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CRUISE FLIGHT THROTTLE OPTIMIZATION BY APPROXIMATE DYNAMIC PROGRAMMING

Maya Dobrovinsky*, Joseph Z. Ben-Asher**
***Faculty of Aerospace Engineering, Technion, Haifa,**
**** Faculty of Aerospace Engineering, Technion, Haifa**

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Abstract

Range maximization for aircraft in cruise flight by the throttle control system is considered. The full problem is decomposed into two sub-problems: finding the optimal cruise speed and the transition to the optimal cruise speed from non-optimal conditions. Dynamic Programming solves the first sub-problem whereby we directly find the most economic cruise velocity. For the second sub-problem, Approximate Dynamic Programming method is proposed. Numerical example is used to demonstrate the approach.

1 Introduction

Cruise optimization has the potential for saving significant amount of fuel both for civil and military applications. As the air traffic control requires that aircraft should hold specific altitudes, the optimization problem at a constant altitude is of great importance.

Bryson [1], was probably the first to formulate the problem of cruise flight at a constant altitude in the framework of Optimal Control Theory. In his work, the objective is to optimize a given performance index. He analyzed the following cases: maximum range, minimum direct operating cost and minimum fuel with fixed arrival time. In cruise flight at a constant altitude (and constant heading), the only control variable left is thrust, which appears linearly in the equations of motion, as well as on the performance indices to be optimized; as a consequence, the Hamiltonian of the problem is also linear on the control variable, leading to a singular optimal control problem.

Many researchers have followed Bryson's steps over the past five decades ([2-6] are just a few representative examples). More recently, Pargett and Ardema [7] analyzed the problem of range maximization in cruise flight at a fixed altitude. The problem was formulated with two states - airspeed and mass - and one control - throttle setting. They show that it is a singular optimal control problem with singular arc. The Maximum Principle does not directly provide the optimal solution, so the Kelley condition is used to identify the singular arc. Alternatively, they have use Green's theorem to obtain the same results. Numerical examples, using the Boeing 747-400 aircraft, show that the fuel saving is about 7% , relative to the current constant cruise speed. Rivas and Valenzuela [8] generalized the analysis of the singular optimal control problem, by considering a general drag polar, so that compressibility effects are taken into account. Numerical results are provided for a model of a Boeing 767-300ER aircraft. The results show that compressibility effects are very important; the differences with the incompressible case are shown to be not only quantitative, but also qualitative. Precise modeling of the system is therefore of extreme importance.

The basic approach in the present paper is different from all previous works (known to the writers) on this problem. It proposes the use of dynamic programming (DP) for the cruise

optimization problem. This approach can be used in open or in closed loop. For example, if in flight after a long turn, or after altitude changing (or for any other reason), there exists a difference between the current speed and optimal speed (calculated, as will be shown later, in open loop by dynamic programming), the objective of the proposed algorithm is to reduce this difference, by implementing closed-loop throttle control in a most efficient way.
The full problem is decomposed into two sub-problems: (a) finding the optimal cruise speed, and (b) sub-optimal reaching/tracking the optimal cruise speed, from non-optimal conditions. Dynamic Programming (DP) can solve both sub-problems. However, the computation cost of dynamic programming is very high, as a result of the "curse of dimensionality". Moreover, the models of the aircraft are highly complicated, and those of the environment are not always available a priory. Therefore, for Sub-problem (a) Dynamic Programming is used, whereas for Sub problem (b) we propose the use of Approximate Dynamic Programming (ADP), which is based on Dynamic Programming, but learns on-line from its own mistakes through the reinforcement signal from the obtained performance.
An illustrative example of F-6 aircraft in cruise flight is given, whereby the obtained solution is compared with direct trajectory optimization.

2 Problem Formulation

2.1 Modeling

A cruise flight for an aircraft at a constant altitude is assumed. The point-mass equations of motion are, as follows:

$$\dot{V} = \frac{1}{m}(T(V,R)\cos\alpha - D(V,\alpha))$$
$$\dot{m} = -ff(V,R)$$
$$\dot{x} = V$$
$$mg = L(V,\alpha) + T(V,R)\sin\alpha \qquad (1)$$

where:

V- velocity,
m- mass,
x- distance,
T- thrust force,
L- lift force,
D- drag force,
ff - fuel flow,
R - throttle setting,
α- angle of attack.

The angle-of-attack is determined by the 4th (algebraic) equation, imposing a fixed altitude. Thus, there are three dynamic states, the speed (V), the aircraft mass (m) and the distance (x), and one control – the throttle setting (R). The initial values of speed, aircraft mass, and distance are given.
In this formulation, the drag is a general function $D(V,\alpha)$ of the speed and the angle-of-attack. Similarly, the lift is a general function $L(V,\alpha)$ of the speed and the angle-of-attack. The thrust $T(V,R)$, and the fuel-flow ff(V,R), are both functions of the speed and throttle setting. It is important to note that these four functions are typically quite complex, obtained in a tabular form, and are based on intensive wind-tunnel and engine altitude chamber tests. Thus, in the following modeling and simulations (unless otherwise specified), they will also be represented in a tabular form. The numerical calculations are performed by the interpolation of several arguments.

2.2 Optimization Problem

The maximal range problem consists in finding the optimal throttle control that, for a given terminal mass m_f, maximizes the following performance index:

$$P = \left[\frac{(x_f - x_0)}{(m_0 - m_f)}\right] \qquad (2)$$

Note that, under this formulation, t_f is free, and so are V_f and x_f. Alternatively, a fixed-range problem may be specified, where the consumed fuel will be minimized. Evidently, both problems are equivalent.

3 Optimal Cruise Speed

The aim of this paper is the generation of optimal throttle command for economic cruise (maximal distance for a given amount of fuel) at a given altitude. As discussed above, the optimal control problem is typically a singular one and its solution cannot be easily obtained, certainly in closed loop (a requirement for real-time applications).

Due to the complexity of the problem, a two-step approximate solution is proposed. First, a simpler optimal cruise speed search is sought by Dynamic Programming. In the second step (next section), sub-optimal throttle autopilots that reach the desired speed and track this speed are designed by Approximate Dynamic Programming. It will be demonstrated that this two-step solution approximates well the optimal solution.

3.1 Finding the Optimal Cruise Speed by Dynamic Programming

The fuel consumption per kilometer, for varying speed and mass, are used as the basis for the optimal speed determination in aircraft cruise flight. The program of an optimal cruise speeds, as a function of the mass *m,* and the altitude *H* (i.e. *V (m, H)*), is thereby constructed. The Dynamic Programming (DP) method is proposed for the optimal velocity search. Dynamic Programming is a step-by-step planning of the multistage process, whereby at each stage a single step is optimized. Continuity of the velocity profile will be enforced by the following approach. The aircraft mass is considered in discrete values (steps), between the given initial and terminal values (Fig. 1). A cost function is defined as the sum, over all steps, of the fuel expenditure per unit distance. For each value, the DP approach chooses the optimal speed that takes us to the next value while minimizing the residual cost. Throughout each step, the aircraft, starting with the flight speed under consideration, reaches and maintains the previously calculated reference speed. In this way the continuity of the velocity is guaranteed.

