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MEASUREMENT OF THE CROSS-COUPLING DERIVATIVES DUE TO PITCHING IN THE HIGH REYNOLDS NUMBER BLOWDOWN WIND TUNNEL

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Abstract

The T-38 wind tunnel measurements of the cross-coupling derivatives using the forced oscillation technique are presented. The data reduction is based on a conversion of the secondary deflection vectors into causative aerodynamic moment vector. The amplitudes and phase shifts of the causative moments are calculated in frequency domain by applying the cross-power spectral density

1 Introduction

The stability parameters can be obtained from model experiments in many different types facilities. Some of them are: out-door free flight tests with either rocket-propelled or radio-controlled gliding models, wind-tunnel free-flight tests with freely launched or remotely controlled models or spin-tunnel experiments. All these techniques have one common disadvantage, they are not suitable for experiments at high Reynolds number. Thus main use of these techniques is for visual studies of the stability characteristics and motions of the aircrafts and missiles, all at low Reynolds number.

The best way to obtain model-scale dynamic stability at realistic Reynolds and Mach numbers is to perform captive-model experiments in high Reynolds number wind tunnels. The T-38 wind tunnel of the Military Technical Institute (VTI) in Belgrade is a blowdown-type pressurized wind tunnel [1]. Mach number in range 0.2-4.0 can be achieved in the test section. One of the main advantages of the T-38 wind tunnel is its high Reynolds number capability: up to 110 million per meter.

For measurement of stability derivatives in the T-38 wind tunnel the forced oscillation technique is used [2]. During these measurements external force and moment between a model and forcing part of the dynamic apparatus are measured. A model motion is angular and sinusoidal, and stability derivatives are obtained from in-phase and out-phase components of the measured aerodynamic reactions. Using this method, in principle both force and moment derivatives can be measured. The measurement can be made in same degree of freedom (primary degree of freedom) as the imparted oscillation resulting in damping derivatives. Also, the measurement can be made in the other degrees of freedom (secondary degrees of freedom) resulting in cross and cross-coupling derivatives [3].

2 Equations of angular motion for captive-model

In the body axis system (Fig.1.) oscillatory motion of the symmetrical model in the roll, pitch and yaw are given by equations (1)-(3):

$$
\begin{aligned}
& I_x \dot{p} + \left(f_{\varphi} - \left(L_p + L_{\dot{\beta}} \sin\alpha\right)\right)p + \left(K_{\varphi} - L_{\beta}\sin\alpha\right)\varphi = \\
& = I_{xz}\dot{r} + \left(L_r - L_{\dot{\beta}}\cos\alpha\right)r - L_{\beta}\psi\cos\alpha + \\
& + \left(L_q + L_{\dot{\alpha}}\right)q + L_{\alpha}\theta + L_P
\end{aligned}
\qquad (1)
$$

$$\begin{aligned} &I_y\dot{q} + \left(f_\theta - \left(M_q + M_{\dot{\alpha}}\right)\right)q + \left(K_\theta - M_\alpha\right)\theta = \\ &= \left(M_r - M_{\dot{\beta}}\cos\alpha\right)r - M_\beta\psi\cos\alpha + \\ &+ \left(M_p + M_{\dot{\beta}}\sin\alpha\right)p + M_\beta\varphi\sin\alpha + M_P \end{aligned} \tag{2}$$

$$\begin{aligned} &I_z\dot{r} + \left(f_\psi - \left(N_r - N_{\dot{\beta}}\cos\alpha\right)\right)r + \left(K_\psi + N_\beta\cos\alpha\right)\psi = \\ &= I_{xz}\dot{p} + \left(N_p + N_{\dot{\beta}}\sin\alpha\right)p + N_\beta\varphi\sin\alpha + \\ &+ \left(N_q + N_{\dot{\alpha}}\right)q + N_\alpha\theta + N_P \end{aligned} \tag{3}$$

