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A TIME SERIES MODEL OF ANGLE OF ATTACK AND PITCH RATE FOR A WIDE RANGE OF ALTITUDE AND VELOCITY

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Abstract

This paper presents a dynamic time-series model representing the longitudinal attitude movement of aircraft for a wide range of altitude and velocity. The model consists of a linear time-series equation of angle of attack and that of pitch rate, with coefficients depending on air density and aircraft velocity. It has been developed based on the results of various flight simulations over a wide range of altitude, velocity, and maneuvering movement, using the 6-DOF (six-degrees-of-freedom) nonlinear model of the aircraft. The model is nonlinear in parameters, which have been identified with nonlinear optimization to minimize the same cost function as that used in predictive control. Thus, the model is suitable to predictive control of longitudinal movement. The model accuracy with the 6-DOF model as the reference is considered satisfactory.

Nomenclature

m : aircraft mass
g : gravity acceleration
ρ_a : atmosphere (air) density (dependent on altitude)
V_a : aircraft velocity
F_x, F_y, F_z : aerodynamic force components in the body frame
T_h : engine thrust
L, M, N : aerodynamic moment components in the body frame
U, V, W : velocity components in the body frame
$P, p; Q, q; R, r$: roll rate; pitch rate; yaw rate
ϕ, θ, ψ : Eiler angles (roll, pitch, yaw)
α, β : angle of attack, side-slip angle
I_x, I_y, I_z, J_{xz} : moment of inertia and product of inertia of the aircraft in the body frame
δ_{th}, δ_e : engine thrust command, elevator deflection
h : altitude (height)
$\{\cdot\}^T$: transposed vector or matrix
t_k : time in sampling interval (0.2 s)
$\Delta x(k, k-i)$: difference value of a variable x between two sampling instants of $t_k = k$ and $t_k = k-i$, where $k, i = 1, 2, \cdots$ and $k \geq i$.
$\Delta x(k)$: the same as $\Delta x(k, k-1)$ for $x :\neq \delta_e$, or $\Delta x(k, k-0.5)$ for $x := \delta_e$
$\sigma\{x\}$: standard deviation of a variable x

1 Introduction

To achieve high performances in flight control of an aircraft, it is desirable to apply some model-based control scheme. Model-based predictive control (MBPC), also known as receding-horizon control (RHC), is widely applied because it can incorporate various constraints and its parameters can be tuned fairly easily in an intuitive way.

The aircraft motion is described most accurately by the six degrees-of-freedom (6DOF) nonlinear dynamic model [4], as far as aerodynamic coefficients and other parameters are accurately determined. In some studies on MBPC applications, this nonlinear model is computed online to obtain optimal control inputs [2, 5]. However, online computation of the nonlinear model was reported to suffer heavy computational burden [2, 3]. Moreover, it sometimes involves numerical problems [3]. In addition, it is difficult to apply linear control techniques to systems described by nonlinear models.

Because of these disadvantages, many studies on MBPC applications use a linear state-space model for online computation [1, 6]. In most of the studies, a linear model is obtained by linearizing the nonlinear model at a certain trim point, i.e., at a certain flight condition. When the model is applied to different flight conditions, it becomes necessary to have multiple linear models, each trimmed at each flight condition.

The above linear model, or a set of multiple linear models, has its drawbacks. Firstly, the linear model represents small perturbations from a trimmed flight, steady-state flight with constant velocity. Large maneuvering movements are out of scope of the model.

Secondly, in order to cover a wide range of flight conditions, a large number of linear models, i.e., a large number of sets of model parameters, are required. Let n denote the number of model parameters at each trimmed point, and m denote the number of trimmed points, then the total number of parameters becomes $n \times m$. This number could be quite large, making it rather difficult to tune or modify parameter values by using actual data. On the other hand, Keviczky, et al. [3] reported that using multiple models in online computation may result in a significant computational overhead coming from the need for interpolation over different linear models.

