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ITERATIVE LEARNING IDENTIFICATION AGAINST NON-ZERO INITIAL STATES AND ESTIMATION OF AERODYNAMIC DERIVATIVES

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Abstract

This paper presents two techniques in iterative learning identification (ILI) when the zero initial state condition is not achieved. One is to obtain acceptable impulse responses. The other is to measure the response error to the exclusion of non-zero initial state factor. This paper proposes an estimation technique using the least-squares (LS) method for the former and introduces discarded data in measurement of the response error for the latter. The ILI with the proposed techniques is applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the proposed techniques is demonstrated in numerical simulations.

1 Introduction

Recently, a system identification technique using iterative learning control, [1], [2] called iterative learning identification (ILI) in this paper, has been developed for continuous-time systems. [3] - [8] Compared to system identification techniques based on the least-squares (LS) approaches, [9], [10] an advantage of the ILI technique is that it is robust against insufficient excitation because data used in parameter update computation are newly obtained at each iteration. Moreover, the measurement noise does not directly influence the estimated parameters because the derivatives of command signal rather than those of measured output are used in ILI. The procedures of ILI are roughly given as follows. Step 1: construct an iterative learning control system (ILCS) for identification, step 2: obtain the impulse responses in the ILCS, step 3: perform tracking control of the ILCS and measure response error, and step 4: update parameters to be identified. Step 3 and 4 are iteratively repeated until convergence of the parameters is accomplished. In ILI techniques which have been developed so far, [3] - [8] it has been assumed in step 2 and 3 that all initial state variables in the ILCS are set to be zero. It is called as *zero initial state condition* in this paper. As a matter of fact, the update law in step 4 does not guarantee the convergence of the parameters if the impulse responses and the response error are not obtained accurately. The zero initial state condition has therefore been required in ILI. It may be possible to achieve the zero initial state condition when the system to be identified is stable. In practice, however, it is not easy to realize the zero initial state condition in many cases such that disturbances and/or noises are included in the ILCS. Especially, when the system to be identified is unstable, the zero initial state condition cannot be achieved because a stabilizing controller has to be operated at the beginning of measuring the impulse responses and the response error.

To overcome this problem, this paper presents two techniques in ILI when the zero initial state condition is not achieved. One is to obtain acceptable impulse responses in step 2. This paper proposes a technique where the impulse responses are estimated by the LS method. [9] The other is to measure the response error so as to exclude factors due to non-zero initial state in step 3. A basic idea for this subject is that the response error data are sampled after the factors due to non-zero initial state are sufficiently reduced. To do this, discarded data are introduced in measurement of the response error. The ILI with the proposed techniques is applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the proposed techniques is discussed in numerical simulations.

2 ILI with Non-Zero Initial States

2.1 Identified system and parameters

The system to be identified in this paper is a multi-input and multi-output SS linear time invariant (LTI) system

$$\begin{cases} \dot{x}(t) = A_p(\eta)x(t) + B_p(\eta)u(t) \\ y(t) = C_p(\eta)x(t) + D_p(\eta)u(t) + v(t) \end{cases} \quad (1)$$

where $x(t) \in \boldsymbol{R}^{n_x}$ is the state, $u(t) \in \boldsymbol{R}^{n_u}$ the input, $y(t) \in \boldsymbol{R}^{n_y}$ the output and $v(t) \in \boldsymbol{R}^{n_y}$ is the noise included in $y(t)$. Additionally, $\eta \in \boldsymbol{R}^{q}$ is a q-dimensional vector that consists of state-space (SS) parameters to be identified and is called the SS parameter vector. The transfer function from $u(t)$ to $y(t)$ is represented by

$$P(p) \triangleq \frac{N(p)}{D(p)} \triangleq \frac{1}{D(p)} \begin{bmatrix} N_{11}(p) & \dots & N_{1n_u}(p) \\ \vdots & & \vdots \\ N_{n_y1}(p) & \dots & N_{n_yn_u}(p) \end{bmatrix} \quad (2)$$

$D(p)$ and $N_{ij}(p)$ are denominator and numerator polynomials of $P(p)$, respectively. p is the differential operator; that is,

$$p^l u(t) \triangleq \frac{d^l u(t)}{dt^l}. \quad (3)$$

In ILI, a command signal vector, denoted as $h(t) \in \boldsymbol{R}^{n_u}$, is needed to generate the reference for the output and the controlled input. It is assumed that the elements of $h(t)$ are smooth and are differentiable by n_x times. Using $h(t)$ and its derivatives in an ILCS whose structure is given by a tracking control system and will be shown in the following subsection, responses are measured at a specified time interval. The SS parameter vector is updated so as to reduce the response error. That is, ILI estimates the SS parameters by performing the tracking control and updating the SS parameter vector iteratively.