As a numerical example, calculations were performed for the F-6 aircraft in cruise flight at the altitude of 5000 ft (the simulated full aircraft model is given in [9]). To this end, a six-degree-of-freedom (6DOF) simulation code has been employed. All aerodynamic coefficients, thrust, fuel consumption, etc., were used in a detailed tabular form. The initial and terminal masses were 32000 lb and 27000 lb, respectively. The incremental mass value is 1000 lb. The feasible speed range is between 540 ft/s to 710 ft/s, with incremental values taken every 10 ft/s. As a first step, the (terminal) optimal speed for the minimal value $m_{n.}$ is evaluated. The fuel expenditure per unit distance I_n is minimized by a static optimization problem:

$$I_n = ff / V \qquad (3)$$

Fig. 2 shows that, for 27000 lb, the optimal speed is V=610 ft/s.

After the cost at the end point is calculated, intermediate calculations are initiated. The calculation process is performed in the backward direction. Thus, the speed transition during fuel usage of 1000 lb is evaluated for the transition from State *(i-1)* to State *(i)* (the state (i-1) being defined by mass *W(i-1)=W(i)+1000 lb*). Different speeds for *(i-1)* are considered, and the corresponding transitions to state *(i),* with its previously calculated speed is evaluated. A simplified throttle autopilot for the speed control (standard proportional + integral) is used during transitions.

For each speed transition, the incremental cost function is calculated by the following formula:

$$r = \frac{\Delta Fuel}{\Delta X} \qquad (4)$$

where:

ΔX - aircraft distance during transition.

$\Delta Fuel$ - amount of fuel used during transition.

Finally, the optimal speed is selected by the minimization of the residual cost:

$$I_{n-1} = r_{n-1\to n} + I^*_n \qquad (5)$$

This process continues until the maximal (initial) mass is reached (Fig. 3). The resulting optimal cruise speed as function of mass is shown in Fig. 4.

3.2 Optimal program based on quasi-static flight

To check the validity of the optimal program *V(m)*, calculated by DP, it is compared with the standard quasi-static flight calculations.
The specific fuel consumption is defined as

$$c = \frac{\dot{m}}{T} \tag{6}$$

Assume (for trimming purposes only) $\alpha=0$, and let $\beta = D/L$, then

$$T = D = \beta L = \beta W; \quad W \equiv mg \tag{7}$$

The fuel used per unit distance becomes:

$$\frac{dm}{dx} = \frac{\dot{m}}{V} = \frac{cT}{V} = \frac{c\beta W}{V} \tag{8}$$

As the aircraft travels a distance *dx*, its mass changes from *m* to *m*+*dm*, where

$$dm = -\left(\frac{c\beta W}{V}\right)dx \tag{9}$$

The specific range x_s is defined as the distance traveled per unit mass of fuel used, thus

$$x_s \equiv -\frac{dx}{dm} = \frac{V}{c\beta W} \tag{10}$$

The optimal speed can be found such that x_s is maximized. Note that the results of these calculations are based only on trimmed state (quasi-static check) and, therefore, are approximate. Fig. 5 compares (for F-6 at 5000 ft) the optimal speed program calculated by DP and by the quasi-static check formula. As shown, the optimal speeds calculated by DP method are higher than the quasi-static optimal speeds. The DP method provides the more accurate results, since it takes into account the fuel expenditure during transitions. This fact will be substantiated below by comparing the results with the numerical optimal results.

4 Cruise Speed Transition

In this step, the problem consists in constructing a closed loop control law for the throttle setting, when a deviation of the cruise speed from the optimal (pre-planned) speed is present. That may be the case when, for example, the aircraft needs to increase its speed after a long turn (in patrolling aircraft, this could happen very frequently), or after a long climb. The problem here is how to do it optimally.

4.1 Formulating the Speed Transition Problem

Rewriting the equations of motion for cruise at constant altitude:

$$\dot{V} = \frac{1}{m}(T(V,R)\cos\alpha - D(V,\alpha))$$
$$\dot{m} = -ff(V,R)$$
$$\dot{x} = V$$
$$mg = L(V,\alpha) + T(V,R)\sin\alpha \tag{11}$$

Define the right-hand side of first equation as:

$$\frac{1}{m}(T(V,R) - D(V,\alpha)\cos\alpha) \equiv f_1 \tag{12}$$

and the right-hand side of the second equation as:

$$-ff(V,R) \equiv f_2 \tag{13}$$

The optimal problem for the speed transition to the optimal speed *V(m)* consists in minimizing the cost

$$I = I_1(1-\varepsilon) + \varepsilon \cdot I_2 \tag{14}$$

where

$$I_1 = \int_0^{t_{trans}} f_2 dt \tag{15}$$

$$I_2 = \int_0^{t_{trans}} (V(t) - V(m))^2 dt \tag{16}$$

t_{trans} is the transition time and ε is a weighting coefficient.
The first term represents the consumed fuel, whereas the second term determines the difference between the actual speed and the

desired speed. The problem consists in finding the throttle setting *R* (preferably in feedback form) that provides the minimal value for the cost (14).

4.2 Solving the Speed Transition Problem by DP

The first approach to this sub-problem, as in the previous sub-problem, may be Dynamic Programming (Fig. 6). Thus, when a deviation of the speed V from the optimum speed of flight is identified, a search for an optimal throttle setting is performed. This throttle setting should provide the minimum value for the cost (14), which becomes, in discrete form

$$I = (1-\varepsilon)\cdot\sum_j f_2\Delta t + \varepsilon\cdot\sum_j (V - V_{req}(m))^2 \Delta t \tag{17}$$

The optimal cost is a function of the 2-dimensional state *(V,m)* (also called "optimal return function"). Similar to the above (5), the optimal throttle is selected by the minimization of the residual cost. As this is a standard well-known technique, it will not be presented here (see [9] for the details).

For example, F-6 aircraft flying at constant altitude $h=5000ft$ is considered. The initial speed is $V_0=500$ *ft/sec*, and the initial mass m_0=29500 lb. The quasi-static optimal speed for these altitude and mass is $V_{req}=$ *610 ft/sec* (Fig. 5), so an initial deviation from the optimal speed of 110 ft/sec is present. (Similar results have been obtained for the DP optimal speed of 630 ft/sec.) The design parameter is ε=0.5. Dynamic Programming throttle control was used to smoothly decrease this deviation.