To overcome the drawbacks of both nonlinear and linear models, this paper presents a new linear time-series model representing longitudinal attitude movements of an aircraft. The model is linear in variables: angle of attack (α), pitch rate (q) and elevator deflection (δ_e). The coefficient of each time-series term is expressed by a nonlinear continuous equation of air density (ρ_a) and aircraft velocity (V_a) with parameters to be identified. The model is nonlinear in these parameters.

The features and advantages of the model are described below:

A1 One can design various linear control systems such as MBPC, robust control, etc., by using this model, because the model is linear in variables and because the coefficients can be set constant at each control instant by setting all the ρ_a and V_a values at the current value.

A2 The model can be applied to a wide range of flight conditions and to large maneuvers, because the model has been identified by using simulation results of these flights.

A3 The model is most suitable to MBPC, because the model identification is designed to minimize the same cost function as that used in MBPC.

The author previously proposed a control of flight trajectory for a fixed-wing aircraft [7]. The control mainly employs PID (Proportional, Integral, and Differential) control scheme. In order to achieve better control performances, however, replacing the PID control with MBPC has been considered desirable. The model presented here will be used in the MBPC in the proposed control.

2 The 6DoF nonlinear dynamic model and linearized model

Considering the relative motion of the body frame of fixed-wing aircraft against the inertial frame, the 6DoF nonlinear dynamic model is expressed as [4]:

$$F_x = m(\dot{U} + QW - RV + g\sin\theta) - T_h \quad (1)$$
$$F_y = m(\dot{V} + RU - PW - g\cos\theta\sin\phi) \quad (2)$$
$$F_z = m(\dot{W} + PV - QU - g\cos\theta\cos\phi) \quad (3)$$
$$\bar{L} = \dot{P}I_x - \dot{R}J_{xz} + QR(I_z - I_y) - PQJ_{xz} \quad (4)$$
$$M = \dot{Q}I_y + PR(I_x - I_z) + (P^2 - R^2)J_{xz} \quad (5)$$
$$N = \dot{R}I_z - \dot{P}J_{xz} + PQ(I_y - I_x) + QRJ_{xz} \quad (6)$$
$$\dot{\phi} = P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta \quad (7)$$
$$\dot{\theta} = Q\cos\phi - R\sin\phi \quad (8)$$

The aerodynamic forces and moment of longitudinal movements (F_x, F_z, M) largely depend on dynamic pressure $\bar{q}$ $(= 0.5\rho_a V_a^2)$, V_a, and δ_e.

Most of model-based flight control algorithms use a linear state-space model. The linear model is derived by linearizing the nonlinear model of (1) to (8) at a certain steady state

[4]. The linear state-space model for longitudinal movements is expressed by

$$\dot{\vec{X}} = A\vec{X} + B\vec{Y} \qquad (9)$$
$$\text{where } \vec{X} = [u \ \ \alpha_d \ \ \theta_d \ \ q]^T, \qquad (10)$$
$$\vec{Y} = [\delta_{thd} \ \ \delta_{ed}]^T \qquad (11)$$

and where the components of $\vec{X}$ and $\vec{Y}$ are deviations from the steady-state values of $[U \ \ \alpha \ \ \theta \ \ Q]^T$ and $[\delta_{th} \ \ \delta_e]^T$, respectively; A and B are matrices of constant values.

The above linear model is valid only for small perturbations from a steady state. This suggests that, in order to control the flights of large maneuvers, it is desirable to use a different model which covers such flights. When MBPC is employed, a time-series model with an appropriate sampling interval is required.

3 Linear time-series model for longitudinal attitude movements

The aerodynamic forces and moments acting on the aircraft depend on altitude (or air density ρ_a) and velocity V_a, making it necessary to express the coefficients of a linear model to be functions of these variables. These functions are desirable to be simple, having as small parameters as possible.

Fig. 1 and 2 shows the step responses of α and q for various values of altitude (or ρ_a) and V_a. It shows that dynamics and steady-state gains change with ρ_a and V_a nonlinearly. From the step responses, the sampling interval of α and q were set at 0.2 s. On the other hand, that of the manipulated variable δ_e was set at 0.1 s so that the controller could act quickly against abrupt disturbances.

