29th Congress of the International Council of the Aeronautical Sciences
St. Petersburg, Russia; September 7-12, 2014

OPTIMALITY CRITERIA FOR STRUCTURAL DESIGN OF AIRFRAMES WITH DYNAMIC REQUIREMENTS

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Keywords: *natural frequency, optimality criteria, airframes*

Abstract

We suggest the new optimality criteria for designing of aircraft structures with natural frequency limitations. These criteria were developed with taking into account the specificity of aircraft operation. We show that our criteria allow to obtain effective projects.

1 Introduction

In structural design often arises a problem of achievement of a given value of one or several natural frequencies. For aviation structure, such demands are linked with the aero elasticity phenomena, which are considered as the impermissible. The design problem in this case is formulated as following.

To define such stiffness performances of structural elements which without violation of strength requirements supply a given value of the lowest natural frequency of certain natural mode, for example, the lowest torsional or the lowest bending natural mode, and have a minimum volume of structural material.

Problems of such type were solved by many researchers, see in particular [1], however the received optimization criteria possess difficultly understandable physical meaning, does not discover properties of optimum designs and consequently is difficultly implanted in practical design. The reason to that, in our opinion, concludes in an excessive generality of the statement of problem, which are not considering the specificity of job of particular structures.

For aviation structures is typical, that the airframe has on itself heavy passive masses, namely: fuel, a power plant, a payload, aggregates and systems. For wings of modern airliners the part of mass of the load-bearing elements responsible for making its structure stiffness is 10…15 % from total wing mass, and for fuselages this part is equal 18…22 % from total fuselage mass. Therefore, at obtaining optimality criteria it is possible not to take into account the influence of redistribution of mass of a structural material onto natural frequencies, what allows to fill optimality criteria with accurate physical meaning and to discover properties of optimum designs. However, again we will underscore, that it is possible only in the presence of the large passive masses, which are not participating in optimization.

2 Optimality Criteria

2.1 Wings with large and middle aspect ratio

In this case the mode of structures deformation is defined enough exactly by the beam theory. Therefore we use the theory of thin-web beam and as object function for a flexural vibration mode we used the function of function from flexural stiffness *B(z)* along the wingspan *z*:

$$F = \int_0^L B(z)\,\mathrm{d}z\,; \tag{1}$$

We will determine natural frequency through a ratio of the Lord J. Rayleigh [2].

$$\omega^2 = \frac{\int_0^L B(z)\left[\dfrac{d^2 f(z)}{dz^2}\right]^2 dz}{\int_0^L m(z)\, f^2(z)\, dz} \tag{2}$$

Here *f(z)* is form of natural oscillations, and *f(z)* and *B(z)* do not depend from each other. Also,

m(z) is fixed per unit length mass distribution, *L* - length of a beam.
To minimize a functional

$$F = \int_0^L B(z)\, dz \;\Rightarrow\; \min, \qquad (3)$$

by isoperimetric condition

$$\omega^2 = \omega^2_0, \qquad (4)$$

where ω_0 - given frequency of any bending form of natural oscillations.
The extended functional with taking into account conditions (4) looks like

$$\Phi = \int_0^L B(z)\, dz + \lambda_1\left(\omega^2 - \omega_0^2\right), \qquad (5)$$

Where λ_1 - Lagrange multiplier. A necessary condition of functional (5) stationarity is the equality to zero of its total variation: $\delta\Phi = 0$, that gives:

$$\delta\Phi = \int_0^L \delta B(z)\, dz + \lambda_1\left\{ \frac{\int_0^L \delta\left[B(z)\, f''(z)^2\right] dz}{\int_0^L m(z)\, f^2(z)\, dz} - \frac{\int_0^L B(z)\, f''(z)^2 dz}{\left[\int_0^L m(z)\, f^2(z)\, dz\right]^2}\cdot 2\int_0^L m(z)\, f(z)\, \delta f(z)\, dz \right\} = 0. \qquad (6)$$

After some transformations, we get:

$$\delta\Phi = \int_0^L \left[1 + \lambda_1 \frac{f''(z)^2}{\int_0^L m(z)\, f^2(z)\, dz}\right] \delta B(z)\, dz + \frac{2\lambda_1}{\int_0^L m(z)\, f^2(z)\, dz} \times$$
$$\times\left\{ \int_0^L \left[\left\langle B(z)\, f''(z)\right\rangle'' - \omega^2\, m(z)\, f(z)\right] \delta f(z) - B(z)\, f''(z)\, \delta f'(z)\Big|_0^L - \left[B(z)\, f''(z)\right]'\, \delta f(z)\Big|_0^L \right\} = 0. \qquad (7)$$

