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INVERSE METHODS FOR AERODYNAMIC DESIGN USING THE NAVIER-STOKES EQUATIONS

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Abstract

The inverse method for aerodynamic design using the Navier-Stokes equations is developed. First intended for single airfoil design, it was extended for solving more complicated inverse problems, including two-dimensional ones for multielement airfoil and three-dimensional problems.

The method belongs to the class of residual correction methods. Airfoils or 3D wing section shapes corrections are defined from the residuals between current and desired pressure distributions.

The proposed algorithm provides fast convergence and high accuracy of the results. Examples of solving inverse problems of different complexity are given.

1 Introduction

Inverse methods of aerodynamics determine the geometry of the aircraft elements for a given pressure distribution and are a powerful tool of aerodynamic design. They allow to improve aerodynamics of the element: to eliminate or reduce shocks, to improve flow behavior at the specified location or to implement pressure distribution favorable for the development of a laminar or turbulent boundary layer. But the inverse problem is incorrect in general case, so not every given pressure distribution can be realized physically. The solvability conditions are precisely defined only for 2-D potential flows [1–2]. Engineering approaches are to be used for real viscous, transonic, and, in particular, 3-D flows.

One of the most common approaches to the construction of inverse methods is based on the principle of residual correction. According to this principle the deformation of the airfoil surface is determined in some way through the residual between the calculated and the target (c_p^*) pressure distributions [3–12]. The problem is solved iteratively by alternate calls of the direct analysis method and the geometry correction block (Fig.1). The run time of the geometry correction block is usually only a small fraction of the direct calculation time. Due to the iterative nature of the solution, the surface deformation rule can be quite rough. But evidently, the more accurately the geometry correction is determined, the smaller is the total number of required iterations.

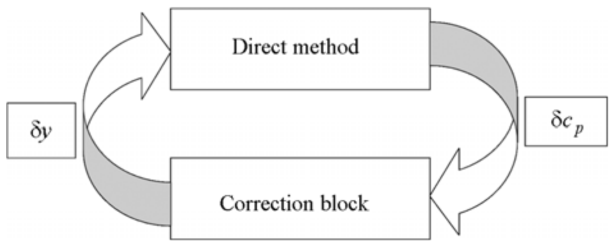

Fig.1 Residual correction method.

The well-developed methods of direct flow problems solution are used in the residual correction procedures. Such procedures are not associated with a particular direct method, but use it as a "black box". So the latter can easily be replaced in further development by a more perfect one. The development of direct methods usually precedes the development of "pure" inverse methods based on the solution of the flow problem with boundary conditions of $c_p = c_p^*$ type. So the residual correction principle allows solving the inverse problems for a wider class of flows and configurations.

The inverse methods for aerodynamic design using the Navier-Stokes equations belonging to the class of residual correction methods have been developed by the authors and are presented in this article. The details of the algorithm are described and some examples of solving the inverse problems of different complexity are given.

2 Algorithm description

The rapid development of computational fluid dynamics has recently led to the widespread implementation of direct RANS methods, closed by some turbulence model [13-17]. Currently these methods represent a reasonable compromise between the complexity of equations and the required computational and time resources. Although the use of simpler equations leads to a significant acceleration of calculations, it is not of universal nature and requires the careful justification because of the risk to miss some important physical flow property, which ultimately affects locally or even globally the aerodynamics of the aircraft.

The inverse methods using the Navier-Stokes equations, proposed in this paper, also belong to the class of residual correction methods. The principle of hierarchical levels is used to create an effective algorithm. According to this principle it is appropriate to apply the inverse method of the lower level as a geometry correction block. Here “level” means the complexity of equations to be solved or of the geometry forms to be considered. Typical geometry levels in aerodynamics are airfoil, wing, wing+fuselage, and complete configuration. Typical equation complexity levels are the Laplace equation, the full potential equation, Euler equations, and Navier-Stokes equations. As a rule, the transition to the next level increases the computational costs by one to two orders of magnitude.

The principle of hierarchical levels allows gradually increasing the complexity of inverse problems with clear understanding of the physics of the phenomena and rational distribution of the calculations at different levels in order to minimize the total expenditure of computer time. Applying this principle, it is possible to create n-level systems for complex inverse problems, using in fact only one simple inverse method at the lowest level. For example, the inverse method for a wing in transonic flow developed in TsAGI, namely, TRAWDES [4], consists of three levels (Fig. 2).