Preliminary identification experiments were done to examine the accuracy of various different models. The results show that models with variables $\Delta\alpha$, Δq and $\Delta\delta_e$ are better than models with α, q and δ_e in accuracy improvement. Thus, a linear time-series model was determined as

$$\Delta\alpha(k) = \sum_{i=1}^{n_{\alpha q}} \{a_{\alpha\alpha i}\Delta\alpha(k-i) + a_{\alpha q i}\Delta q(k-i)\} + \sum_{i=1}^{n_e} b_{\alpha i}\Delta\delta_e(k-0.5(i-1)) \qquad (12)$$

$$\Delta q(k) = \sum_{i=1}^{n_{\alpha q}} \{a_{q\alpha i}\Delta\alpha(k-i) + a_{qqi}\Delta q(k-i)\} + \sum_{i=1}^{n_e} b_{qi}\Delta\delta_e(k-0.5(i-1)) \qquad (13)$$

where
$$a_{xyi} = f_{xyi}\rho_a^{g_{xyi}}(0.01V_a)^{h_{xyi}} \qquad (14)$$
$$b_{xi} = f_{xei}\rho_a^{g_{xei}}(0.01V_a)^{h_{xei}} \qquad (15)$$
$$x,\ y := \alpha \text{ or } q, \qquad (16)$$

Let eqns. (12, 13) be named as the α model and the q model, respectively. k denotes time in sampling interval of 0.2 s. f_{xzi}, g_{xzi} and h_{xzi} ($z := y$ or e) are parameters to be identified with the data of flight simulations or actual flights. Nonlinear identification is necessary as the model is nonlinear in parameters. $n_{\alpha q}$ and n_e (≥ 1) are model orders. Smaller model orders are preferable as far as the model accuracy is satisfactory.

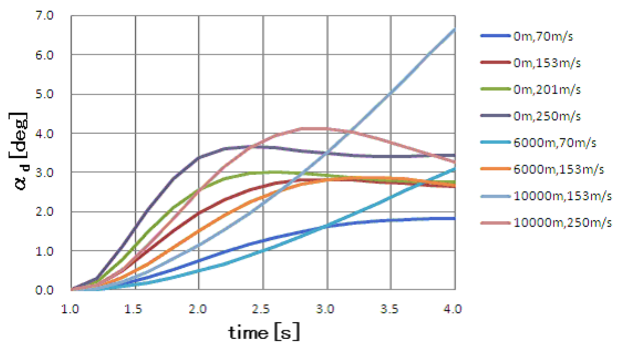

Fig. 1 Step responses of α_d (deviation of α); δ_e deviation: -1 deg at t =1 s

4 Parameter identification

4.1 Flight simulation for identification

The data for identification should include a wide range of flights. The following three groups of data (Data-C, Data-S and Data-P) were obtained from flight simulations by using the 6 DOF nonlinear model. The aircraft was chosen to be F-

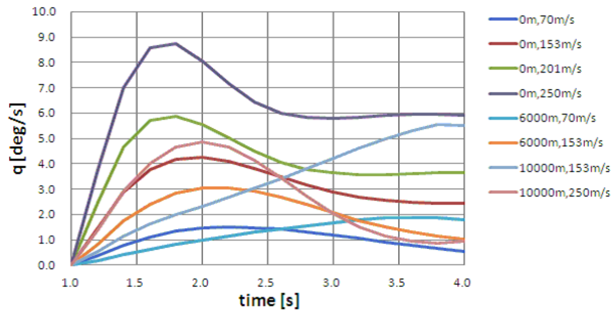

Fig. 2 Step responses of q;
δ_e deviation: -1 deg at t =1 s

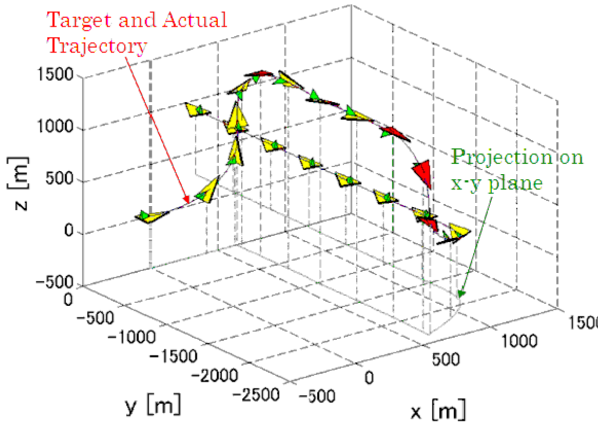

Fig. 3 Flight simulation for Data-C

16 fighter, considering high maneuverability and availability of aerodynamic data [4].