As the variations *δB(z)* and *δf(z)* are mutually independent, following the basic lemma of calculus of variations we receive optimality criterion

$$\frac{f''(z)^2}{\int_0^L m(z)\, f^2(z)\, dz} = -\frac{1}{\lambda_1}; \qquad (8)$$

and differential equation of bending natural oscillations

$$\left[B(z)\, f''(z)\right]'' - \omega^2\, m(z)\, f(z) = 0 \qquad (9)$$

with boundary conditions

$$B(z)\, f''(z)\, \delta f'(z)\Big|_0^L = 0; \qquad (10)$$

$$[B(z)\, f''(z)]'\, \delta f(z)\Big|_0^L = 0. \qquad (11)$$

It is impossible to get the solution of a system (8)...(11) in the closed form, therefore we select some properties, which allow create designing algorithm for beam systems.
From a boundary problem (9)...(11) forms *f(z)* are determined within a constant coefficient, therefore by an appropriate normalization it is possible to achieve, that

$$\int_0^L m(z)\, f^2(z)\, dz = 1 \qquad (12)$$

at all variations *B(z)*. Then the optimality criterion (8) will accept a view:

$$f''(z)^2 = -\frac{1}{\lambda_1} = const. \qquad (13)$$

Following [3] on the basis of criterion (8) it is possible to receive a recursion formula for assignment of new bending stiffness *B(z)*, ensuring fulfilment of the given requirement. Let's multiply left and right member (8) on $B^s(z)$ and after elementary conversions we receive

$$B(z)_{l+1} = k_1\, B(z)_l\, \left| f''(z) \right|^{2/s}, \qquad (14)$$

here k_1 - some constant coefficient, *l* – iteration number, and s determines the size of a step.
The second derivative of beam sagging is a curvature of a bending neutral axis. Therefore, the *new bending stiffness need to be assigned proportionally to curvature of the form of bending oscillations, and by made assumptions the structure with stiffness distribution ensuring constant curvature will be optimum.*

Let's consider now torsional forms of natural oscillations. The extended functional for the taking into account the isoperimetric condition (4) will be written as

$$\Phi = \int_0^L C(z)\, dz + \lambda_2 \left\{ \frac{\int_0^L C(z)\ \varphi(z)^2\ dz}{\int_0^L i(z)\ \varphi(z)^2\ dz} - \omega_0^2 \right\}, \quad (15)$$

here *C(z)* - torsional stiffness, *φ(z)* - form of natural torsional vibrations, *i(z)* - per unit length polar moment of inertia of structural weights concerning centers of torsion. For thin-wall beams, which are carrying torsional stresses on Bredt [2], criterion function (15) determine volume of a structure material.

Repeating the mathematical manipulation, similar earlier conducted, allows to receive a final kind of optimality criterion

$$\varphi'(z)^2 = -\frac{1}{\lambda_2} = const, \quad (16)$$

And formula for assignment of torsional stiffness

$$C(z)_{l+1} = k_2\ C(z)_l \left| \varphi'(z) \right|^{2/s}, \quad (17)$$

Where k_2 - some constant coefficient, and s, as well as in (14), determines the size of a step.

The equations (14) and (17) can be extended. Really, according to the technical theory of a beam [2] bending and torsion moments are accordingly

$$M_{bend} = B(z)\ \frac{d^2 y}{dz^2}; \quad (18)$$

$$M_{tors} = C(z)\ \frac{d\theta}{dz}; \quad (19)$$

here *y(z)* - sag, and θ - angle of elastic twisting.

Comparing (14), (17) with (18), (19) we can see, that ***new stiffness, bending or torsional, it is necessary to assign proportionally to moment, bending or torsion, scaled on strained state appropriate to this or that form of natural oscillations.***

$$A(z)_{l+1} = k\ \left| M(z)_l \right|^{2/s}. \quad (20)$$

here *A(z)* - bending or torsional stiffness of a structure, *M(z)* - moment from deformation under any form of natural oscillations.

2.2 Wings with small aspect ratio

For the description of bending systems, such as wing panels with small curvature, wings with small aspect ratio, etc. the models based on plate theory are most common. Therefore, we deduce optimality criterion for isotropic and orthotropic plates with assumption that the deformation mode of a structure can be defined by the theory of thin plates.