The results are compared with a regular (proportional) controller for throttle of the form:

$$R = k\cdot(V_{req} - V) \tag{18}$$

The throttle setting and the flight velocity, for both autopilots) are presented in Fig. 7 and Fig. 8, respectively. The comparison of the dynamic programming throttle controller with the regular (proportional) law shows an advantage in weight after a throttle maneuver: the difference in weight is 15 lb. Since the total fuel consumption for this transition is about 100 lb, it results in 15% fuel saving.

The dynamic programming method used here required a predefined transition time (40 sec in the above example), and a backward calculation process to be started from this time. In general, it is difficult to know the optimal time for the transition from current speed to optimal speed and we may need to search for different values. A second limitation is that the computation cost of dynamic programming is very high; a grid for speeds and masses should be defined, and the calculations are performed for each combination. As a result, we are faced with "curse of dimensionality" problem. To resolve the problem, an Approximate Dynamic Programming method is considered next.

4.3 Solving the Speed Transition Problem by SARSA

SARSA (State-Action-Reward-State-Action) controller [10] aims in finding optimal throttle control to minimize the cost (14). The penalty function for each step of transition is defined as follows:

$$r = (1-\varepsilon)\cdot\Delta Fuel + \varepsilon\cdot(V - V_{ref})^2 \tag{19}$$

The scheme of the SARSA algorithm is presented in Fig. 9.

SARSA is based on learning the state-action value function (Q-function), which is the residual cost, obtained after using the control (action) $a_t=a(t)$ for the state $s_t=s(t)$. It can be employed either in real time using flight data, or off-line, using simulated data. During the learning process, SARSA does not assume that the optimal policy is imposed after a one-time step. The update rule is:

$$Q(s_t,a_t) = Q(s_t,a_t) + \lambda(r_{t+1} + \gamma Q(s_{t+1},a_{t+1}) - Q(s_t,a_t)) \tag{20}$$

where r_{t+1} is the cost of transition from state t to state $t+1$, chosen according to a greedy policy (i.e. minimizing r_{t+1} without considering the future). The design parameter λ is the learning rate, and γ is a discount factor (unity in our case).

SARSA uses the concept of learning in episodes, in which there is a terminal state, and the episode terminates when this terminal state is reached.
In our case, the state space is divided into a finite number of intervals for the mass and the speed. Q values are initialized for all states to some arbitrary values. For each episode, the following sequence is repeated:

- Initialize time t, start the episode with initial speed
- Select throttle action a_t via greedy exploration on r_{t+1}
- Simulate the action a_t, and let the next state be s_{t+1}
- Select the action a_{t+1} via greedy exploration
- Update Q-factor $Q(s_t,a_t)$ using (20)
- If s_{t+1} is a terminal state, terminate the current episode; otherwise, continue within the episode.

For example, the F-6 cruise flight at 5000ft altitude is reconsidered. The initial speed is 600ft/s, and the optimal speed, for this aircraft mass (32000 lb.), is 660ft/s.
First we set $\varepsilon=1$. Thus only the difference between the current and the reference speed appears in the penalty function:

$$r=(V-V_{ref})^2\cdot\varepsilon \quad (21)$$

Fig. 10, Fig. 11 and Fig. 12 present the *Q*-function, calculated after 1 episode, 2 episodes, and 10 episodes, respectively. Note that this is presented for a given initial mass (32000 lb). As episodes number increases, the *Q*-function is calculated more accurately.
By changing ε, the relation between fuel used and difference between reference speed and current speed, we get different penalty functions. Fig. 14 and Fig. 15 present the Q-functions and the controls for two other value of ε: 0.4 and 0.8, respectively. Clearly, the results depend on this design parameter.

4.4 Comparison with Optimal Numerical Results

In order to validate the current approach, some reference solutions have been obtained using a Pseudo-Spectral (PS) method. PS optimal control is a direct computational method for solving complex nonlinear optimal control problems. The PS method approximates the state and control variables by a set of polynomials with domain over the entire elapsed time. The states and controls are represented by discrete values at N+ 1 points. These points are calculated numerically**.** In this research work the code GPOPS [11] has been used.
The following are the initial conditions:

$$V_0=500\,{ft}/{s}$$

$$m_0=32000\,lb$$

$$x_0=0\,ft$$

Figs. 16 -18 compare the mass, velocity and the throttle, respectively, for the PS numerical solution and the DP with $\varepsilon=1$. As seen from the graph, the solutions for both methods are very close.
<u>Remarks</u>:

a. The small difference between the results is inevitable, because the model used in the PS solution is an approximated model (for the drag, lift, fuel flow, and thrust). It is very complicated, and difficult in implementation, to use the full aircraft model in the GPOPS tool.
b. As a result, the optimal speed from DP is 650 ft/sec, whereas GPOPS predicts 675 ft/sec. The "true" optimal cruise lies between these values. Note that the quasi-static optimization (Section 3b) yields an even slower cruise speed (640 ft/sec).

To estimate the benefits of optimal cruise, a constant mean-value speed of 650 ft/sec is also considered. After 1 hour and 12 min. (4360 sec) the mass for constant speed flight reaches 28000 lbs. At this time, DP and GPOPS masses are about 28300 lbs. The fuel saving by the optimal solutions is, therefore, 7.5% (similar to the results of [7]). Moreover, the cruise range under constant speed (consuming 4000 lb. of fuel) is 2.82 10^6 ft. For the same amount of fuel, the DP and GPOPS solutions cruise ranges reach 2.96 10^6 ft. - an increase of 4.8%(!). This result clearly justifies the effort of using optimal policies.

5 Conclusions

Range maximization in cruise flight by the throttle control system is a singular optimal control problem. Rather than employing singular control techniques, the problem was solved by Dynamic Programming and Approximated Dynamic Programming. To overcome the curse-of-dimensionality, the full problem was decomposed into two sub-problems: finding the optimal cruise speed and the transition to the optimal cruise speed from non-optimal conditions. Dynamic Programming solves the first sub-problem off line to produces a flight plan (velocity as a function of altitude and mass). Approximate Dynamic Programming method, working on-line, is proposed for the second sub-problem. After a few flights, the aircraft will learn what the best policy is and will adjust itself to it. In case of changes in aircraft properties, or external environment, the aircraft will keep adapting its policy. The flight trajectory will become near optimal in fuel usage**.**

