accordance with influence of flow conditions on C_L^{α} (fig. 15).

The numerical investigations for given MRA shows that experimental aeroelasticity supplies (boundaries) receiving in transonic wind tunnel must be corrected due to influence of aerodynamic load (incidence angle) and viscosity (Reynolds number). Both parameters lead to decrease of aeroelasticity suppliers in total up to 20% of critical dynamic pressure under approaching to cruise conditions. It is significant result for practice.

5 Conclusion

Actual problems of safety ensuring under aeroelasticity requirements are considered in the paper in connection with some nonlinearities and uncertainties

It is shown that the flight static structural deformations can have a significant impact on the frequencies and mode shapes of elastic oscillations of extra high aspect ratio wing and can lead to appreciable decrease of flutter speed. This effect doesn't lead to onset of new dangerous flutter forms, and the decrease of flutter speed due to static deformations is not exceeded the accuracy of our analysis for modern airliners. Therefore these effects may be ignored at certification of modern airliners under flutter requirements.

Structural nonlinearities in "engine-pylon-wing" attachment units include considerable uncertainties in characteristics of flutter related to engine oscillations. This uncertainty can be increased due to FCS effect on flutter. To decrease uncertainty level without a loss in flight performance we have to complicate the computational and experimental research and to increase their volume.

Also we have shown that aeroelasticity safety margins determined experimentally in transonic WT have to refined additionally with taking into account the effect of air loads (angle of attack) and viscosity (Reynolds number). Both these parameters decrease aeroelasticity safety margin when approaching the actual (full-scale) flight regime and the sum of these effects can lead to reduction of safety margin for critical dynamic pressure up to 20%.

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