As a criterion function we shall accept the functional

$$G = \iint_\Omega D(x,y)\ dx\ dy. \quad (21)$$

Hereinafter: *D(x,y)* - bending stiffness; Ω - plane area of a plate.

The minimization of such criterion function in general does not provide a minimum of a structure material volume, however for sandwich plates with variable, but not varied during optimization structural depth, and with filler absolutely rigid for shift deformation, the structural weight is determined by thickness of carrying layers, so both the minimum of a function (21) and volume of a material coincide.

Natural frequency *ω*, expressed through the ratio of the Lord J. Rayleigh, will be written as:

$$\omega^2 = \frac{\iint_\Omega D\left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left(\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} - \left\{\frac{\partial^2 w}{\partial x\, \partial y}\right\}^2\right)\right] dx\ dy}{\iint_\Omega \rho w^2\ dx\ dy}.$$

The frequency *ω* depends on stiffness distribution *D(x,y)* and on a vibration mode *w(x,y)*. Here *ρ(x,y)* - fixed mass distribution and μ - Poisonn ratio.

In the total, we have the following task of optimum designing. To find such stiffness distribution *D(x,y)*, which one delivers a minimum to a functional

$$G = \iint_\Omega D(x,y)\, \mathrm{dx\ dy} \Rightarrow \min \quad (23)$$

by isoperimetric limitation

$$\omega_0^2 - \omega^2 = 0, \quad (24)$$

here ω_0 – is required natural frequency.

Let's solve the task by the method of Lagrange multiplier. Let's record the extended functional as

$$L = \iint_\Omega D(x,y)\ dx\ dy + \lambda\left(\omega_0^2 - \omega^2\right), \quad (25)$$

here λ - is Lagrange multiplier.

Necessary condition of a stationarity of the functional (25) is the equality to zero its total variation, that is

$$dL = 0, \qquad (26)$$

that gives

$$\delta L = \delta \iint_{\Omega} D(x,y)\, dx\, dy + \lambda \left(\delta\omega_0^2 - \frac{\delta \iint_{\Omega} D\left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\mu)\left(\frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left\{ \frac{\partial^2 w}{\partial x\, \partial y} \right\}^2 \right) \right] dx\, dy}{\iint_{\Omega} \rho w^2\, dx\, dy} \right) = 0. \qquad (27)$$

After bulky but uncomplicated mathematical manipulations, we get the optimality criteria:

$$\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2\,(1-\mu) \left[\frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] = \frac{1}{\lambda} = const,$$

physical sense of which is not prime.

In a differential geometry is the concept of ***main surface curvatures*** defined as eigenvalues of a matrix of curvatures in the Cartesian coordinate system [4]:

$$K = \begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} & \dfrac{\partial^2 w}{\partial x \partial y} \\ \dfrac{\partial^2 w}{\partial x \partial y} & \dfrac{\partial^2 w}{\partial y^2} \end{bmatrix}. \qquad (29)$$

It is known, that the sum of main curvatures is equal the spur of matrix, and their product is equal matrix determinant (29). Having substituted these ratios in the optimality condition (28), we receive

$$K_1^2 + K_2^2 + 2\mu K_1 K_2 = \frac{1}{\lambda} = const\,, \quad (30)$$

here K_1 and K_2 - greatest and least (main) curvatures of plate surface deformed in according with the form $w(x,y)$. The bending moments M_1 and M_2 on a direction of main curvatures are determined as

$$M_1 = -D\,(K_1 + \mu K_2); \qquad M_2 = -D\,(K_2 + \mu K_1). \qquad (31)$$

With taking into account (31) optimality criterion (28) takes its final form:

$$\frac{M_1 K_1 + M_2 K_2}{D} = -\frac{1}{\lambda} = const. \qquad (32)$$

Thus, *with according to made assumptions*, ***in a structure having a minimum of bending stiffness and having required value of any natural frequency, the sum of products of main curvatures of the appropriate form and moments on direction of these curvatures divided on bending stiffness, should be constant in any point (x, y) of structure.***

Following [3] on the basis of criterion (32) it is possible to receive a recursion formula for assignment of new stiffness *D(x,y)*, ensuring fulfilment of the given requirement. Let's multiply left and right member (32) on $D^2(x,y)$ and after elementary conversions we receive

$$D_{\nu+1} = r\sqrt{D_\nu \left| M_1 K_1 + M_2 K_2 \right|}\,; \qquad (33)$$

here r - some constant coefficient, and ν - iteration number.