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Contact Author Email Address

mailto: yossiba@technion.ac.il

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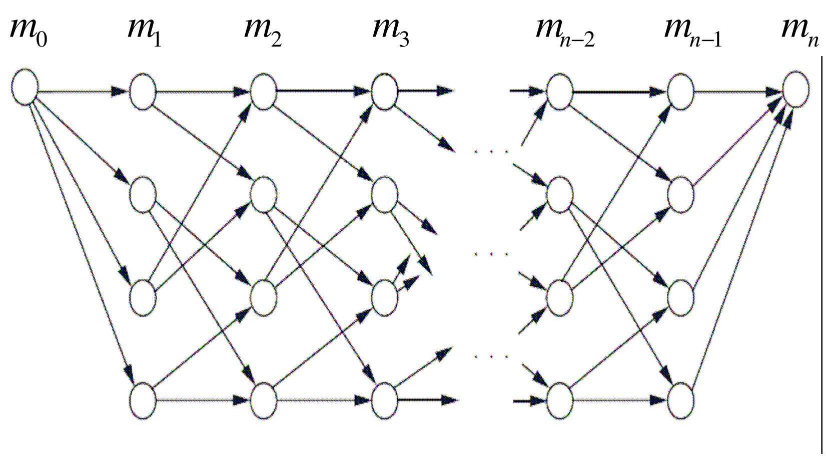


Fig. 1. Dynamic Programming process for Sub-problem (a)

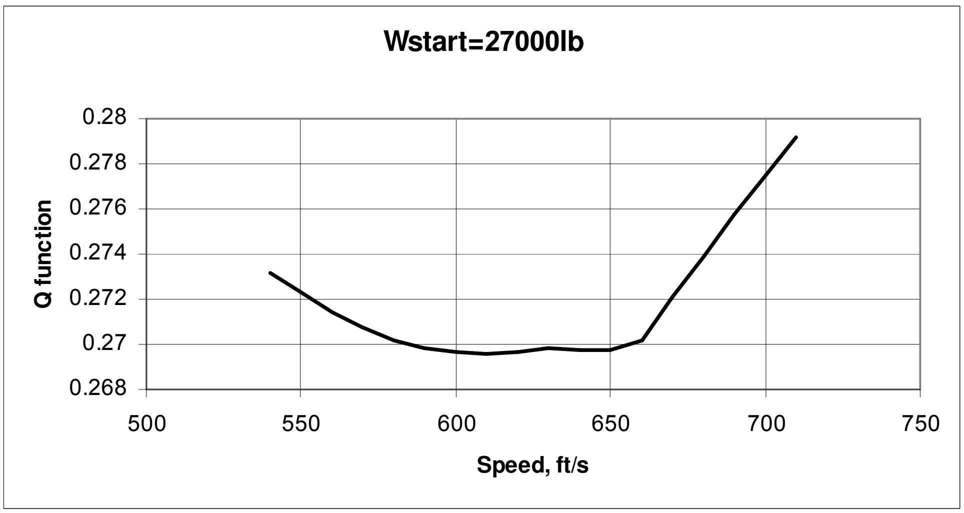
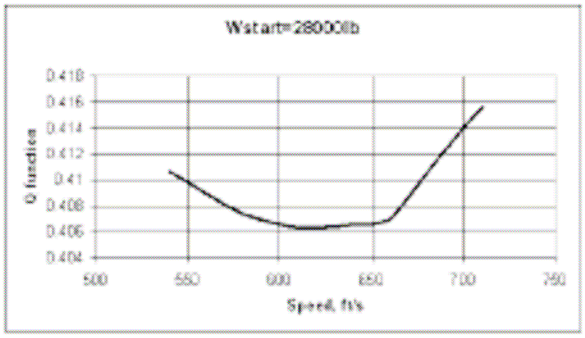
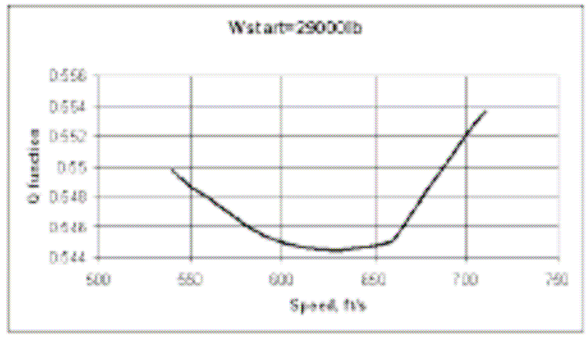


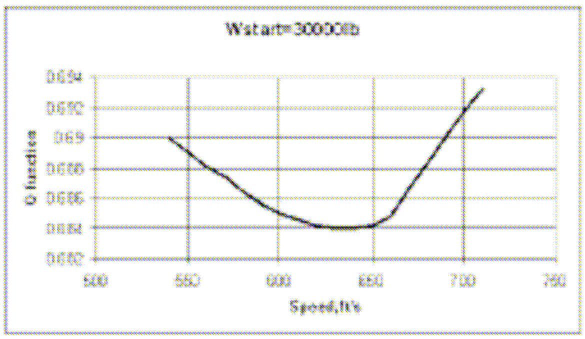
Fig. 2: Cost value at the terminal state



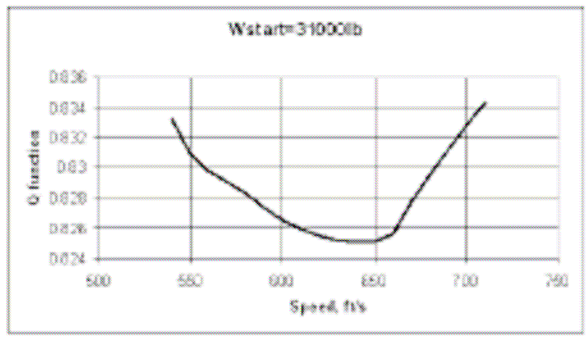
(a) Cost function for step (n-2) different start speed: Start conditions: mass/Weight = 28000 lb, V=540 - 710ft/s



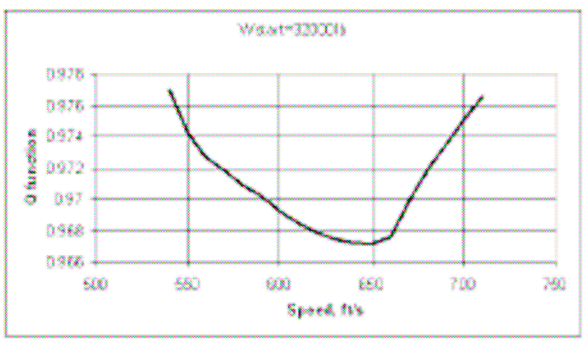
(b) Cost function for step (n-3) for different start speeds: Start conditions: mass/weight = 29000 lb, V=540 - 710ft/s



(c) Cost function for step (n-4) for different start speed: Start conditions: mass/weight = 30000 lb, V=540 - 710ft/s