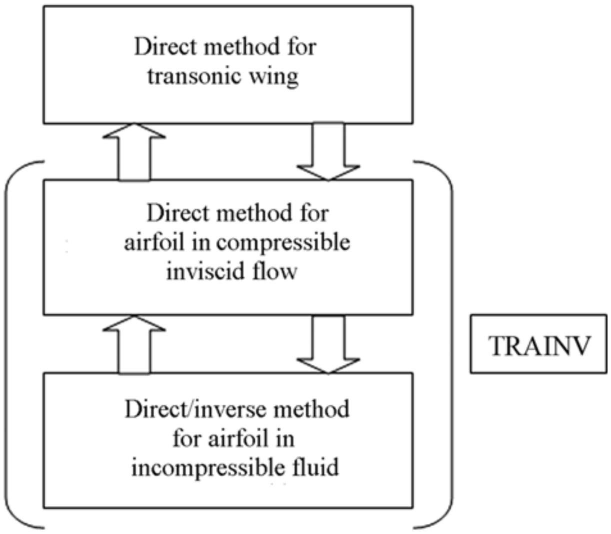

Fig.2 Principle of hierarchical levels.

The inverse method for multielement airfoil proposed in the present article consists of two levels (Fig. 3). The upper level includes direct RANS method for multielement airfoil by ANSYS CFX software package use [18] and the lower level is the direct/inverse panel method for multielement airfoil in incompressible fluid.

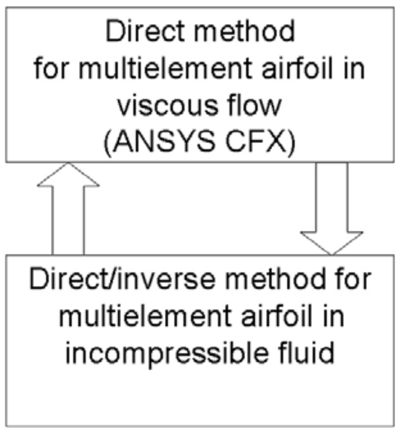

Fig.3 Inverse problem algorithm for multielement airfoil

By turn the single airfoil inverse method consists of three levels (Fig. 4). The upper level includes a direct RANS method for airfoil. On the second level the direct calculation of the compressible inviscid flow over airfoil by the full-potential method is performed. The first (lowest) level is a direct/inverse method for airfoil in an incompressible fluid. The first and the second levels together constitute the inverse method for airfoil in inviscid compressible flow, the TRAINV program [3].

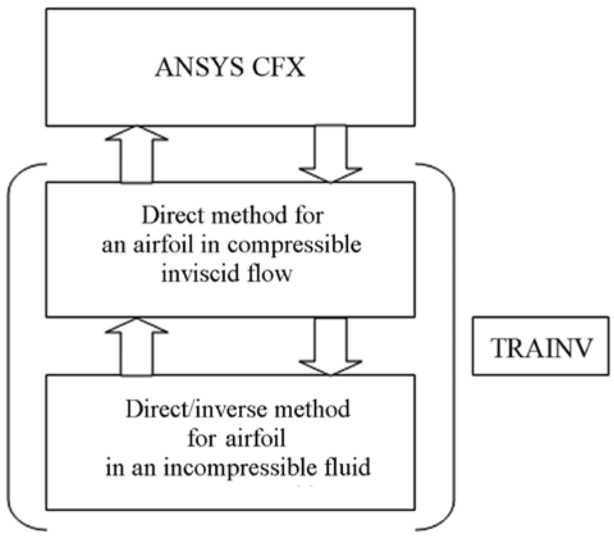

Fig.4 Inverse problem algorithm for single airfoil

The wing inverse method connects four levels (Fig. 5). The upper level is a direct RANS method for wing. On the third level the transonic inviscid flow over wing by means of full-potential code BLWF [4] is calculated. The second level is the direct calculation of compressible inviscid flow over airfoil. The direct/inverse method for airfoil in incompressible fluid is the lowest level. In this algorithm three lower levels compose the inverse method for a transonic wing, namely, the TRAWDES program mentioned above.