Data-C : two controlled-flight simulations along the same target trajectory starting at 0 m and at 6000 m above sea level. The simulation starting at 0 m is shown by Fig. 3. It includes level, pull-up, vertical upward and downward, and downward spiral flight with wind disturbance.
Remark: Large non-symmetric (lateral and/or directional) movements are irrelevant and removed from Data-C, whereas small to medium non-symmetric movements are included so that the identified model of longitudinal movements can be applied to various non-symmetric maneuvers.

Data-S : step response simulations against a change of elevator deflection δ_e at various altitudes and velocities, as shown by Figs. 1 and 2.

Data-P : triangular-pulse response simulations against a change of elevator deflection δ_e at various altitudes and velocities, as shown by Figs. 4 and 5.

In the above data, physical values change in the following ranges:

Altitude above sea level: $0 \sim 10000$ [m]
(air density ρ_a: $0.41 \sim 1.23$ [kg/m^3])
Velocity V_a: $35 \sim 250$ [m/s]
Angle of attack α: $-5.8 \sim 51.7$ [deg]
Pitch rate q: $-15.1 \sim 33.4$ [deg/s]
Elevator deflection δ_e: $-16.9 \sim 24.9$ [deg]

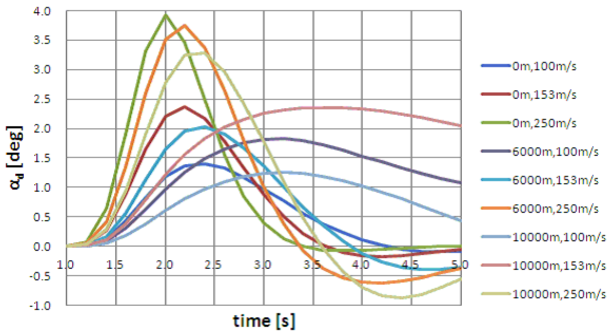

Fig. 4 Triangular pulse responses of α_d (deviation of α);
δ_e deviation: 0 (1 s) $\rightarrow$ -2 deg (1.5 s) $\rightarrow$ 0 (2 s)

The number of data for identification is 751. Each data includes more than ten time-series values of each variable: α, q, δ_e, ρ_a and V_a.

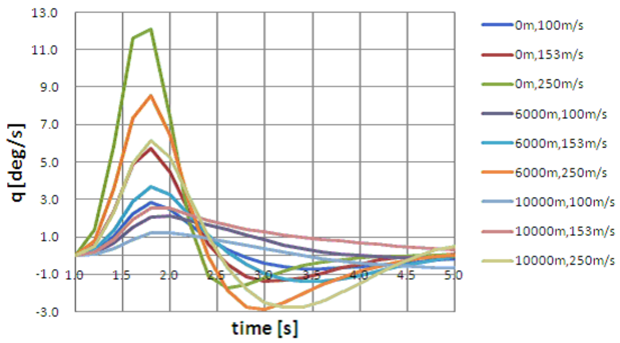

Fig. 5 Triangular pulse responses of q;
δ_e deviation: 0 (1 s) $\rightarrow$ -2 deg (1.5 s) $\rightarrow$ 0 (2 s)