(d) Cost function for step (n-5) for different start speed: Start conditions: mass/weight = 31000 lb, V=540 - 710ft/s



(e) Cost function for step (n-6) for different start speeds: Start conditions: mass/weight = 32000 lb, V=540 - 710ft/s, V^*_{n-5} = 650 ft/s

Fig. 3: Cost value at intermediate states

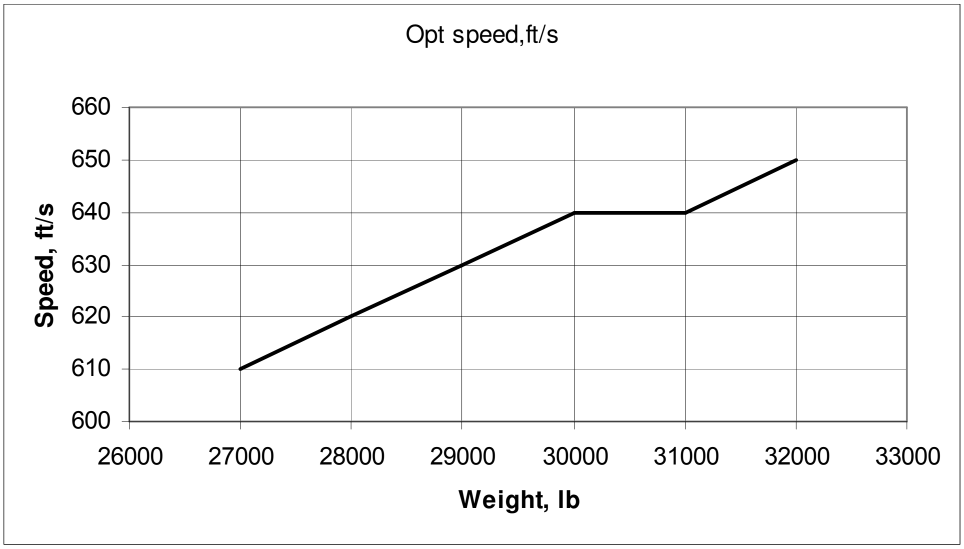


Fig. 4: Optimal speed as function of mass

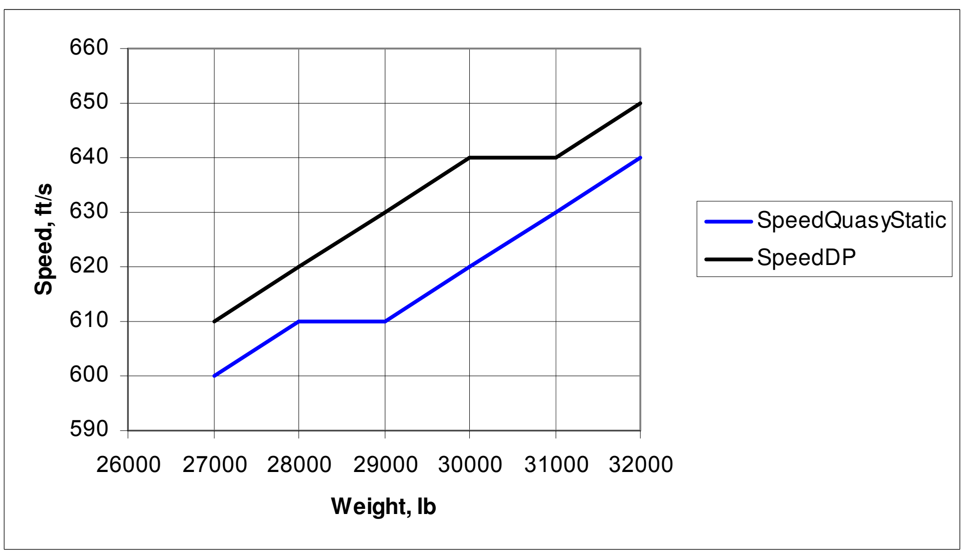


Fig. 5: Optimal speed for DP and quasi static check

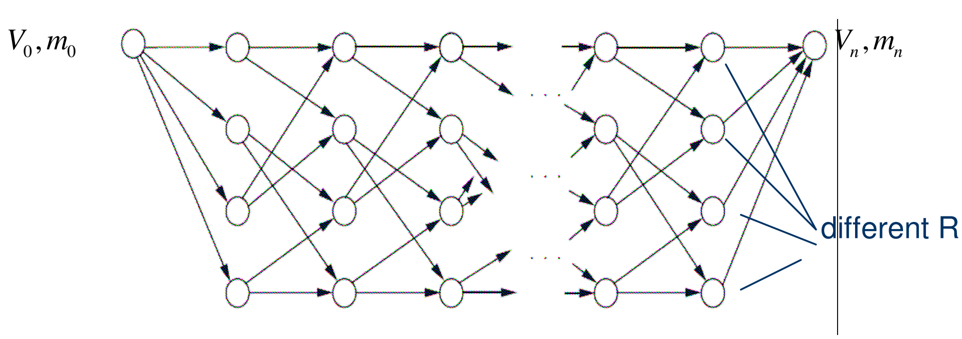


Fig. 6. Dynamic programming process for Sub-problem (b)