Each of above mentioned inverse methods is performed in the process of iterations. The following steps are performed on each k-th iteration:

- The structured mesh is built around the geometry obtained on the previous (k–1)-th iteration, the flow over it is calculated by CFX, and the pressure distribution residual $\delta c_p^{\,k}$ is determined as the difference between the current and target pressure distributions.
- The inviscid flow around the same geometry is calculated.
- The desired pressure distribution for inviscid flow over current geometry is formed by adding the difference $r\cdot\delta c_p^{\,k}$ (where $r \leq 1$) to the pressure distribution from step 2.
- The inverse problem for the inviscid flow is solved with the target pressure distribution from step 3 and the k-th airfoil (wing) geometry is obtained.

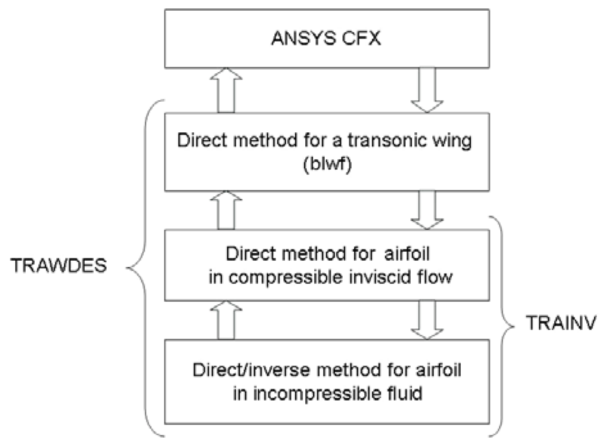

Fig.5 Inverse problem algorithm for a wing

For the obtained geometry it is necessary to solve again the direct RANS problem, to find the pressure distribution, etc. The pressure residual of airfoil or of each wing section is as follows:

$$\varepsilon_{c_p} = \sqrt{\frac{1}{N}\sum_{i}^{N}(c_{p_i} - c_{p_i}^{*})^2}$$

and is calculated at each iteration, which defines the disagreement between calculated and target pressure distributions. The process is repeated several times until the convergence or the achievement of the minimum residual. In practice sufficient convergence corresponds to the level of $\varepsilon_{cp} \leq 0.01 \div 0.02$.

3 Calculation Examples

The geometry recovery tests are often used to verify the inverse methods. In such tests the target pressure distribution is taken from the direct calculation of the existing geometry, and the initial geometry is arbitrary. Such tests allow monitoring the convergence of the method not only on pressure distribution, but also on geometry. Note that for the general inverse problem the required geometry is unknown, moreover, in general, there is no evidence of its existence, i.e., the inverse problem is often incorrect.

Figure 6 shows an example of the geometry recovery test for the target pressure distribution obtained by fully turbulent RANS calculation of NACA 4412 airfoil at transonic condition M = 0.67, α = 2°, Re = $4{\cdot}10^6$, where the shock exists on the upper surface. Iterations start from the NACA 0010 airfoil at the initial angle of attack α = 1°. Twelve iterations were required to solve the design problem (each iteration includes one step of direct calculation and one step of solving the inverse inviscid problem). Since the corrector describes well the physics of flow, the result is quite good even at the first iteration. Fairly good agreement of the pressure distribution was obtained on the seventh iteration, but five additional iterations are necessary to obtain a good geometric convergence. The graph of the pressure residual decrease and the angle of attack convergence graph are also presented in the figure.

In the following example the target pressure distribution was chosen arbitrarily on the upper surface (Fig. 7) to implement the shock-free airfoil at M = 0.67. The pressure distribution on the bottom surface was not changed. As the initial approximation the NACA 4412 airfoil was taken. The solution with acceptable accuracy has been obtained after ten iterations.

The third example is an inverse problem solution for multielement airfoil (Fig. 8). The target pressure distribution on flap was chosen arbitrarily. The GA(W)-1 airfoil at M=0.2, Re=$2.1{\cdot}10^6$ with flap deflected at δ=20° was used as initial approximation. The geometry of flap was changed in the process of iterations and the geometry of main element was fixed. Four iterations were required to get solution of the design problem with quite good convergence on pressure distribution. In the process of solving the total lift coefficient increased from 3.34 to 3.4, which of almost 90% belongs to main element. The residual reduction graph is also shown.