For real wing structures the formula (33) does not give a capability to construct iterative designing algorithm, because the actual wings are structurally orthotropic plates having different stiffness in a direction of spars and ribs, but in (33) we control only alone stiffness parameter. Therefore, criterion (32) has practical value only for isotropic plates of variable thickness and allows optimizing structures such as sandwich-panels, blades of turbines, sandwiching shells etc.

The orthotropic plate is featured both stiffness parameters and angle of orientation of orthotropy axes. We separate the task of optimization for orthotropic plate onto two independent tasks: 1) task of optimization stiffness parameters at constant orientation angle of orthotropy axes; 2) task of optimization of orthotropy axes orientation angle at constant distribution of a load-carrying material.

First task connected with material distribution determination when wing skeleton is known. The second task supposes the definition of direction of spars and ribs.

At first we receive the optimality conditions for the first of these tasks. For simplification of a criterion making let's direct the coordinates axis Ox and Oy along orthotropy axes. As design variables we shall accept bending $D_{11}(x,y)$, $D_{22}(x,y)$ and torsional $D_{66}(x,y)$ stiffneses of an orthotropic plate [5]. For such plate the square of a natural frequency is determined by Lord J. Rayleigh ratio:

$$\omega^2=\frac{\iint\limits_\Omega\left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dxdy}{\iint\limits_\Omega \rho w^2\,dx\,dy}.$$

The Poisson's constants along orthotropy axes express as

$$\mu_1=\frac{D_{12}}{D_{11}};\qquad \mu_2=\frac{D_{12}}{D_{22}}. \qquad (35)$$

With taking into account (35) is possible to eliminate stiffness D_{12} from (34). Then

$$\omega^2=\iint\limits_\Omega\left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}+\mu_1\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2}+D_{22}\left(\frac{\partial^2 w}{\partial y^2}+\mu_2\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial^2 w}{\partial y^2}+\right.$$

$$\left.+4D_{66}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]dx\,dy\Big/\iint\limits_\Omega \rho w^2\,dx\,dy. \qquad (36)$$

The criterion function should depend on a structural mass and be invariant concerning a direction of orthotropy axes. Therefore, we use such linear combination of plate stiffnesses, which one will be invariant relatively a rotation of orthotropy axes. In this case it is possible to optimize separately stiffness distribution and direction angle of orthotropy axes.

There are known [5] two invariants of stiffness of orthotropic plates:

$I_1 = D_{11}+D_{22}+2D_{12};\ I_2 = D_{12} - D_{66}.$ (37)

Let's make from these invariants a function, having eliminated stiffness D_{12}. In the total we receive

$I = I_1 - 2I_2 = D_{11} + D_{22} + 2D_{66}.$ (38)

The invariant I depends only on design variables D_{11}, D_{22}, D_{66} and does not change by rotation of orthotropy axes.

Let's formulate now following task of optimization. Among functions $D_{11}(x,y)$, $D_{22}(x,y)$, $D_{66}(x,y)$ to find such, which one provides minimum for a functional

$$J=\iint\limits_\Omega\left(D_{11}+D_{22}+2D_{66}\right)\,dx\,dy\Rightarrow\min, \qquad (39)$$

by isoperimetric limitation

$$\omega_0^2-\omega^2=0; \qquad (40)$$

Here ω is determined by Lord J. Rayleigh ratio (36) for an orthotropic plate at earlier made assumptions.

The extended functional after grouping terms containing like stiffness, we write in the form:

$$L=\iint\limits_\Omega D_{11}\left\{1-\lambda\frac{\left(\frac{\partial^2 w}{\partial x^2}+\mu_1\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2}}{\iint\limits_\Omega \rho w^2 dxdy}\right\}dxdy+\iint\limits_\Omega D_{66}\left\{1-\lambda\frac{\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2}{\iint\limits_\Omega \rho w^2 dxdy}\right\}dxdy+$$

$$+\iint\limits_\Omega D_{22}\left\{1-\lambda\frac{\left(\frac{\partial^2 w}{\partial y^2}+\mu_2\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial^2 w}{\partial y^2}}{\iint\limits_\Omega \rho w^2 dxdy}\right\}dxdy+\lambda\ \omega_0. \qquad (41)$$