Hereafter for convenience, the iteration number is denoted as k. The signal vectors, estimated parameters and polynomials of the transfer functions at the k-th iteration are denoted as $(\cdot)^{(k)}$. Their true values are denoted as $(\cdot)^*$.

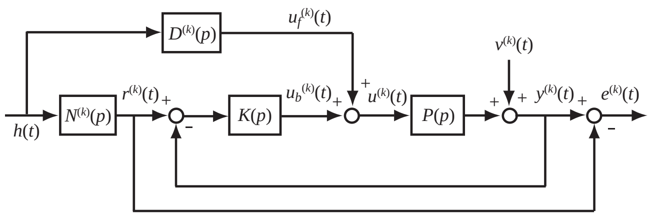

Fig. 1 An iterative learning control system (ILCS) for identification.

2.2 Response error with initial state

Figure 1 shows an iterative learning control system (ILCS) for identification. Here, $K(p)$ is an $n_u \times n_y$ feedback controller for stabilization. It does not matter whether the structure of $K(p)$ is known or not. $u_b^{(k)}(t) \in \boldsymbol{R}^{n_u}$ is the k-th iteration feedback input; $u_f^{(k)}(t) \in \boldsymbol{R}^{n_u}$ is the k-th iteration feedforward input generated by feeding the command $h(t)$ into the k-th iteration estimated denominator polynomial $D^{(k)}(p)$; $r^{(k)}(t) \in \boldsymbol{R}^{n_y}$ is the k-th iteration reference for $y^{(k)}(t)$ and is generated by feeding the command $h(t)$ into the k-th iteration estimated numerator polynomial $N^{(k)}(p)$.

The response error, denoted as $e^{(k)}(t) \in \boldsymbol{R}^{n_y}$, is defined as the difference between $y^{(k)}(t)$ and $r^{(k)}(t)$. Letting $x_{cl}^{(k)}(0)$ be the initial state of the closed-loop at the k-th iteration, the response error at the k-th iteration is given by

$$\begin{aligned} e^{(k)}(t) &\triangleq y^{(k)}(t) - r^{(k)}(t) \\ &= Y(p)u_f^{(k)}(t) - S(p)r^{(k)}(t) \\ &\quad + S(p)v^{(k)}(t) + f_{cl}(t)x_{cl}^{(k)}(0) \end{aligned} \tag{4}$$

where

$$S(p) \triangleq (I_{n_y} + P(p)K(p))^{-1},$$
$$Y(p) \triangleq (I_{n_y} + P(p)K(p))^{-1}P(p).$$

$f_{cl}(t)$ is an $n_y \times (n_x + n_c)$ time-function matrix which is constructed by the state transition matrix of the closed-loop. The zero initial state condition; that is, $x_{cl}^{(k)}(0) = 0$ is achieved when the system to be identified is stable and no external signal except $h(t)$ is fed into the ILCS before performing tracking control but not when disturbances and/or noises are always included in the ILCS. Especially, when the system to be identified is unstable, the zero initial state condition cannot be achieved because a stabilizing controller has to be operated at the beginning of measuring the impulse responses and the response error.

2.3 Procedures of ILI

The procedures of ILI are given as follows. [7], [8]

Step 1: The SS parameter vector η to be identified is defined. Construct an ILCS as shown in Fig. 1. If the system is unstable, provide a stabilizing controller $K(p)$. Otherwise, $K(p)$ may be omitted.

Step 2: Obtain the impulse responses of $S(p)$ and $Y(p)$, respectively. Set $k = 1$.

Step 3: Perform tracking control of the ILCS and measure the response error $e^{(k)}(t)$ for $t = 0, T_s, \cdots, NT_s$, where T_s is the sampling time and N is the number of sampled data.

Step 4: Update the SS parameter vector $\eta^{(k)}$ by the following law

$$\eta^{(k+1)} = \eta^{(k)} + H^{(k)}\mathbf{e}^{(k)} \tag{5}$$

where

$$\mathbf{e}^{(k)} \triangleq \begin{bmatrix} e^{(k)}(0) \\ \vdots \\ e^{(k)}(NT_s) \end{bmatrix},$$
$$H^{(k)} \triangleq -\alpha^{(k)} \left\{ (\Lambda\Psi^{(k)})^T (\Lambda\Psi^{(k)}) \right\}^{-1} (\Lambda\Psi^{(k)})^T,$$
$$\Lambda \triangleq [-\mathbf{G}_S\Gamma_b \;\; \mathbf{G}_Y\Gamma_a], \quad \Psi^{(k)} \triangleq \frac{\partial\theta(\eta^{(k)})}{\partial\eta^T}.$$

$\alpha^{(k)}$ is a non-decreasing gain with respect to the iteration number k. $\mathbf{G}_S$ and $\mathbf{G}_Y$ are block lower triangular matrices which consist of impulse responses of $S(p)$ and $Y(p)$, respectively. θ is the TF parameter vector which consists of coefficients of $D(p)$ and $N_{ij}(p)$. That is, $\Psi^{(k)}$ is the gradients of θ with respect to $\eta^{(k)}$. Γ_a and Γ_b are constructed by $h(t)$ and its derivatives. More details are given in Refs. [7] and [8].