The next two examples are the wing geometry recovery tests: the first one is a test for subsonic wing at M=0.2 and the second one is a test for transonic wing at M=0.68. Reynolds number in both cases is Re = $4{\cdot}10^6$. The geometry of base wing is determined by three sections: 2y/span=0, 0.333 and 1. Target geometry corresponded to airfoil NACA 4415 with twist 3° at 2y/span=0, airfoil NACA 4412 with twist 0° at 2y/span=0.333 and airfoil NACA 4410 with twist -3° at 2y/span=1. The untwisted wing with airfoil NACA 0010 at each base section was taken as initial geometry.

At the first example at M=0.2 the target pressure distribution was obtained by calculation of flow over target geometry wing at angle of attack α=5°, and the initial wing was started from α=0°. Three iterations were needed to obtain the solution. In figure 9 the pressure distributions and geometry of base sections with twist are shown. The obtained geometry, twist, angle of attack and corresponding pressure distributions are almost identical to the target ones.

At the second 3-D example the similar problem was solved, but M=0.68, and the target angle of attack been reduced to α=2°were used. Convergence has been obtained in three iterations (see Fig. 10).

4 Conclusions

A new algorithm to use the RANS-codes for inverse problem solutions have been developed. The solution is performed iteratively in the framework of the residual correction procedure. The software package ANSYS CFX is used as a direct analysis method. Different methods previously developed at TsAGI for solving