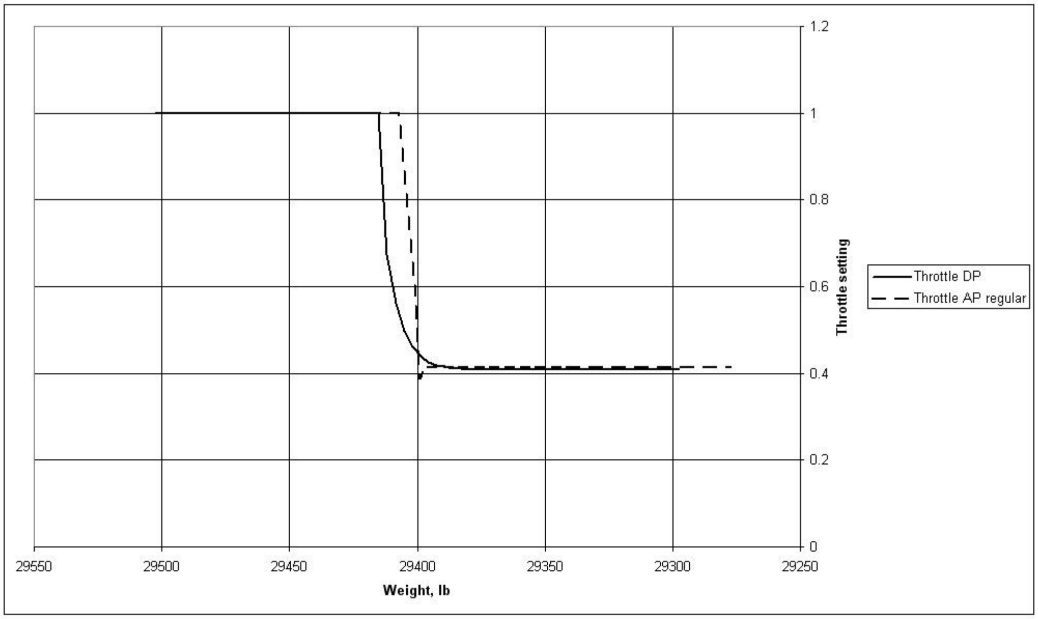


Fig. 7. The comparison between throttle settings of two types of autopilot

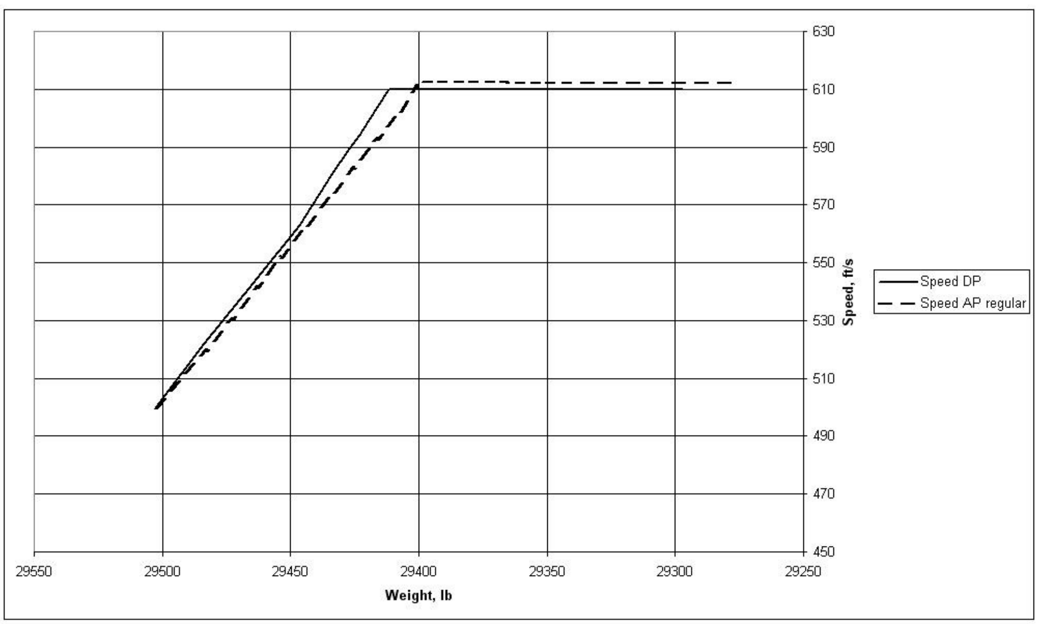


Fig. 8. The comparison between fuel used by two types of autopilots

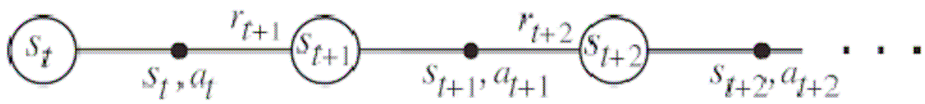


Fig. 9 SARSA scheme

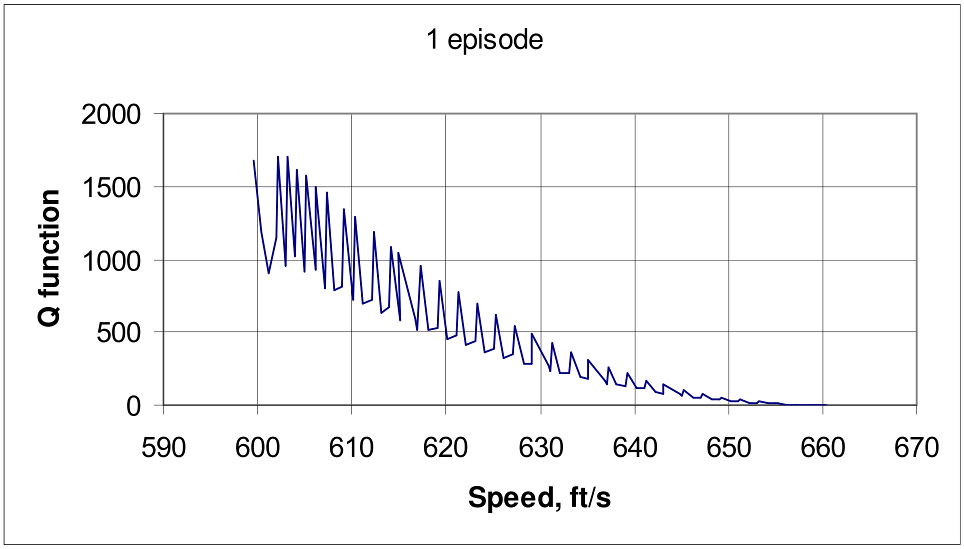


Fig. 10: Q-function for transition from V=600 ft/s to V=660ft/s; 1st episode

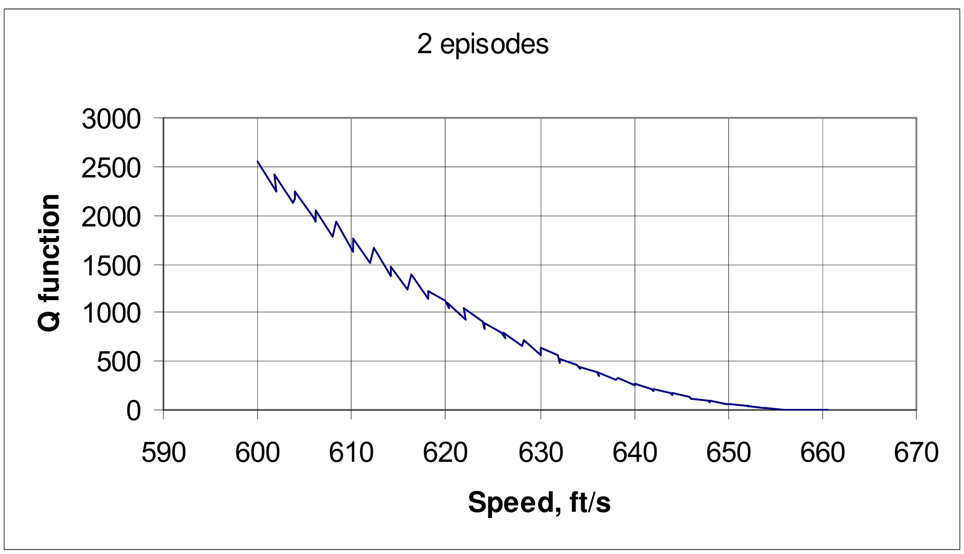


Fig. 11: Q-function for transition from V=600 ft/s to V=660ft/s, 2nd episode

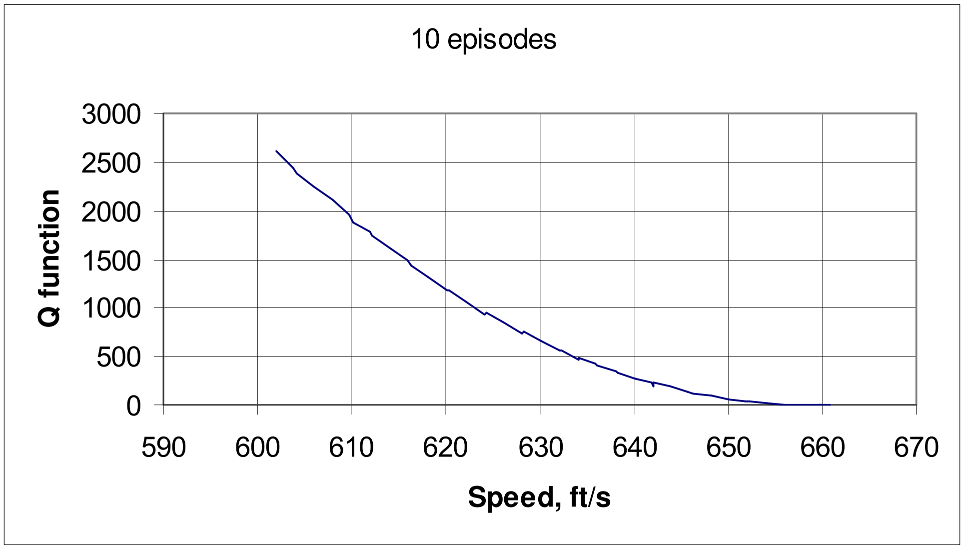


Fig. 12: Q-function for transition from V=600 ft/s to V=660ft/s; 10th episodes

The control after 10 episodes is presented in Fig. 10:

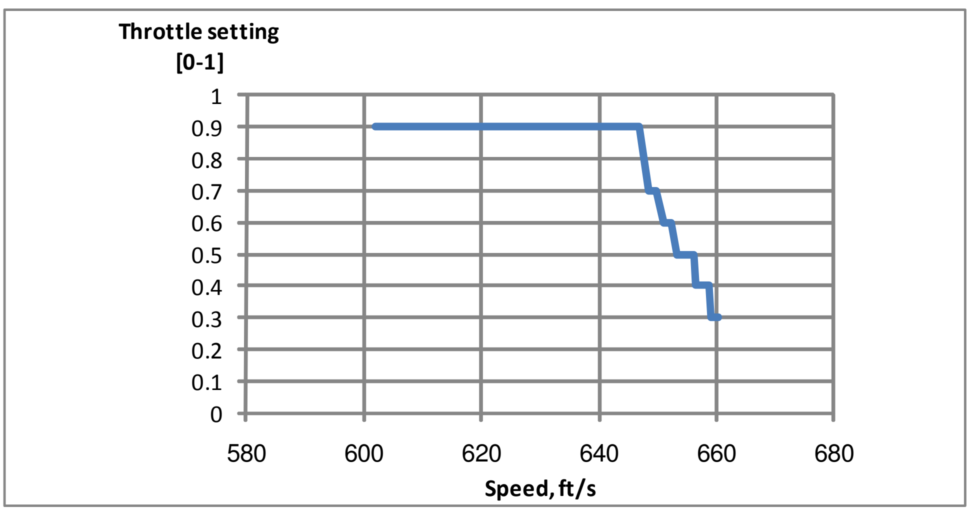


Fig. 13: Control as function of speed for transition from V=600ft/s to V=660ft/s

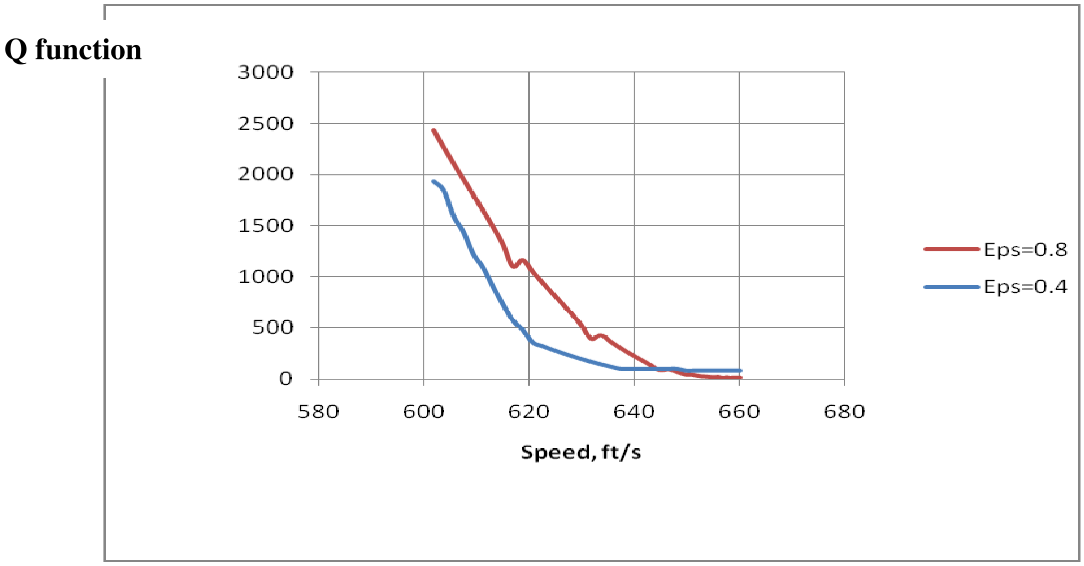


Fig. 14: Q-function dependence on ε

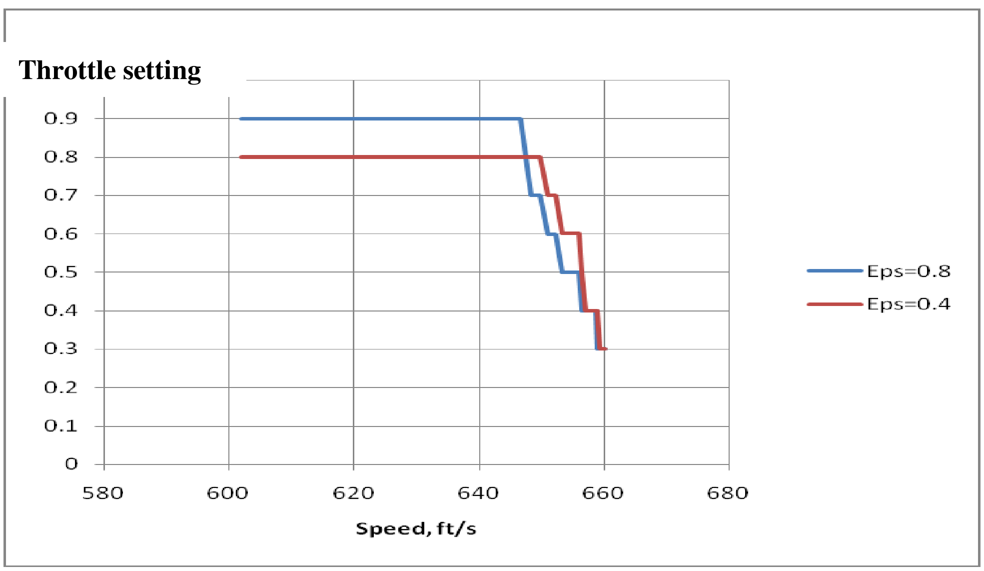


Fig. 15: Control as function of ε