Step 5: Judge the convergence of the response error and the SS parameters. If iteration continues, set $k+1 \to k$ and go to step 3. Otherwise, stop.

Since $H^{(k)}$ in Eq. (5) contains the impulse responses of $S(p)$ and $Y(p)$ obtained at step 2, they should be obtained as precisely as possible. Otherwise, the SS parameters are not estimated accurately. One of methods for obtaining the impulse response is to use a pseudo-impulse input which will be described in the following section. In this method, the zero initial state condition is required. Moreover, if the response error $e^{(k)}(t)$ includes non-zero initial state $x_{cl}^{(k)}(0) \neq 0$, the convergence of the SS parameter by the update law Eq. (5) is not guaranteed. The rest of this paper presents two techniques when the zero initial state condition is not achieved. For the former problem, section 3 will show a technique where the impulse responses are estimated by the LS method. For the latter, section 4 will show a technique where the response error is measured so as to reduce the factors due to non-zero initial states.

3 Impulse Response Estimation by LS Method

This section explains estimation of the impulse response by the LS method in terms of the sampled signals and impulse responses of an LTI system.

3.1 Sampled signal and impulse response

For discretizing a stable LTI system by the 0-th order held with the sampling time T_s, its SS representation is written as

$$\begin{cases} x((i+1)T_s) = Ax(iT_s) + Bu(iT_s) \\ y(iT_s) = Cx(iT_s) + Du(iT_s) \end{cases} \tag{6}$$
$$x \in \boldsymbol{R}^{n_x},\ \ u \in \boldsymbol{R}^{n_u},\ \ y \in \boldsymbol{R}^{n_y}$$

When the input $u(t)$ is given by the j-th *pseudo-impulse input sequence* which is fired at the m-th sampling

$$u(iT_s) \triangleq \begin{cases} [0\ \cdots\ 0]^T & (i < m) \\ [\delta_{j,1}\ \cdots\ \delta_{j,n_u}]^T & (i = m) \\ [0\ \cdots\ 0]^T & (i > m) \end{cases} \tag{7}$$

where $\delta_{k,l}$ is Kronecker's δ-function

$$\delta_{k,l} \triangleq \begin{cases} 1 & (k = l) \\ 0 & (k \neq l), \end{cases} \tag{8}$$

the state and the output for $i \geq m$ are given by

$$\begin{cases} x((i+1)T_s) = A^{i+1}x(0) + B_j \\ y(iT_s) = CA^i x(0) + D_j \end{cases} \quad (i = m),$$
$$\begin{cases} x((i+1)T_s) = A^{i+1}x(0) + A^{i-m}B_j \\ y(iT_s) = CA^i x(0) + CA^{i-m-1}B_j \end{cases} \quad (i > m) \tag{9}$$

where B_j and D_j are the j-th column vectors of B and D, respectively. Expanding the above for $m = 0, 1, \cdots, i$, $j = 1, \cdots, n_u$, the output $y(iT_s)$ for an arbitrary input sequence

$$\{u(0),\ u(T_s),\ \cdots,\ u(iT_s)\} \tag{10}$$

is expressed as follows.

$$y(iT_s) = f(iT_s)x(0) + \sum_{m=0}^{i} g(mT_s)u((i-m)T_s) \tag{11}$$

where

$$f(iT_s) \triangleq CA^i,$$
$$g(iT_s) \triangleq \begin{cases} D & (i = 0) \\ CA^{i-1}B & (i > 0) \end{cases}$$

$g(iT_s)$ is the impulse response matrix. If the zero initial state condition $x(0) = 0$ is achieved, the first term in the right hand side of Eq. (11) is zero. The impulse response vector for the j-th input channel, denoted as $g_j(iT_s) \in \boldsymbol{R}^{n_y}$ can be then obtained using the j-th pseudo-impulse input sequence of Eq. (7) where m is given by $m = 0$; that is, $y(iT_s) = g_j(iT_s)$. Conversely, if the zero initial state condition is not achieved, $y(iT_s)$ includes the first term. $x(0)$ and/or $f(iT_s)$ are usually unknown. It is therefore impossible to obtain the impulse response using the pseudo-impulse input sequence when the zero initial state condition is not achieved.