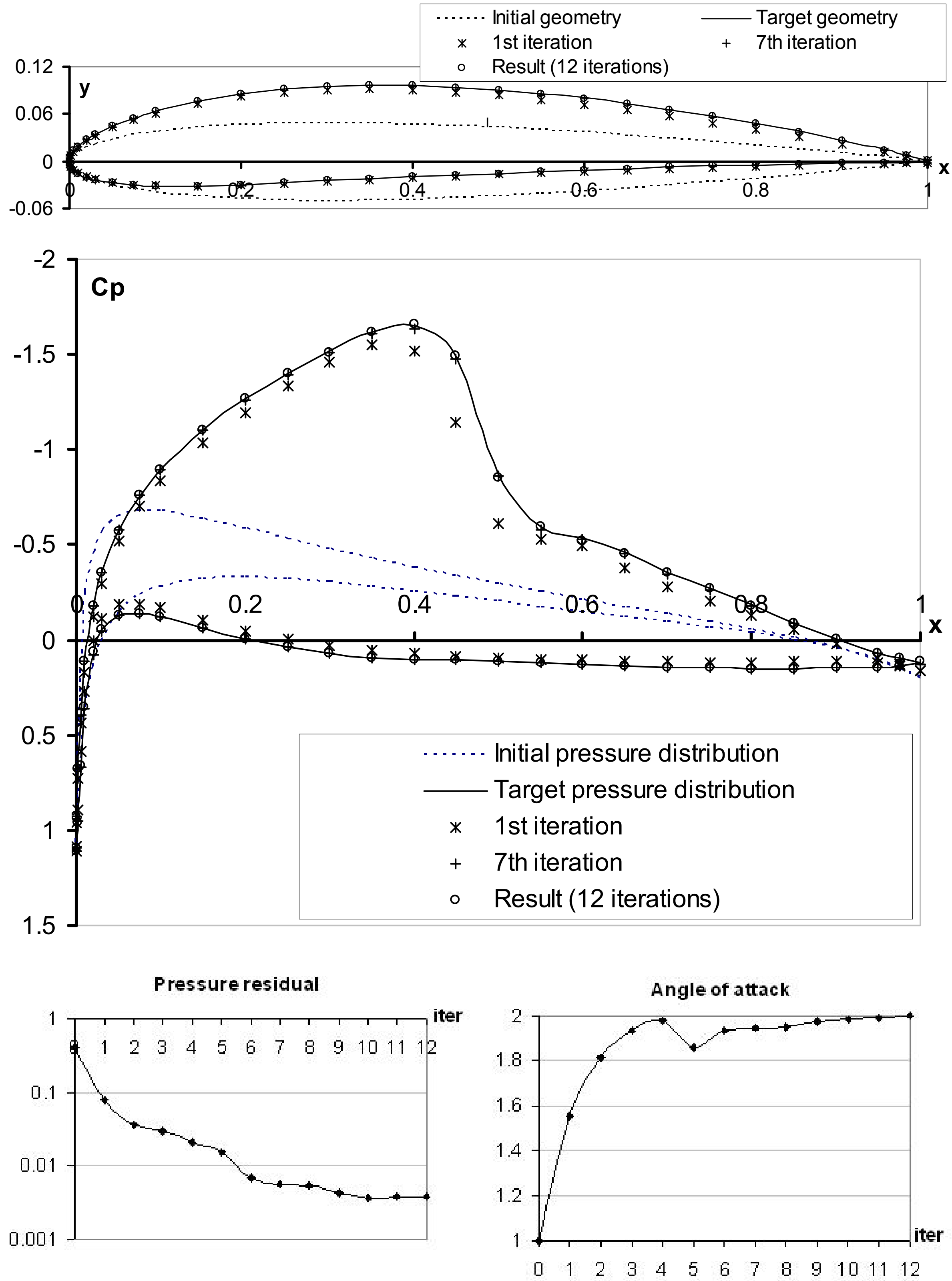

Fig. 6 Geometry recovery test for airfoil in transonic flow $M_\infty = 0.67$, $\alpha = 2°$.

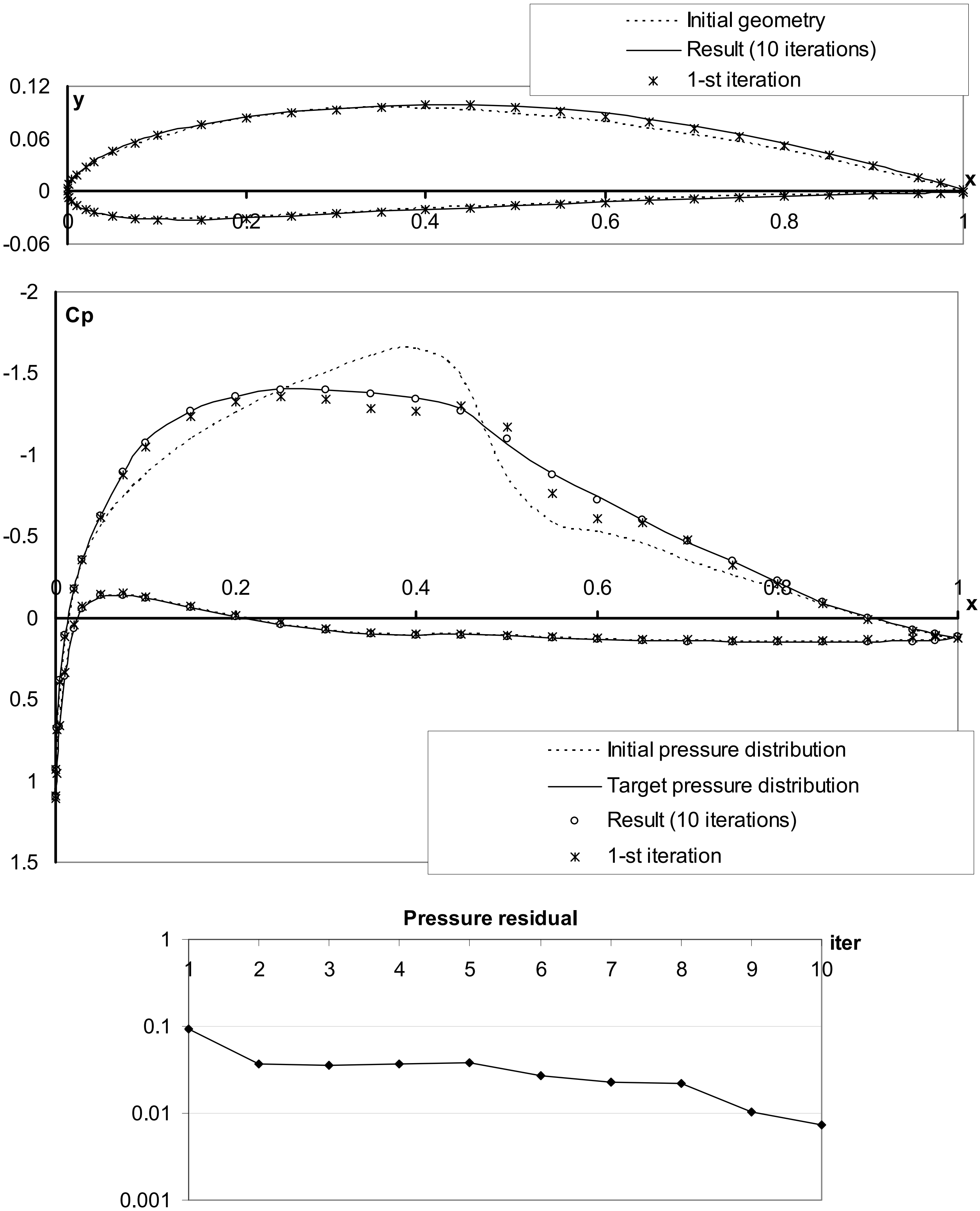

Fig. 7 Inverse problem solution for airfoil, target pressure distribution is set arbitrary.