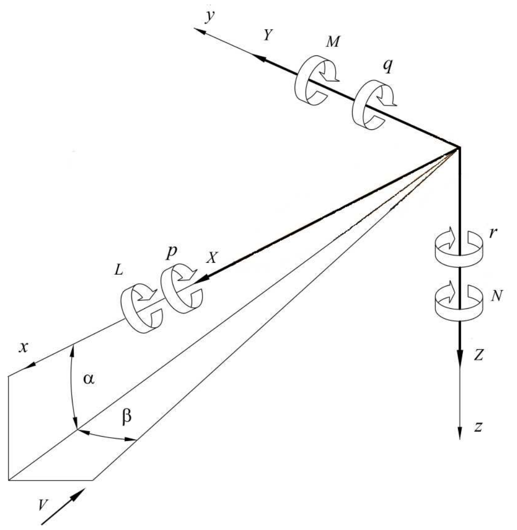

Fig.1. Body axis system

Primary oscillatory motion in pitch induced secondary yawing and rolling moments. These secondary moments can be expressed by equations (4) - (6):

$$\theta = |\theta| e^{i\omega t} \tag{4}$$

$$\bar{N} = |\bar{N}| e^{j(\omega t + \eta_\psi)} = \left(N_q + N_{\dot{\alpha}}\right)q + N_\alpha\theta \tag{5}$$

$$\bar{L} = |\bar{L}| e^{j(\omega t + \eta_\varphi)} = \left(L_q + L_{\dot{\alpha}}\right)q + L_\alpha\theta \tag{6}$$

Secondary yawing and rolling moments are expressed as a sum of two contributions depending on q and θ respectively. Solving equations above expressions for the static and dynamic derivatives of the yawing and rolling moments due to oscillation in pitch are defined:

$$N_\alpha = \frac{|\bar{N}|}{|\theta|}\cos\eta_\psi \tag{7}$$

$$N_q + N_{\dot{\alpha}} = \frac{|\bar{N}|}{|\theta|}\frac{\sin\eta_\psi}{\omega} \tag{8}$$

$$L_\alpha = \frac{|\bar{L}|}{|\theta|}\cos\eta_\varphi \tag{9}$$

$$L_q + L_{\dot{\alpha}} = \frac{|\bar{L}|}{|\theta|}\frac{\sin\eta_\varphi}{\omega} \tag{10}$$

For each derivative, the amplitude of the primary motion, oscillation frequency, the amplitude of the causative moment and phase shift between them is required. Measurements of the dynamic stability derivatives using forced oscillation technique cannot be performed directly. The aerodynamic damping is obtained as a difference between the total damping obtained in the presence of the aerodynamic load and mechanical damping which is obtained in absence of aerodynamic load. Mechanical damping is measured in the tare run. In the tare run a model is oscillate with the same frequency and amplitude as during the wind-on run, but the wind tunnel is not running.

3 Basic principle of the apparatus

The apparatus for the stability derivatives measurements in the T-38 wind tunnel, shown in Fig. 2, provides a primary oscillatory motion in pitch with secondary (resulting) oscillatory motions in yaw and roll. With the apparatus rotated 90° around its longitudinal axis (in relation to both model and internal wind tunnel balance) a primary oscillatory motion in yaw is provided with secondary oscillatory motions in pitch and roll.

Performance parameters of the apparatus are:

- Model oscillation amplitude: 0.25-1.5°
- Model oscillation frequency: 1-15 Hz
- Sting: elliptical cross-section 50x72 mm
- Maximum normal force: 10 000N

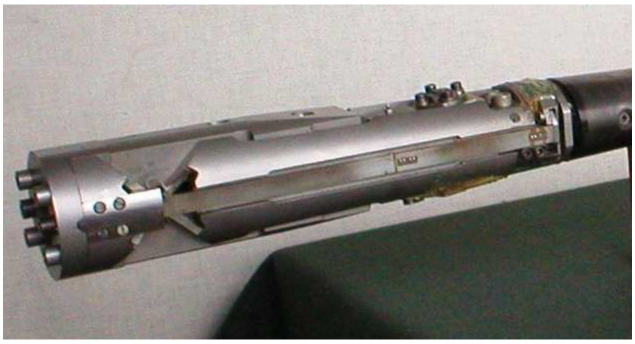

Fig.2. Pitch/yaw apparatus

The apparatus presented is a fourth generation dynamic apparatus designed and manufactured in the VTI [4]. It is characterized by high structural rigidity and high load capacity. An important requirement for any forced oscillation apparatus for damping derivatives measurement is structural rigidity. An additional reason for structural rigidity requirement of the pitch/yaw apparatus is fact that the T-38 wind tunnel is blowdown wind tunnel. A common characteristic of blowdown wind tunnels is the appearance of very large loads at the beginning and the end of a wind tunnel run.