We will operate the same, as for isotropic plates, and receive the optimality criterion:

$$\left(\frac{\partial^2 w}{\partial x^2}+\mu_1\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2}=\left(\frac{\partial^2 w}{\partial y^2}+\mu_2\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial^2 w}{\partial y^2}=2\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2=\frac{1}{\lambda};$$

With usage equations

$$\begin{cases} M_x=-D_{11}\left(\dfrac{\partial^2 w}{\partial x^2}+\mu_1\dfrac{\partial^2 w}{\partial y^2}\right);\\ M_y=-D_{22}\left(\dfrac{\partial^2 w}{\partial y^2}+\mu_2\dfrac{\partial^2 w}{\partial x^2}\right);\\ M_{xy}=-2D_{66}\left(\dfrac{\partial^2 w}{\partial x\partial y}\right);\end{cases} \qquad (43)$$

we record the optimality criterion as:

$$\frac{M_x\frac{\partial^2 w}{\partial x^2}}{D_{11}}=\frac{M_y\frac{\partial^2 w}{\partial y^2}}{D_{22}}=\frac{M_{xy}\frac{\partial^2 w}{\partial x\partial y}}{D_{66}}=-\frac{1}{\lambda}=const\,. \qquad (44)$$

Thus, with taking into account the made assumptions, ***in optimum by criterion (44) structures having required value of any natural frequency, the product of the appropriate form curvature on a direction orthotropy axes and moment on the same direction divided on appropriate stiffness, should be constant in any point (x,y) of structure.***

Based on optimality criterion (3.150) it is possible to receive recursion formulas for assignment of new stiffness:

$$\begin{cases} D_{11}^{\nu+1}=r\sqrt{D_{11}^{\nu}\left|M_x\dfrac{\partial^2 w}{\partial x^2}\right|};\\ D_{22}^{\nu+1}=r\sqrt{D_{22}^{\nu}\left|M_y\dfrac{\partial^2 w}{\partial y^2}\right|};\\ D_{66}^{\nu+1}=r\sqrt{D_{66}^{\nu}\left|M_{xy}\dfrac{\partial^2 w}{\partial x\partial y}\right|};\end{cases} \qquad (44)$$

here r - some constant coefficient, and ν - number of iteration.

The solution of task 2 based on searching the direction of orthotropy axes, which gives maximal natural frequency value by given stiffness distribution. We receive that ***in an optimum orthotropic structure a direction of maximum stiffness and direction of maximum curvature should coincide in any point.***

2.3 Designing algorithm for wings

With usage of obtained results is possible to construct simple algorithm of searching of distribution of a material ensuring given natural frequencies.

1. Let there is an initial stiffness distribution D_{ii}^{0} on structure elements (ii = 11, 22, 66).
2. We calculate frequencies and forms of natural oscillations of structure.
3. We shall route axes of stringer sets along lines of main curvatures of the surface, deformed on a vibration mode of a design or into direction, defined by the technological requirements, if those are available.
4. For constrained frequencies on expressions (44) we calculate new values $D_{ii,j}^{\nu+1}$ (ν - number of iteration, j - number of frequency).
5. We shall select maximum $D_{ii,\max}^{\nu+1}$ from all values assigned in the previous block:

$$D_{ii,\max}^{\nu+1} = \max_{j} \left(D_{ii,j}^{\nu+1}\right). \qquad (45)$$

6. If there is the strength limitation such as $D_{ii,j}^{\nu+1} \geq \left[D_{ii,\sigma}\right]$ then we select $D_{ii,1}^{\nu+1}$ maximal from $D_{ii,\max}^{\nu+1}$ and $\left[D_{ii,\sigma}\right]$:

$$D_{ii,1}^{\nu+1} = \max\left(D_{ii,\max}^{\nu+1}, \left[D_{ii,\sigma}\right]\right). \qquad (46)$$

7. If there is the technological limitation such as $D_{ii,j}^{\nu+1} \geq \left[D_{ii,h}\right]$ then we select $D_{ii}^{\nu+1}$ maximal from $D_{ii,1}^{\nu+1}$ and $\left[D_{ii,h}\right]$:

$$D_{ii}^{\nu+1} = \max\left(D_{ii,1}^{\nu+1}, \left[D_{ii,h}\right]\right). \qquad (47)$$

8. We go to point 2 with stiffness distribution $D_{ii}^{\nu+1}$.

We are continuing evaluations in cycle 2 - 8 either until fulfilment of the frequencies limitations, or until stabilization material distribution among elements. The second case show that within the framework of optimality criterion (44) it is impossible to find material distribution ensuring given frequencies and it is required to change a distribution law of passive masses, or proportionally increase stiffness for all load-bearing elements.