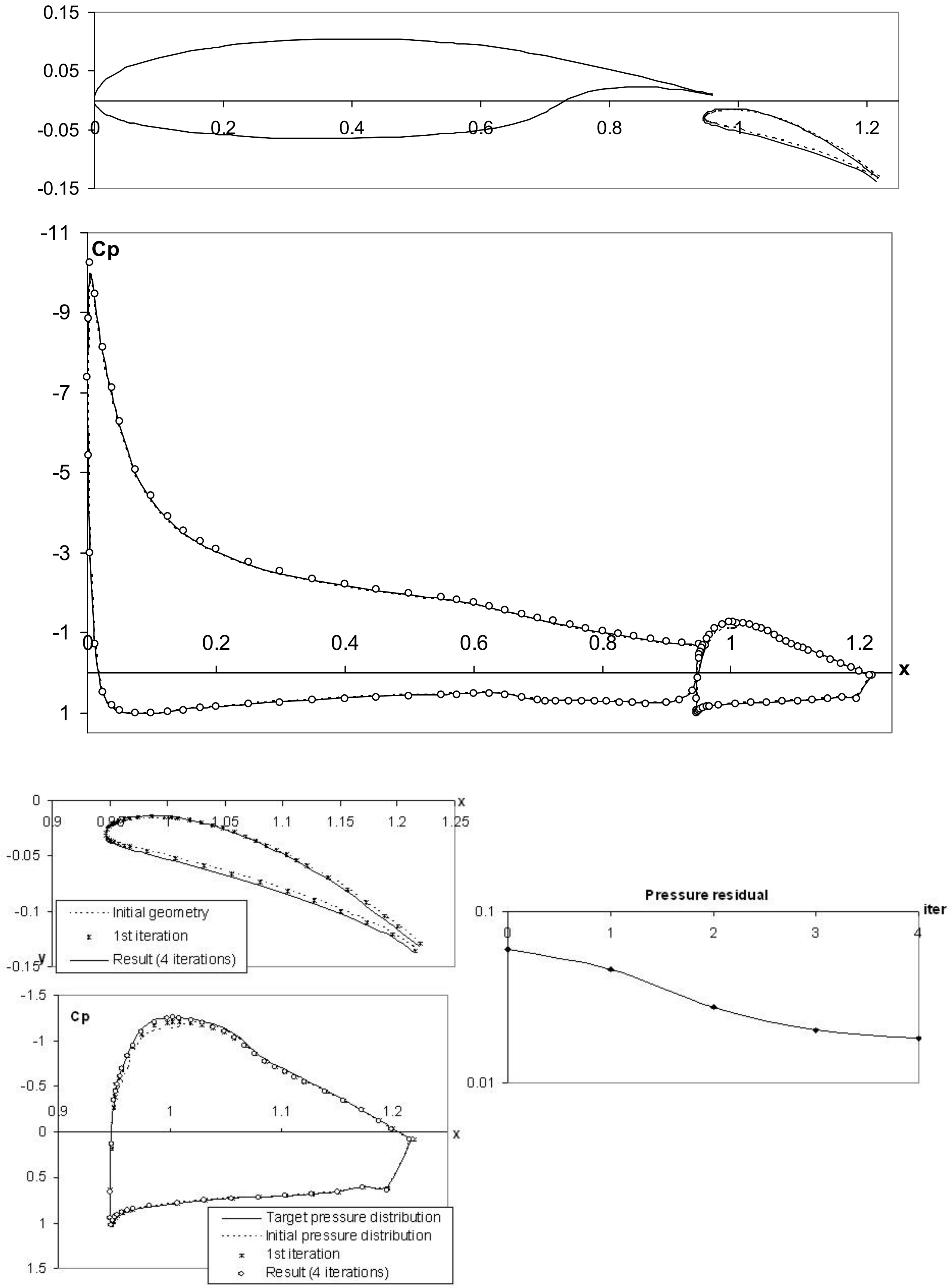

Fig. 8 Inverse problem solution for multielement airfoil.