3.2 LS estimation

To obtain acceptable finite impulse response even if the zero initial state condition is not achieved, this paper estimates the impulse response by the LS method. For simplicity, consider a multi-input and single-output system ($n_u \geq 1$, $n_y = 1$). Let the number of the estimated impulse response be $M+1$. For $i \leq M$, $y(iT_s)$ is given by Eq. (11). While for $i > M$, $y(iT_s)$ is approximated by finite impulse response

$$y(iT_s) \simeq f(iT_s)x(0) + \sum_{m=0}^{M} g(mT_s)u((i-m)T_s) \tag{12}$$

where $u(t) = 0$ $(t < 0)$. Equation (12) includes an error due to finite number of the impulse response. Expressing the sampled data vectors for $i = 0, 1 \cdots, N$ by the boldface letters, the sampled data vector of the output is given by

$$\mathbf{y} \simeq \mathbf{U}\mathbf{g} + \mathbf{f}x(0) \tag{13}$$

where

$$\mathbf{y} \triangleq \begin{bmatrix} y(0) \\ \vdots \\ y(NT_s) \end{bmatrix}, \mathbf{f} \triangleq \begin{bmatrix} f(0) \\ \vdots \\ f(NT_s) \end{bmatrix} \in R^{N+1},$$

$$\mathbf{g} \triangleq \begin{bmatrix} g^T(0) \\ \vdots \\ g^T(MT_s) \end{bmatrix} \in R^{n_u(M+1)}$$

$$\mathbf{U} \triangleq \begin{bmatrix} u^T(0) & & 0 \\ \vdots & \ddots & \\ u^T(MT_s) & \dots & u^T(0) \\ \vdots & & \vdots \\ u^T(NT_s) & \dots & u^T((N-M)T_s) \end{bmatrix}$$
$$\in R^{(N+1)\times n_u(M+1)}$$

Since the system in Eq. (6) is stable, $g(iT_s)$ is asymptotically reduced according to increase of i. Therefore, the equation error due to finite number of the impulse response is decreased. $f(iT_s)$ is also asymptotically reduced according to increase of i; that is, $\mathbf{f}x(0)$ is reduced. Then, in this paper, the impulse response is estimated by giving N and M sufficiently large numbers. Applying the LS method to Eq. (13), the estimated impulse response vector $\mathbf{g}^h \in R^{n_u(M+1)}$ is obtained as

$$\mathbf{g}^h = (\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{y}. \tag{14}$$

$u(t)$ must be given so that $\mathbf{U}$ is the full column rank. As one of candidates, $u(t)$ is given by a random signal. To extend the above for multi-output systems, Eq. (14) is applied for each output channel.

4 Response Error Measurement with Discarded Data

As pointed out in section 2, the fourth term in the right hand side of Eq. (4), $f_{cl}(t)x_{cl}^{(k)}(0)$ is added in $e^{(k)}(t)$ if the zero initial state condition is not achieved. As a result, the parameters to be identified and the response error do not appropriately converge. It is therefore desirable to reduce the term due the non-zero initial state as small as possible. Since the closed-loop of the ILCS is stable, $f_{cl}(t)x_{cl}^{(k)}(0)$ is reduced according to increase of time. Then, the response error sampling is modified by introducing discarded data.

Let l be the number of the discarded data. Letting $t=0$ be the start time at which tracking control of the ILCS is performed, the response error is measured from $t=lT_s$. Then, the response error is written in the form of the sampled data vectors denoted by the boldface letters as follows.

$$\mathbf{e}^{(k)} \simeq \mathbf{G}_Y\mathbf{u}_f^{(k)} - \mathbf{G}_S\mathbf{r}^{(k)} + \mathbf{G}_S\mathbf{v}^{(k)} + \mathbf{F}_{cl}x_{cl}^{(k)}(0) \tag{15}$$

where

$$\mathbf{e}^{(k)} \triangleq \begin{bmatrix} e^{(k)}(lT_s) \\ \vdots \\ e^{(k)}((l+N)T_s) \end{bmatrix}, \mathbf{u}_f^{(k)} \triangleq \begin{bmatrix} u_f^{(k)}(0) \\ \vdots \\ u_f^{(k)}((l+N)T_s) \end{bmatrix},$$
$$\mathbf{r}^{(k)} \triangleq \begin{bmatrix} r^{(k)}(0) \\ \vdots \\ r^{(k)}((l+N)T_s) \end{bmatrix}, \mathbf{v}^{(k)} \triangleq \begin{bmatrix} v^{(k)}(0) \\ \vdots \\ v^{(k)}((l+N)T_s) \end{bmatrix},$$
$$\mathbf{G}_S \triangleq \begin{bmatrix} g_S(lT_s) & \dots & g_S(0) & \dots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ g_S((l+N)T_s) & \dots & g_S(NT_s) & \dots & g_S(0) \end{bmatrix},$$
$$\mathbf{G}_Y \triangleq \begin{bmatrix} g_Y(lT_s) & \dots & g_Y(0) & \dots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ g_Y((l+N)T_s) & \dots & g_Y(NT_s) & \dots & g_Y(0) \end{bmatrix},$$
$$\mathbf{F}_{cl} \triangleq \begin{bmatrix} f_{cl}(lT_s) \\ \vdots \\ f_{cl}(l+N)T_s) \end{bmatrix} \tag{16}$$

where $g_S(t)$ and $g_Y(t)$ are the impulse response matrices of $S(p)$ and $Y(p)$, respectively. That is, $u_f^{(k)}(t)$ and $r^{(k)}(t)$ are input to the ILCS during $t \in [0,\ (l+N)T_s]$, while $e^{(k)}(t)$ is measured during $t \in [lT_s,\ (l+N)T_s]$.