3 Theory application

Let's consider a hypothetical swept wing to which the engine is attached on a pylon. The computational model consists from wing, pylon and an engine nacelle and is displayed on Fig. 1.

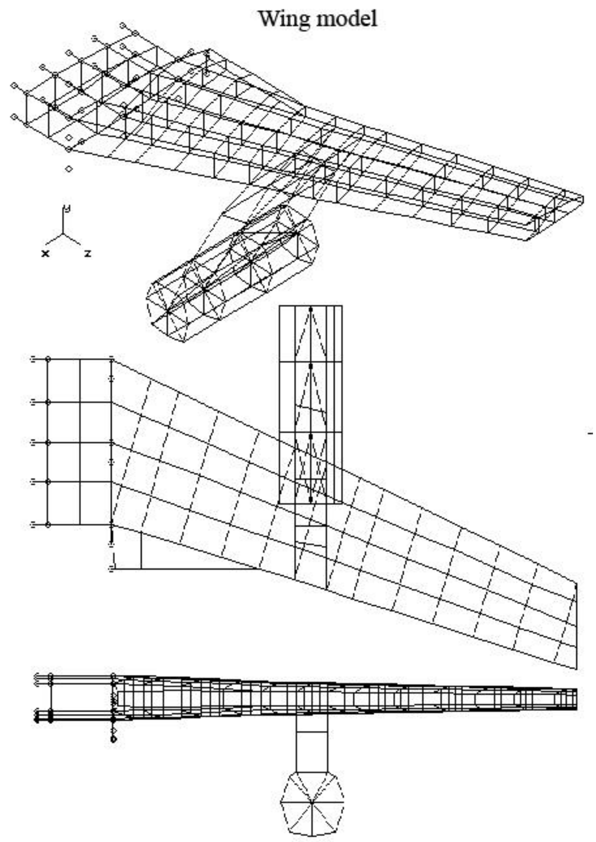

Fig. 1. Wing model

The wing has a span 10m, aspect ratio λ = 4.38, wing taper ratio η = 2.24. The engine is disposed on distance z=2490мм from an airplane axis. Sweepback angle at the leading edge is $\chi = 28^0$, along front spar $\chi = 25^0$. With the unit load on a wing equal p_0 = 450 dN/m^2 aircraft all-up mass is m_0=10280kg. With thrust-to-weight ratio $\bar{p}$ = 0.31 the engine thrust is

1600dN and its mass is m_{eng} = 320kg (the specific mass of the engine is 0.2).
With such initial data the structural optimization gives volume of full-strength design for half wing $V_{strength}$ = 0.0509m^3, that with the material density ρ=2780kg/m^3 leads to a mass of a structural material $m_{strength}$=142kg. The volume of a material of the pylon and the engine nacelle are included in this volume and, accordingly, in mass. Material distribution is shown on Fig. 2

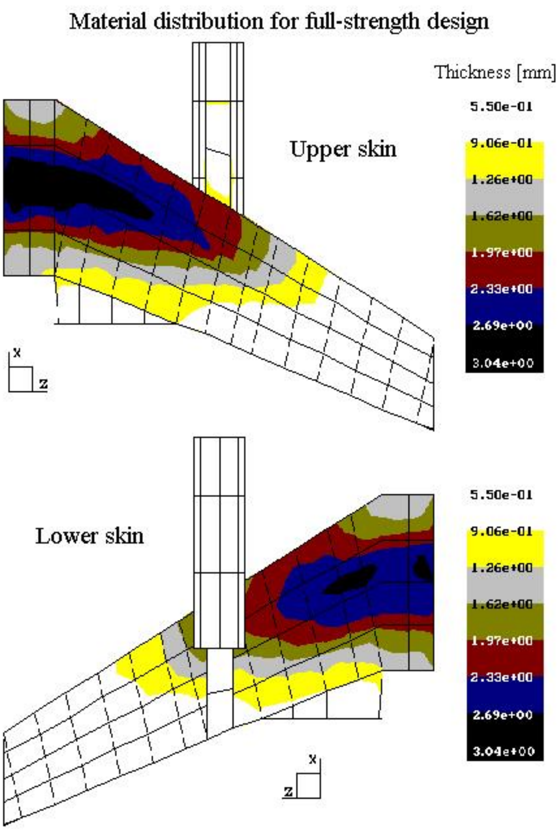

Fig. 2. Wing skin thickness for full-strength design

We take this material distribution as an initial allocation and compute natural modes and frequencies, which are presented in Table 1 and on Fig. 3.