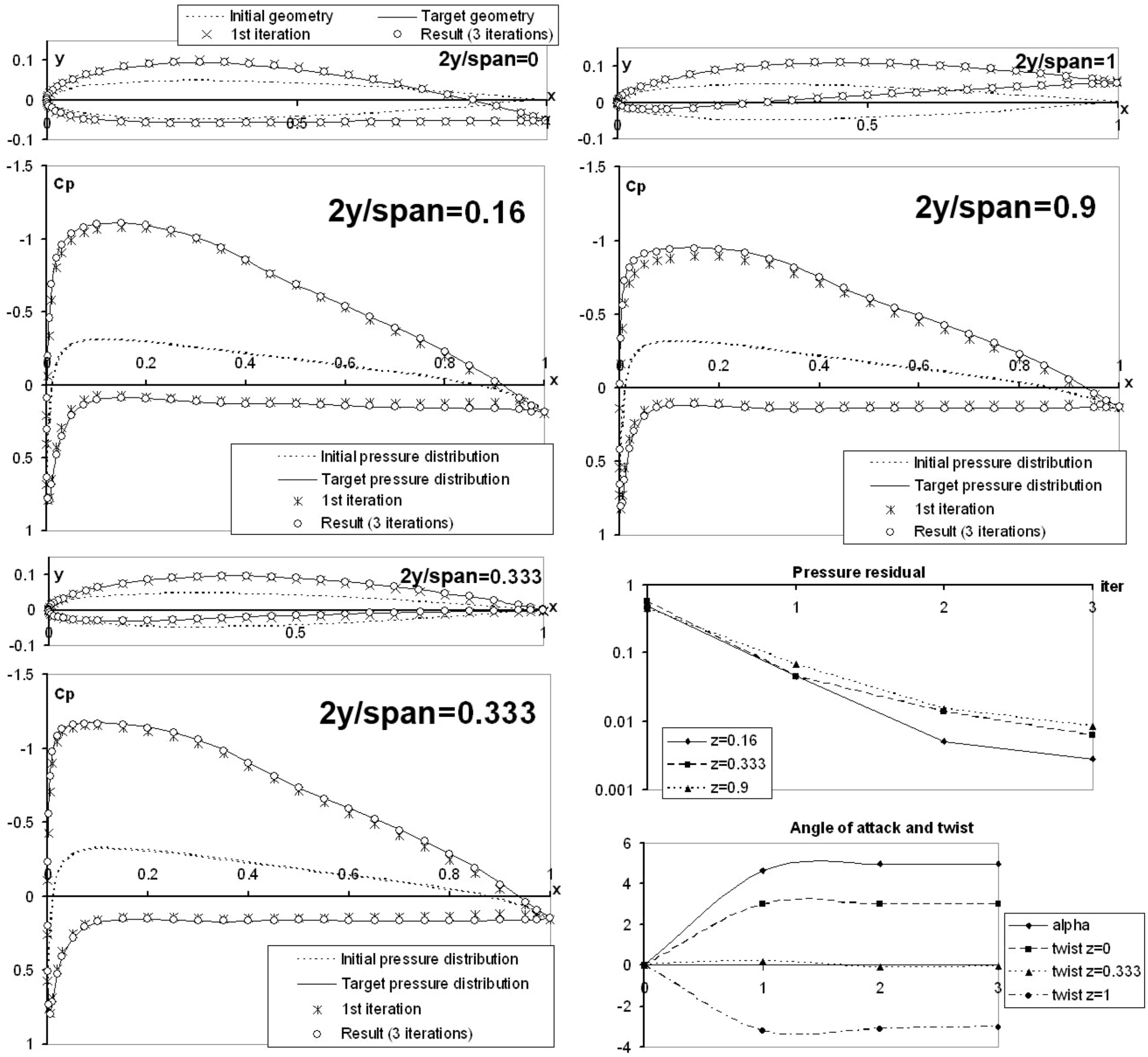

Fig. 9 Geometry recovery test for a wing at M=0.2

the inverse problem in an inviscid flow–TRAINV, TRAWDES–were used as a corrector.