Including the techniques which have been described in sections 3 and 4 into the procedures of ILI, step 2 and 3 shown in section 2.3 are modified as follows.

Step 2': Obtain the impulse responses of $S(p)$ and $Y(p)$; that is, $g_S(t)$ and $g_Y(t)$ for $t =$

$0, T_s, \cdots, (l+N)T_s$ and construct $\mathbf{G}_S$ and $\mathbf{G}_Y$ in Eq. (16). Set $k=1$.

Step 3': Perform tracking control of the ILCS and measure the response error $e^{(k)}(t)$ for $t = lT_s, \cdots, (l+N)T_s$.

The larger l is, the more the factors due to non-zero initial states are reduced but the longer the measurement time becomes. An index for designing l will be given in estimation of the aerodynamic derivatives in the following section. The necessity of the feedback controller $K(p)$ will be also mentioned.

5 Estimation of Aerodynamic Derivatives in Lateral Linear Model of Aircraft

The ILI with the proposed techniques is applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft in this section. The SS representation of the lateral motion of aircraft is given in the form of Eq. (1) where the state and input vectors x and u are given by [11]

$$x \triangleq [\beta \ \ \phi \ \ p \ \ r]^T, \ \ u \triangleq [\delta_a \ \ \delta_r]^T. \tag{17}$$

Here, x consists of the side slip angle β, the roll angle ϕ, the roll rate p (not the differential operator of the transfer function here) and the yaw rate r. u consists of the aileron deflection angle δ_a and the rudder deflection angle δ_r. These variables represent the deviation from the equilibria. A_p and B_p are given as

$$A_p = E_p^{-1} F_p, \ \ B_p = E_p^{-1} G_p, \tag{18}$$

where

$$E_p \triangleq \begin{bmatrix} V_a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -I_{xz}/I_{xx} \\ 0 & 0 & -I_{xz}/I_{zz} & 1 \end{bmatrix}, \ G_p \triangleq \begin{bmatrix} 0 & Y_{\delta_r} \\ 0 & 0 \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix},$$

$$F_p \triangleq \begin{bmatrix} Y_\beta & g\cos\Theta_0 & Y_p & Y_r - V_a \\ 0 & 0 & 1 & \tan\Theta_0 \\ L_\beta & 0 & L_p & L_r \\ N_\beta & 0 & N_p & N_r \end{bmatrix}.$$

V_a is the flight velocity and Θ_0 is the pitch angle at the equilibrium. I_{xx} and I_{zz} are the moments of inertia in the x-axis and z-axis, respectively. I_{xz} is the product of inertia. Y_β, Y_p, etc., are the aerodynamic derivatives to be identified.

The output y is defined by the following two cases.

(O1) Two-outputs:

$$y \triangleq [\beta \ \ \phi]^T \tag{19}$$

(O2) Four-outputs:

$$y \triangleq [\beta \ \ \phi \ \ p \ \ r]^T = x \tag{20}$$

The SS parameter vector η is constructed by the aerodynamic derivatives that are assigned in advance. In this paper, the following three cases are examined.

Case 1: The output y is given by (O1). η is constructed by

$$\eta = [L_\beta \ \ L_p \ \ N_\beta]^T \in \boldsymbol{R}^3. \tag{21}$$

Case 2: The output y is given by (O1). η is constructed by

$$\eta = [N_r \ \ L_{\delta_a} \ \ N_{\delta_r}]^T \in \boldsymbol{R}^3 \tag{22}$$

Case 3: The output y is given by (O2). η is constructed by

$$\eta = [Y_\beta \ \ Y_r \ \ L_\beta \ \ L_p \ \ L_r \ \ N_\beta \ \ N_p \ \ N_r \ \ Y_{\delta_r} \ \ L_{\delta_a} \ \ N_{\delta_r}]^T \in \boldsymbol{R}^{11}. \tag{23}$$