The pitch/yaw apparatus includes the following elements:

- Sting support system.
- Elastic suspension system which consists of a pair of cross-flexures. The cross-flexures were selected for the main pivot, because they represent the best combination of a good definition of axis of oscillation, sufficient angular deflection and high enough load-carrying capability.
- Five-component balance.
- Hydraulic driving mechanism to impart the oscillatory primary motion of the model.
- Sensor to detect the primary oscillatory motion.
- Servomechanism which ensures a constant-amplitude primary motion.

The combination of the high aerodynamic loads and very limited space inside a model caused the use of a hydraulic driving mechanism. The driving force is applied to the actuator arm which is connected to the moving end of the elastic suspension system. The hydraulic actuator is controlled by the hydraulic servovalve located at the sting base. The hydraulic servovalve is driven by a signal from the control system. The control system is a part of the T-38 wind tunnel control system and provides an automatic control mode.

The primary oscillatory motion sensor is located on the cross flexures of the elastic suspension system. There are two primary oscillatory motion sensors on each side of the elastic suspension system. Measuring bridges are formed from foil-type strain gauges. These measuring bridges can be used as individual sensor or can be connected in the one sensor.

Fig.3. Five-component strain gauge balance

One of the main parts of the apparatus is internal five-component strain gauge balance. The aerodynamic loads, excitation moment in the primary degree of freedom, as well as secondary reactions caused by the primary motion are measured with this balance. The balance measuring bridges have to ensure the measurement of the very low values of the aerodynamic reactions produced by the model oscillatory motion. Very important requirement for dynamic balances is the optimal signal-to-noise ratio. All measuring bridges of the five-component balance are made from semiconductor strain gauges [5-8].

4 Data reduction

The data reduction for cross-coupling derivatives is based on a conversion of secondary deflection vectors into causative aerodynamic moment vector. The conversion of the secondary deflection vectors ψ and φ into the aerodynamic moments in yaw and roll, $\bar{N}$ and $\bar{L}$, is accomplished by next expressions:

$$|\bar{N}| = |\psi| I_z \left(\left(\omega_\psi^2 - \omega^2 \right)^2 + \mu_\psi \omega^2 \right)^{\frac{1}{2}} \tag{11}$$

$$|\bar{L}| = |\varphi| I_x \left(\left(\omega_\varphi^2 - \omega^2 \right)^2 + \mu_\varphi \omega^2 \right)^{\frac{1}{2}} \tag{12}$$

where $\mu_\psi = \dfrac{\gamma_\psi - \left(N_r - N_{\dot{\beta}} \cos\alpha \right)}{I_z}$

$$\mu_\varphi = \frac{\gamma_\varphi - \left(L_p + L_{\dot{\beta}} \sin\alpha \right)}{I_x}.$$

The moments of inertia (I_x, I_z), damping constants ($\gamma_\psi, \gamma_\varphi$) and natural frequencies (ω_ψ, ω_φ) are determined by separate calibration tests with a model mounted for the oscillations in yaw and roll experiment [9].

The phase shift between causative aerodynamic moment vectors and primary motion is determined from:

$$\begin{aligned} \eta_\psi &= \prec(\psi - \theta) + \varepsilon_\psi \\ \eta_\varphi &= \prec(\varphi - \theta) + \varepsilon_\varphi \end{aligned} \tag{13}$$

where $\varepsilon_\psi = arctg \dfrac{\mu_\psi \omega}{\omega_\psi^2 - \omega^2}$ and $\varepsilon_\varphi = arctg \dfrac{\mu_\varphi \omega}{\omega_\varphi^2 - \omega^2}$.