Table 1. Natural frequencies of the wing.

Number of frequency	Value [1/s]	Form
ω_1	1.559	Bending
ω_2	1.939	Torsional
ω_3	2.203	Bending-torsional
ω_4	4.501	Bending-torsional
ω_5	5.467	Bending-torsional
ω_6	6.154	Bending-torsional

Value of passive masses is 797 kg and includes mass of non-structural elements of the wing, mass of the pylon and the engine nacelle, and also mass of the engine. Thus, the total mass of model presented on fig. 2 is 939 kg; from this mass the share of the structural material, responsible for creation of rigidity is equal 15%.

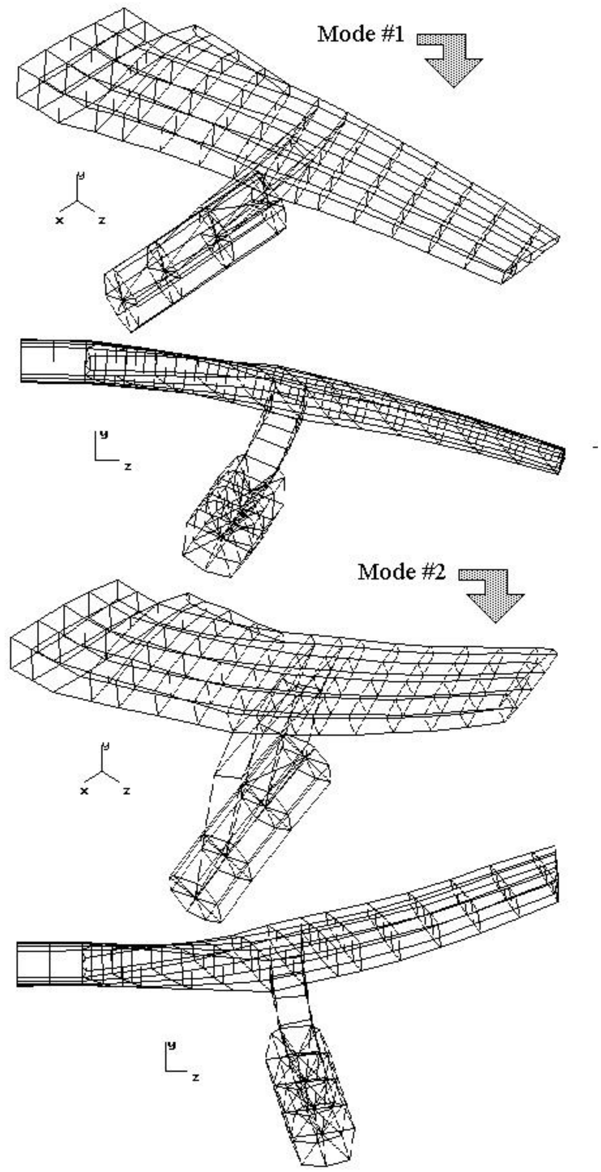

Fig. 3. Modes of natural vibrations.

From vibration modes on fig. 3 is visible, that natural vibrations represent combined moving of system wing - engine and consequently the lowest natural frequency is being determined by the least stiff aggregate in system the engine - pylon - wing. For example, if to increase thickness of skin of pylon from 0.55mm in the full-strength structure till 3mm then we get relative increasing of structural material mass on 1.8 %, and by other things being equal, the first natural frequency will be increased at 6.3 % with the saving the kind of a vibration mode. Let's conduct optimization of allocation of a material with the simultaneous taking into account the strength requirements and dynamic stiffness.

As the limitations of natural frequencies we will take the following values: ω_{10} = 2.027 1/s (magnification on 30%); ω_{20} = 2.908 1/s (magnification on 50%). We will vary thickness of skin of a pylon and a wing, a spars and ribs cap, and also walls of spars. New values of skin thickness we will assign as maximum from the bending and twisting moments in appropriate section, caps of spars - only from bending, and walls of spars - only from torsion moment. The suggested algorithm has converged with an exactitude of 5% after 5 iterations. The repetitive process course is represented in Table 2. The discovered material distribution is displayed on Fig. 4, and allocation of an additional material over the necessary on strength conditions on Fig. 5.