The command signal $h(t) \in \boldsymbol{R}^2$ is given by

$$h(t) = \frac{2^6}{(p+2)^6} w(t) \tag{24}$$

where the elements of $w(t) \in \boldsymbol{R}^2$ are given by white noises. The measurement noise $v^{(k)}(t) \in \boldsymbol{R}^2$ is given by the white noise whose noise signal ratio (NSR) is 20%, where NSR is defined as

$$\mathrm{NSR} \triangleq \frac{||v^{(k)}(t)||}{||y^{(k)}(t)||}. \tag{25}$$

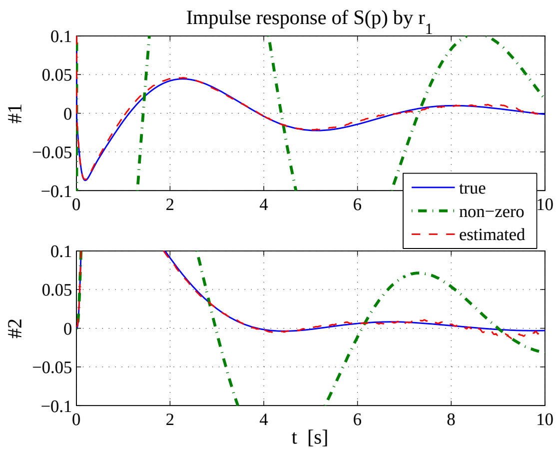

Fig. 2 Impulse response of $S(p)$ by $r_1(t)$.

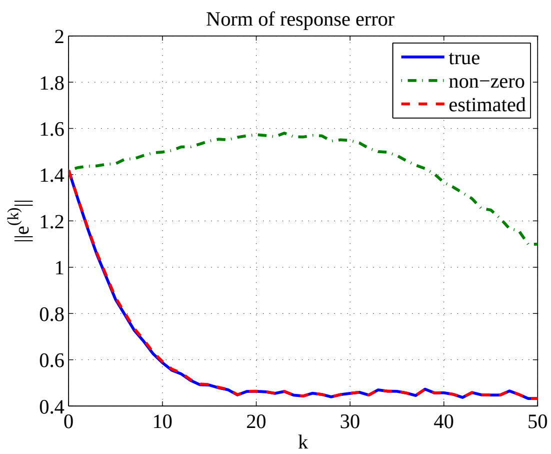

Fig. 4 Norm of response error using three kinds of impulse responses in Case 1.

Numerical data of aircraft considered in this study are referred from Ref. [12]). The flight conditions are given by the altitude $H = 4,000$ [m] and the flight velocity $V_a = 100$ [m/s]. Although the aircraft model to be identified is stable, $K(p)$ is given by an LQG controller whose weighting matrices of the quadratic index are given by $Q = 0.1I_4$ and $R = I_2$, and covariance matrices of disturbance and noise are given by $W = 10^5 B_p B_p^T$ and $V = I_2$.

5.1 ILI using estimated impulse responses

This subsection presents results of estimation of aerodynamic derivatives by ILI using estimated impulse responses which were described in section 3. Figures 2 and 3 show the impulse responses of $S(p)$ by $r_1(t)$ and $Y(p)$ by $u_{f_1}(t)$, respectively where the output $y(t)$ is given by (O1). "true", drawn by the solid-line, means the impulse responses when the zero initial state condition is achieved. "non-zero", drawn by the dash-dotted-line, means the ones with a non-zero initial state $x_p(0) = [\pi/180\ 0\ 0\ 0]^T$ (side slip angle $\beta(0) = 1$ [deg]). "estimated", drawn by the dashed-line, means the ones estimated by the LS method mentioned in section 3.2. The sampling time was $T_s = 0.01$ [sec]. The number of sampled data was $N = 1,000$. It can be seen that a little non-zero initial state of side slip angle caused large differences between "true" and "non-zero". The estimated impulse responses ("estimated") almost approximated the true responses ("true").

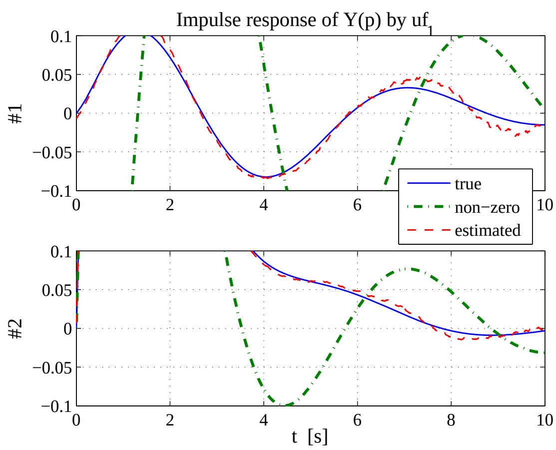

Fig. 3 Impulse response of $Y(p)$ by $u_{f_1}(t)$.