The phase shifts $\prec(\psi - \theta)$ and $\prec(\varphi - \theta)$, are measured phase shift between secondary oscillations and primary motion.

In most cases, signals from the balance bridges used for the measurement of secondary oscillations are seriously contaminated by the noise generated mainly by flow unsteadiness. As a noise level can be several times higher than that of desired signal, it is generally impossible to extract them adequately using conventional narrowband pass filters. Knowledge that the signals are coherent with the primary oscillatory motion permits the use of more sophisticated signal extraction techniques. These are auto-correlation and cross-correlations techniques. Cross-correlation techniques are especially suited to applications where a clean reference signal coherent with the one that needs to be extracted from the noise is available.

In the T-38 wind tunnel tests presented, the amplitudes and the phase shifts of the excitation moment and causative moments were calculated in frequency domain by applying the cross-power spectral density.

5 Experimental results

The T-38 wind tunnel measurement data for cross-coupling derivative of the yawing and rolling moment due to pitching on the Standard Dynamic Model (SDM) for Mach number 0.6 is presented in this paper. The SDM model in the T-38 wind tunnel test section is shown in Fig. 4.

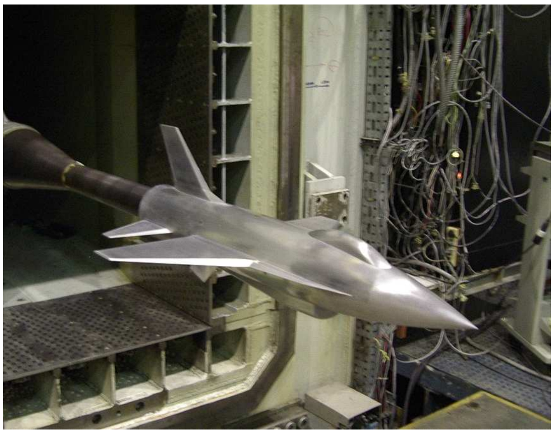

Fig.4. SDM model in the T-38 wind tunnel test section

The model was forced to oscillate in pitch mode at amplitude $\pm 8^\circ$ and frequency 5 Hz. The signals from the balance bridge for pitching moment, Fig. 5, were cross-correlated with reference signal generated by the primary motion sensor.

The cross-correlation function of the excitation moment signal and primary oscillatory motion signal is shown in Fig. 6. Secondary deflections in yaw and roll were measured by balance measuring bridges for yawing and rolling moments. Signals from these measuring bridges are shown in Fig. 7 and Fig. 8.

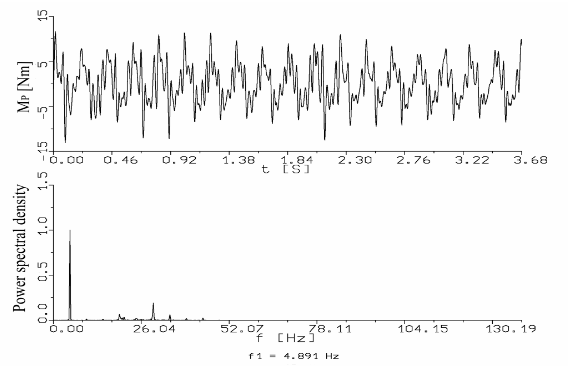

Fig.5. Excitation moment signal and its power spectral density

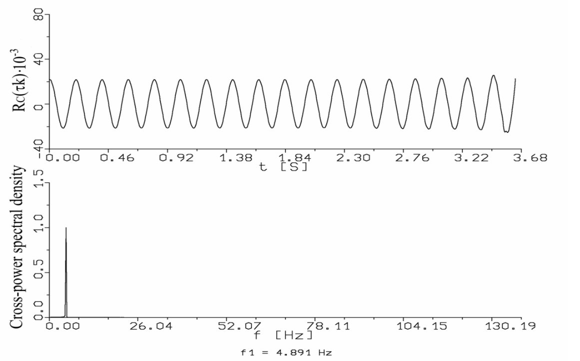

Fig.6. Cross-correlation function of the excitation moment signal and its cross-power spectral density ($\alpha = 0^\circ$)