Examples of inverse problem solution for single airfoil in transonic flow, multielement airfoil in subsonic flow and a wing in subsonic and transonic flow, that demonstrate the high efficiency of the method, are presented. The method can be further developed for more complex inverse problems, including three-dimensional problems for wing+fuselage and even for complete aircraft configuration at cruise and take-off-and-landing regimes.

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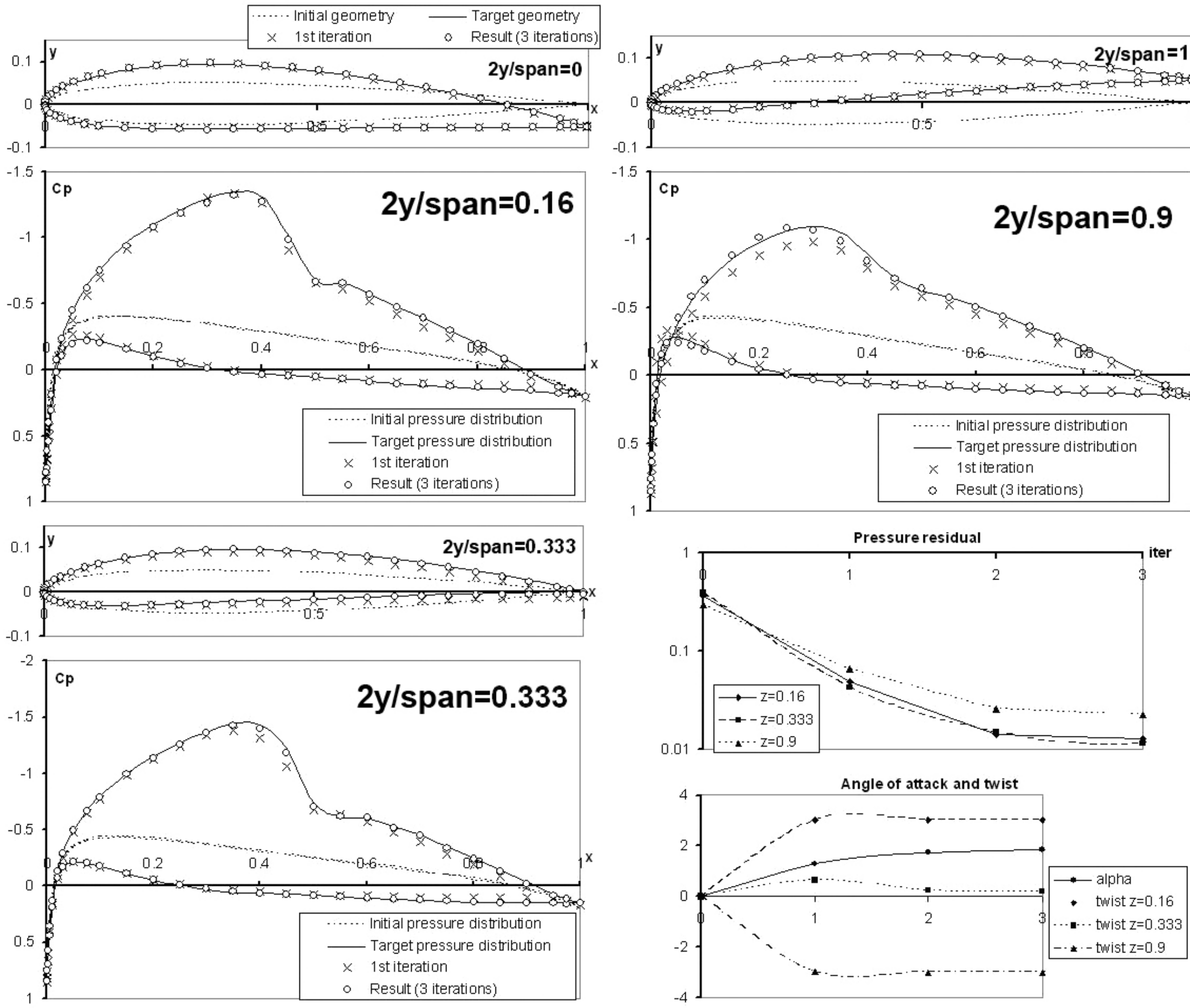

Fig. 10 Geometry recovery test for a wing at M=0.68

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