As results of Case 1, Fig. 4 shows the norm of the response error $\mathbf{e}^{(k)}$ for fifty iterations where the impulse responses are "true", "non-zero" and "estimated". Figures 5-7 show the estimated SS parameters η_i $(i = 1,2,3)$, respectively. The initial SS parameter vector was given by $\eta^{(0)} = [-1\ -1\ -1]^T$. The response error and the SS parameters in the case of "estimated" were almost similar to those in the case of "true". On the other hand, the SS parameters in the case of "non-zero" did not converge to the true values within fifty iterations. Table 1 shows the estimated SS parameters (aerodynamic derivatives) at iteration $k = 50$ using the estimated impulse response in Case 1. Tables 2 and 3 show the results in Cases 2 and 3, respectively.

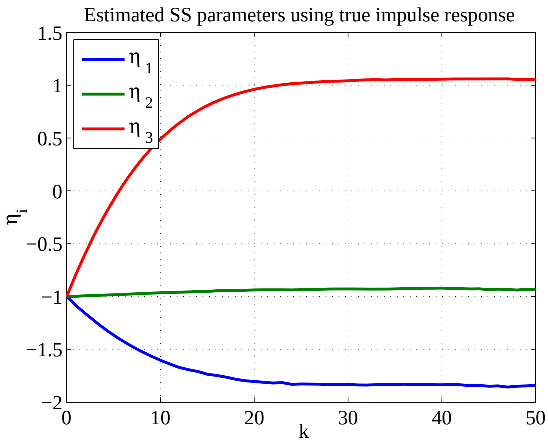

Fig. 5 Estimated SS parameters using “true” impulse response in Case 1.

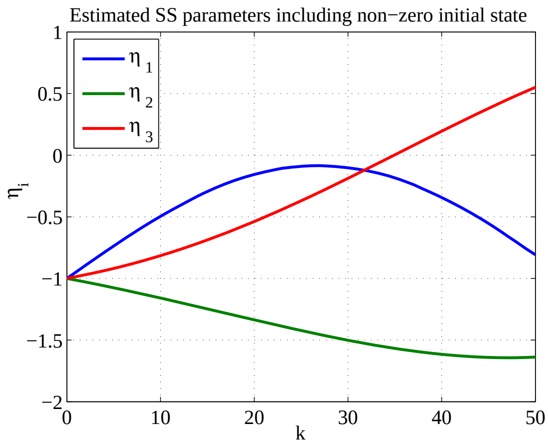

Fig. 6 Estimated SS parameters including “non-zero” impulse response in Case 1.

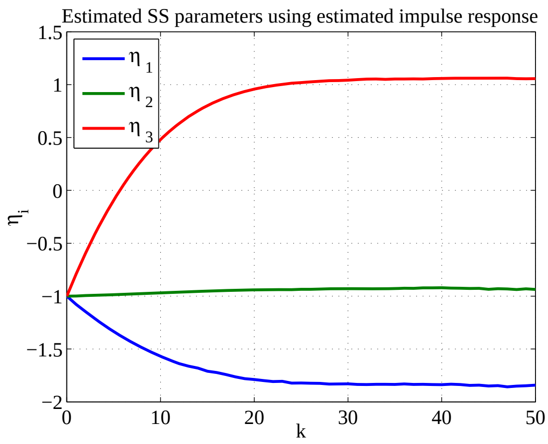

Fig. 7 Estimated SS parameters using “estimated” impulse response in Case 1.

Table 1 Estimated SS parameters at fifty iteration using estimated impulse response in Case 1.

η_i	η^*	$\eta^{(50)}$ (true)	$\eta^{(50)}$ (non-zero)	$\eta^{(50)}$ (estimated)
η_1	-1.8741	-1.8408	-0.8080	-1.8417
η_2	-0.9709	-0.9363	-1.6382	-0.9361
η_3	1.0611	1.0550	0.5511	1.0562

Table 2 Estimated SS parameters at fifty iteration using estimated impulse response in Case 2.

η_i	η^*	$\eta^{(50)}$ (true)	$\eta^{(50)}$ (non-zero)	$\eta^{(50)}$ (estimated)
η_1	-0.2111	-0.1836	-1.8372	-0.1833
η_2	4.5397	4.5831	4.1466	4.5827
η_3	-0.7199	-0.7116	2.3956	-0.7113

Table 3 Estimated SS parameters at fifty iteration using estimated impulse response in Case 3.