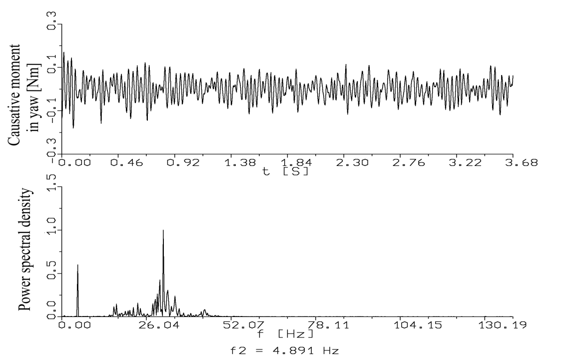

Fig.7. Yawing moment signal and its power spectral density ($\alpha = 0^\circ$)

The frequency of the primary oscillatory motion was detected in the signals from balance measuring bridges for yawing and rolling moment. The semiconductor strain gauges enable determination of the oscillatory components of the signals even if they are deeply hidden in the noise. The phase shifts $\prec(\psi-\theta)$ and $\prec(\varphi-\theta)$ were determined by cross-spectrum density function of secondary deflections signals and primary oscillatory motion signal.

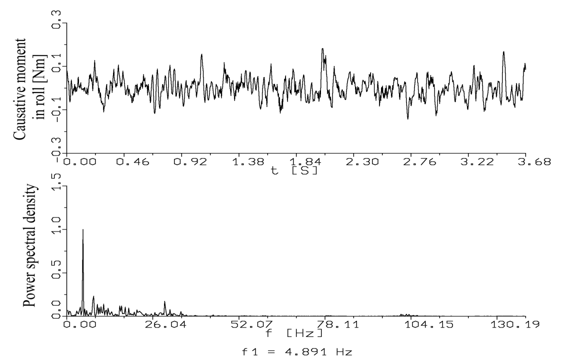

Fig.8. Rolling moment signal and its power spectral density ($\alpha = 0^\circ$)

Test results obtained in the T-38 wind tunnel are compared with published experimental data of the NAE dynamic wind tunnel [10] (Fig.9. and Fig.10.). Non-dimensional cross-coupling coefficients derivatives of the rolling and yawing moment due to pitching are obtained in the following form:

$$C_{nq} + C_{n\dot{\alpha}} = \frac{N_q + N_{\dot{\alpha}}}{q_\infty \bar{c} \, \dfrac{\bar{c}}{2V}}$$

$$C_{lq} + C_{l\dot{\alpha}} = \frac{L_q + L_{\dot{\alpha}}}{q_\infty \bar{c} \, \dfrac{\bar{c}}{2V}} \qquad (14)$$

The nominal wind tunnel parameters at which tests were conducted in the T-38 wind tunnel and NAE dynamic wind tunnel are presented in Table 1. Reynolds number is based on the model reference length ($\bar{c}$).

Table 1

T-38 wind tunnel	M Re	0.6 $4.326 \cdot 10^6$
NAE dynamic wind tunnel	M Re	0.6 $0.897 \cdot 10^6$