4.2 Identification procedure

In MBPC in general, the controller calculates optimal values of manipulated variables which minimize a cost function of future control errors obtained from the predicted values of controlled variables over the so-called prediction horizon. The identification procedure is desirable to minimize the same cost function. Here, from Fig. 1, the prediction errors around 0.8 s $\sim$ 1.0 s ahead were considered appropriate to minimize. Let $\hat{x}(k|k-m)$ denote the value of a variable $x(k)$ predicted m steps earlier by using (12). The cost function $F_{c\alpha}$ for the identification of the α model has been chosen to be

$$F_{c\alpha} = \sum_{n=1}^{N} w_n \sum_{j=0}^{1} \{\hat{\Delta\alpha}(k_n-j, k_n-5|k_n-5) - \Delta\alpha(k_n-j, k_n-5)\}^2 \quad (17)$$

where N is the number of data, 751, and w_n denotes the weight for the n-th data. w_n is set larger for Data-S and Data-P, because α or q in these data have far smaller changes than in Data-C. k_n denotes the latest time of time-series values of the n-th data. $\Delta\alpha(k_n-j, k_n-5)$ denotes the difference of α from the time (k_n-5) to (k_n-j) and $\hat{\Delta\alpha}(k_n-j, k_n-5|k_n-5)$ denotes the value predicted at the time (k_n-5).

In the calculation of $\Delta\alpha$ and Δq, the controller uses time-series values of α and q up to the time (k_n-5), those of δ_e up to the time (k_n-j) and those of ρ_a and V_a at the time (k_n-5). The cost function F_{cq} for the q model identification has been determined similar to eqn. (17). The values of model parameters f_{xzi}, g_{xzi} and h_{xzi} were obtained with nonlinear optimization by using Microsoft Excel Solver.

5 Results of identification

The model orders $n_{\alpha q}$ and n_e were chosen to be 3 and 4, respectively, through identification experiments with various values. This results in the total number of parameters in eqn. (12) or (13) each to be $3\times3\times2+3\times4 = 30$. To reduce the number of identified parameters, the parameters in each bracket [] below were set to have the same value.

$[g_{x\alpha i}, g_{xqi}]$ (different values for $i = 1,2,3$),
$[h_{x\alpha i}, h_{xqi}]$ (different values for $i = 1,2,3$),
$[g_{x\alpha i};\ i = 2,3]$, $[h_{x\alpha i};\ i = 2,3]$
$[g_{xei};\ i = 1,2]$, $[g_{xei};\ i = 3,4]$
$[h_{xei};\ i = 1,2]$, $[h_{xei};\ i = 3,4]$

where x stands for α or q. Thus, the total number of parameters to be identified in the α and the q model each was reduced to 18. The reduction caused only a little and ignorable effect on model accuracy.

Table 1 shows identified parameter values. When more than one parameter have the same value, the table shows the symbol of the first parameter.

Table 1 Values of identified parameters in eqns. (12, 13)

$f_{\alpha\alpha1}$	0.9930	$f_{\alpha\alpha2}$	0.0058	$f_{\alpha\alpha3}$	-0.0867
$f_{\alpha q1}$	0.1742	$f_{\alpha q2}$	-0.0447	$f_{\alpha q3}$	-0.0070
$f_{\alpha e1}$	-0.0181	$f_{\alpha e2}$	-0.0865	$f_{\alpha e3}$	-0.0306
$f_{\alpha e4}$	-0.0038	$g_{\alpha\alpha1}$	-0.2963	$g_{\alpha\alpha2}$	-1.3728
$h_{\alpha\alpha1}$	-0.2821	$h_{\alpha\alpha2}$	-1.1653	$g_{\alpha e1}$	1.1495
$g_{\alpha e3}$	0.7803	$h_{\alpha e1}$	0.8994	$h_{\alpha e3}$	2.9502
$f_{q\alpha1}$	0.0897	$f_{q\alpha2}$	-0.5881	$f_{q\alpha3}$	0.3425
f_{qq1}	0.7593	f_{qq2}	-0.0010	f_{qq3}	0.0507
f_{qe1}	-0.1231	f_{qe2}	-0.4824	f_{qe3}	-0.6328
f_{qe4}	-0.2425	$g_{q\alpha1}$	-0.2220	$g_{q\alpha2}$	0.6798
$h_{q\alpha1}$	-0.1851	$h_{q\alpha2}$	1.6025	g_{qe1}	0.8662
g_{qe3}	0.8793	h_{qe1}	2.1106	h_{qe3}	1.5349