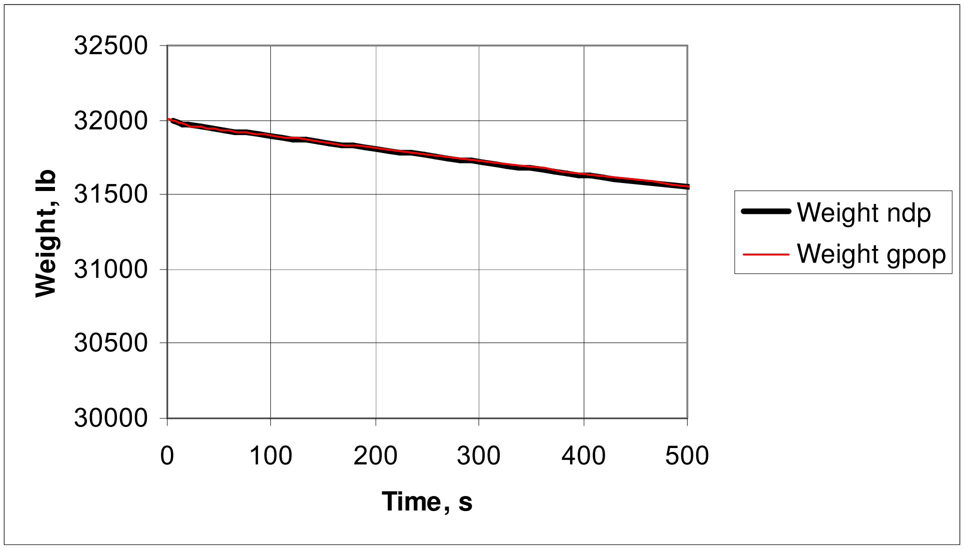


Figure 16 Weight calculated by GPOPS versus DP

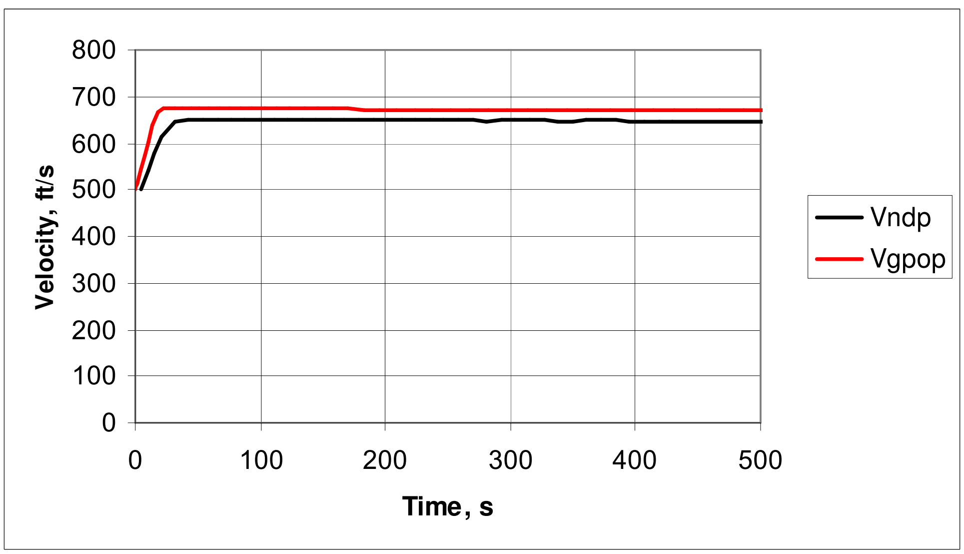


Figure 17: Velocity change with respect to time

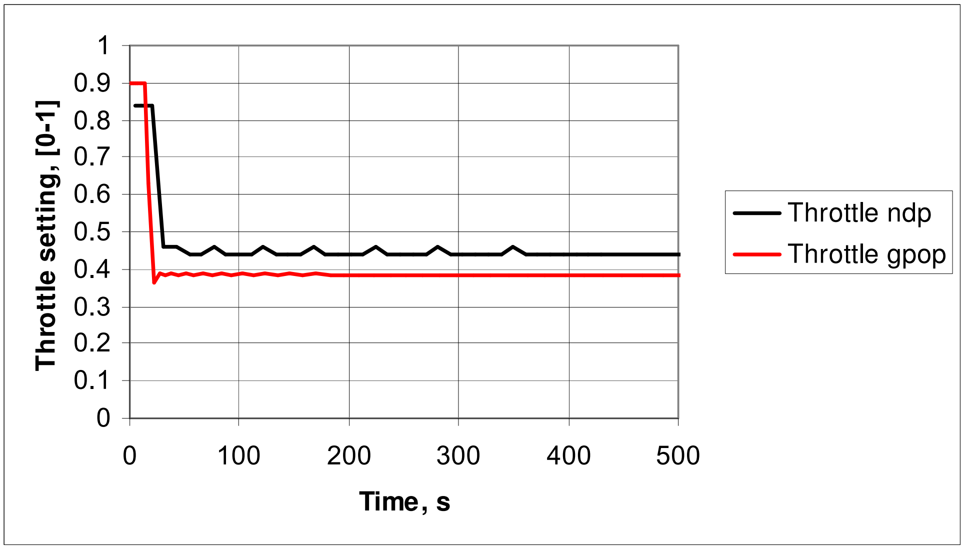


Figure 18: Throttle GPOPS versus DP with respect to time