Table 2. Iterative process course.

Freq #	Initial	1 iter.	2 iter.	3 iter.	4 iter.	5 iter.
ω_1	1.559	2.118	1.906	2.041	1.865	2.039
ω_2	1.939	3.202	2.510	3.009	2.461	2.924
ω_3	2.203	4.415	2.942	4.503	2.885	4.431
ω_4	4.501	6.791	5.976	6.258	5.997	6.294
ω_5	5.457	8.051	7.159	7.013	7.641	7.272
ω_6	6.154	10.883	8.950	10.158	8.835	10.113
V [m^3]	0.0509	0.0956	0.0748	0.0867	0.0782	0.0821
m%	15	25	20.7	23.2	21.4	22.2

It is visible, that the additional material is necessary basically in places of connection of aggregates and a direction change of a structural elements and is placed in areas where it works effectively: for simultaneous magnification of the first natural frequency on 30% and second natural frequency on 50%, relative mass of structural material increased only on 7.2%.

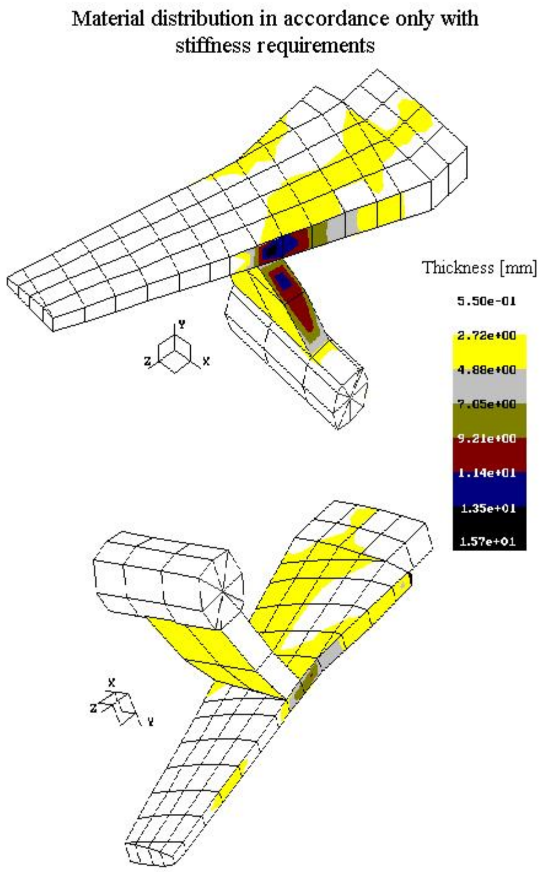

Fig. 4. Material distribution.

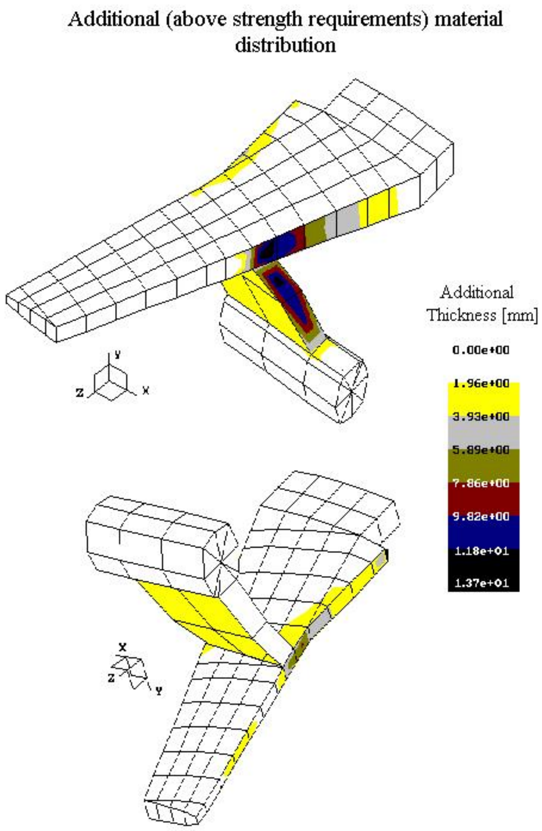

Fig. 5. Additional material distribution.

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