Next, Table 2 shows the standard deviation $\sigma\{\cdot\}$ of 5-steps-ahead (one-second-ahead) prediction errors $e_{\Delta x}$ ($x := \alpha$ or q) and that of variations obtained with the 6 DOF model in the same period. Here, the value of $[\sigma\{e_{\Delta x}\}/\sigma\{\Delta x\}]$ represents model accuracy. The value is between 0.081 (=0.1150/1.4181) and 0.225 (=1.2887/5.7226). The data group of the highest accuracy is Data-P, and that of the lowest is Data-C.

Figs. 6 to 11 show the scatter plots of $\hat{\Delta x}(k, k-5/k-5)$ versus $\Delta x(k, k-5)$, where x stands for α or q and each dot represents the above values of each data. If prediction errors

Table 2 Standard deviations of prediction errors ($e_{\Delta\alpha}$, $e_{\Delta q}$) and variations ($\Delta\alpha$, Δq)

	Data-C	Data-S	Data-P
$\sigma\{e_{\Delta\alpha}(k,k-5/k-5)\}$ [deg]	0.4943	0.1488	0.1150
$\sigma\{\Delta\alpha(k,k-5)\}$ [deg]	3.6541	0.9984	1.4181
$\sigma\{e_{\Delta q}(k,k-5/k-5)\}$ [deg/s]	1.2887	0.4741	0.3187
$\sigma\{\Delta q(k,k-5)\}$ [deg/s]	5.7226	2.3626	3.5044

are all close to zero, the plots would be on the line of 45 degrees upward.

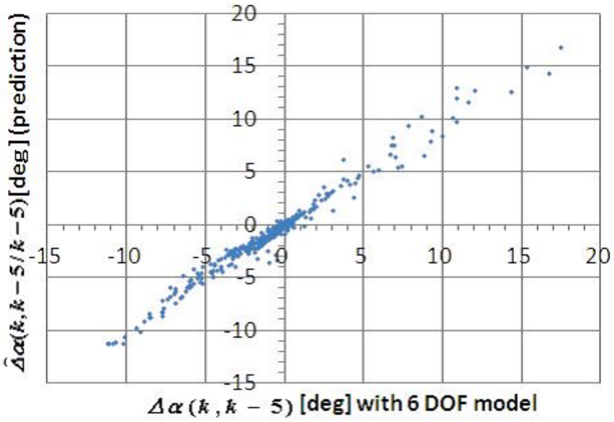

Fig. 6 Prediction accuracy of $\Delta\alpha$ for controlled flights

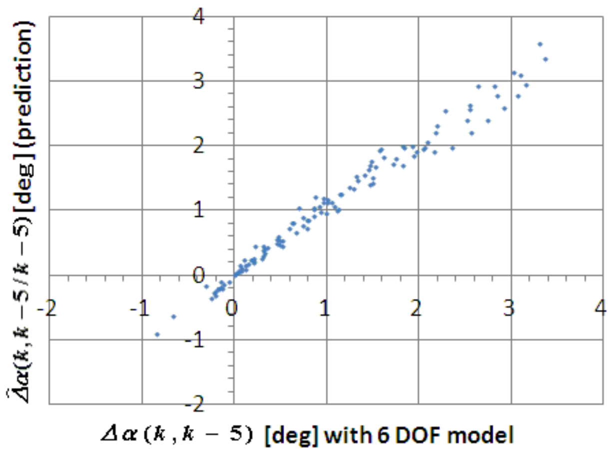

Fig. 7 Prediction accuracy of $\Delta\alpha$ for step responses

From these tables and figures, the accuracy of both the α model and the q model is considered satisfactory to MBPC.