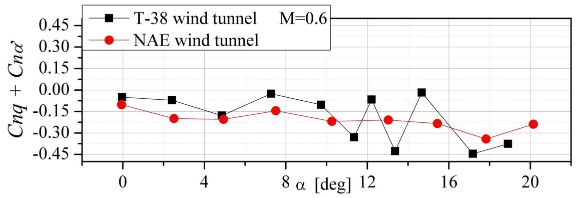

Fig.9. Cross-coupling derivative of the yawing moment due to pitching

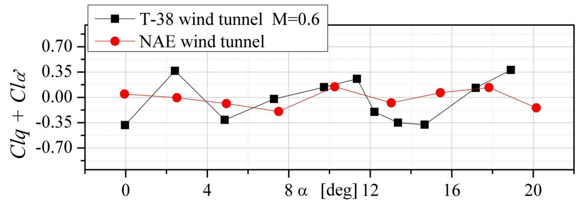

Fig.10. Cross-coupling derivative of the rolling moment due to pitching

6 Conclusions

The wind tunnel measurement of dynamic stability derivatives is a very complicated process. Dynamic wind tunnel balances (as well as transducers on the dynamic apparatuses) have to be designed for large aerodynamic loads in the tests. Also, these balances have to enable measurements of the very small values in the dynamic measurements. In the dynamic measurements using forced oscillation technique stability derivatives are obtained by subtracting the tare run data form the wind-on run data. Because of the fact that these data are small values, the accuracy of the measurements can be reduced more. In the measurement of cross-coupling derivatives in the majority of signals measured in the secondary degrees of freedom, the most significant problem usually is detection of the frequency of the primary oscillations. The measuring bridges on the five-component strain gauge balance, used in the presented tests, enable the determination of the oscillatory components of the signals even they were deeply hidden in the noise.

Acknowledgements

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Nomenclature

$C_{nq} + C_{n\dot{\alpha}}$ - cross-coupling coefficient derivative of the yawing moment due to pitching
$C_{lq} + C_{l\dot{\alpha}}$ - cross-coupling coefficient derivative of the rolling moment due to pitching
$\bar{c}$ - reference length
f_θ - mechanical damping in pitch
f_ψ - mechanical damping in yaw
f_φ - mechanical damping in roll
I_x - roll moment of inertia
I_y - pitch moment of inertia
I_z - yaw moment of inertia
I_{xy}, I_{xz} - centrifugal moment of inertia
K_θ - mechanical stiffness in pitch
K_ψ - mechanical stiffness in yaw
K_φ - mechanical stiffness in roll
L - aerodynamic rolling moment
L_P - excitation moment in roll
$\bar{L}$ - induced rolling moment
$L_p + L_{\dot{\beta}} \sin\alpha$ - direct damping derivative in roll
$L_\beta \sin\alpha$, $L_\beta \cos\alpha$ - rolling moment due to angle of sideslip
$L_r - L_{\dot{\beta}} \cos\alpha$ - cross derivative of the rolling moment due to yawing
$L_q + L_{\dot{\alpha}}$ - cross-coupling derivative of the rolling moment due to pitching
L_α - rolling moment due to angle of attack
M - aerodynamic pitching moment
M_P - excitation moment in pitch
$M_q + M_{\dot{\alpha}}$ - direct damping derivative in pitch
M_α - pitching moment due to angle of attack
$M_r - M_{\dot{\beta}} \cos\alpha$ - cross-coupling derivative of the pitching moment due to yawing
$M_\beta \cos\alpha$, $M_\beta \sin\alpha$ - pitching moment due to angle of sideslip
$M_p + M_{\dot{\beta}} \sin\alpha$ - cross-coupling drivative of the pitching moment due to rolling
N - aerodynamic yawing moment
N_P - excitation moment in yaw
$\bar{N}$ - induced yawing moment
$N_r - N_{\dot{\beta}} \cos\alpha$ - direct damping derivative in yaw
$N_\beta \cos\alpha$, $N_\beta \sin\alpha$ - yawing moment due to angle of sideslip
$N_p + N_{\dot{\beta}} \sin\alpha$ - cross derivative of the yawing moment due to rolling
$N_q + N_{\dot{\alpha}}$ - cross-coupling derivative of the yawing moment due to pitching
N_α - yawing moment due to angle of attack
p - rolling angular velocity
q - pitching angular velocity
q_∞ - dynamic pressure
r - yawing angular velocity
V - free stream velocity
x,y,z - body axis system
X - aerodynamic axial force
Y - aerodynamic side force
Z - aerodynamic normal force
α - angle of attack
β - angle of sideslip
θ - angular oscillatory motion in pitch
ψ - angular oscillatory motion in yaw
φ - angular oscillatory motion in roll
ω - angular velocity
ω_ψ - natural frequency in yaw
ω_φ - natural frequency in roll
γ_ψ, γ_φ - damping constant
μ_ψ - damping coefficient in yaw
μ_φ - damping coefficient in roll
η_ψ - phase shift between primary oscillatory motion and causative aerodynamic moment in yaw
η_φ - phase shift between primary oscillatory motion and causative aerodynamic moment in roll
$|...|$ - amplitude

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