η_i	η^*	$\eta^{(50)}$ (true)	$\eta^{(50)}$ (non-zero)	$\eta^{(50)}$ (estimated)
η_1	-15.5655	-14.3246	18.8118	14.1995
η_2	0.8346	1.2218	1.0674	1.1553
η_3	-1.8741	-1.8937	-3.0481	-1.8938
η_4	-0.9709	-0.9318	-3.7421	-0.9325
η_5	0.2640	0.3227	-7.6283	0.3214
η_6	1.0611	1.0654	0.7729	1.0660
η_7	-0.0894	-0.0897	-0.3780	-0.0897
η_8	-0.2111	-0.2174	-1.0024	-0.2172
η_9	3.1394	2.3137	-34.8455	2.2710
η_{10}	4.5397	4.5004	5.1672	4.5013
η_{11}	-0.7199	-0.7230	-0.7407	-0.7225

5.2 ILI using modified response error sampling

This subsection presents results of the estimation of the aerodynamic derivatives by ILI using modified response error sampling which were described in section 4. It is necessary that the number of discarded data l should be given so as to reduce the influence of the initial state $x_{cl}(0)$. As a technique for designing l from the viewpoints of the damping characteristic of the constructed ILCS, this paper refers the impulse responses of the closed-loop transfer functions such as $S(p)$ and $Y(p)$. Letting $\alpha_d \pm j\beta_d$ be the dominant poles of the closed-loop, the reduction ratio of the amplitude is roughly evaluated as

$$A_m = e^{\alpha_d t}. \qquad (26)$$

For a given A_m, l is then obtained by

$$l = \frac{\log(A_m)}{\alpha_d T_s}. \qquad (27)$$

The dominant poles of the closed-loop in this numerical aircraft model was $-0.2725 \pm j1.0704$. For $A_m = 1.0,\ 0.5,\ 0.3$ and 0.1, l was calculated as $l = 0,\ 254,\ 442$ and 845.

As results of Case 1, Fig. 8 shows the norm of the response error where l is given by $l = 0,\ 254,\ 442$ and 845. The ranges of the initial state variables of Eq. (17) included in the response error were given by

$$|\beta(0)| \le 1 \text{ [deg]},\quad |\phi(0)| \le 1 \text{ [deg]},$$
$$|p(0)| = 0 \text{ [deg/s]},\quad |r(0)| \le 2.5 \text{ [deg/s]}. \qquad (28)$$

The non-zero initial state $x_{cl}^{(k)}(0)$ was varied within the above ranges at each iteration. It was hard in the case of $l = 0$ to judge the convergence of the response error because it was violently varied (Fig. 8). Although the SS parameters with $l = 0$ moved toward their true values, it was hard to judge convergence of the estimates. When l was increased, the convergence was improved. Table 4 shows the estimated SS parameters (aerodynamic derivatives) at iteration $k = 50$ using the modified response error sampling in Case 1.

The SS parameters estimated with $l = 0$ were not greatly different from their true values as shown in Table 4. However, it was hard to judge the convergence of the SS parameters from the iteration histories because they were violently varied. It was therefore effective to introduce discarded data in measurement of the response error in ILI. For the aircraft model considered in this paper, $A_m = 0.1$; that is, $l = 845$ was enough to reduce the influence of the non-zero initial states.

Since the aircraft model was a stable system, it was possible to perform ILI without a feedback controller $K(p)$. If $K(p)$ was not used in ILCS, the number of discarded data for $A_m = 0.1$ was $l = 28441$. This indicates that measurement time of the response error becomes very long. It is therefore desirable to use a feedback controller in ILCS even if the system to be identified is stable.

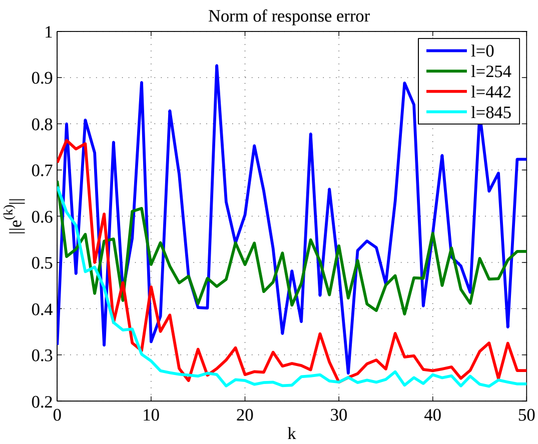

Fig. 8 Norm of response error using modified response error sampling in Case 1.

Table 4 Estimated SS parameters at fifty iteration using modified response error sampling in Case 1.

η_i	η^*	$\eta^{(50)}$			
		$l=0$	$l=254$	$l=442$	$l=845$
η_1	-1.8741	-0.4940	-1.7548	-1.8050	-1.8141
η_2	-0.9709	-0.8471	-0.9375	-0.9118	-0.9259
η_3	1.0611	0.0841	1.1922	1.0429	1.0430

6 Concluding Remarks

This paper has presented two techniques in ILI when the zero initial state condition was not achieved. One was to obtain acceptable impulse responses. The other was to measure the response error to the exclusion of non-zero initial state factor. This paper proposed an estimation technique using the LS method for the former and introduced discarded data in measurement of the response error for the latter. The ILI with the proposed techniques was applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the proposed techniques was demonstrated in numerical simulations.

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