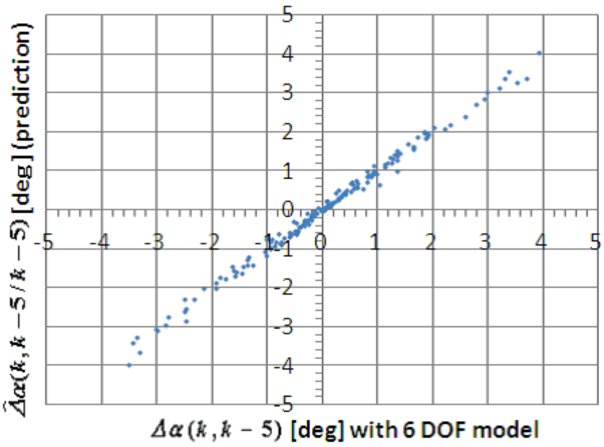

Fig. 8 Prediction accuracy of $\Delta\alpha$ for triangular pulse responses

6 Conclusion

A linear time-series model for α and q has been obtained. The model is linear in variables α and q, and the model dynamics nonlinearly depends on air density and aircraft velocity. The model has good accuracy for a wide range of flight conditions. The MBPC using this model is now being developed. A similar model representing lateral and directional movements could be obtained in the same way.

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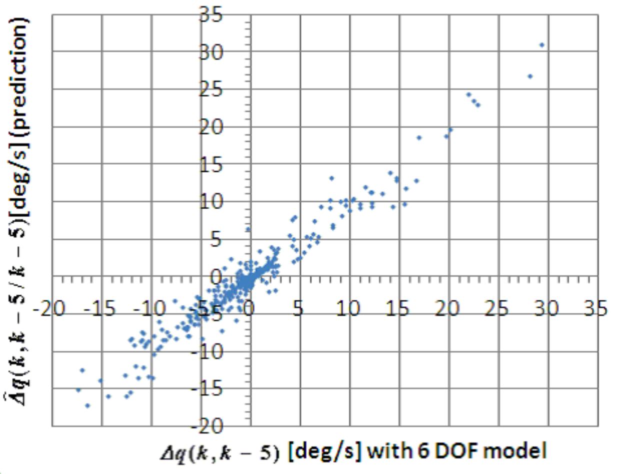

Fig. 9 Prediction accuracy of Δq for controlled flights

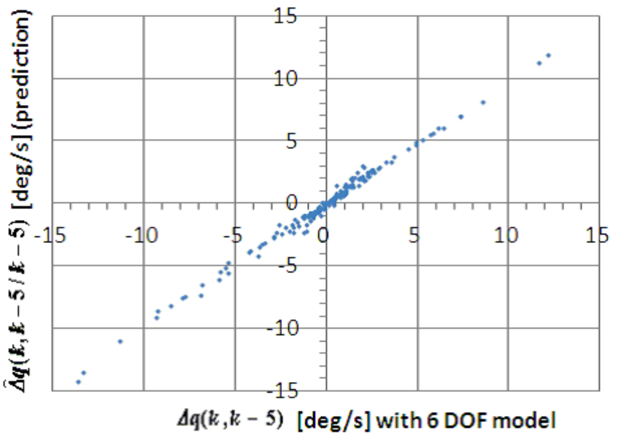

Fig. 11 Prediction accuracy of Δq for triangular pulse responses

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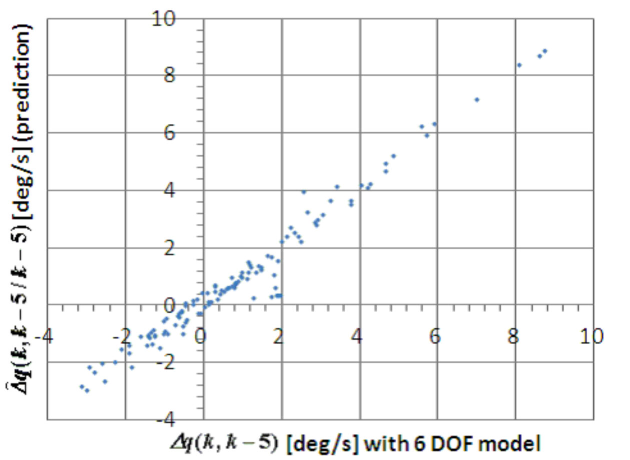

Fig. 10 Prediction accuracy of Δq for step responses

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