NOISE ABATEMENT PATH PLANNING USING VIRTUAL FORCES
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Abstract
This paper presents a study on the development of a real-time path planning algorithm for the synthesis of noise abatement routings based on a so-called virtual force method. More specifically, a “snake method” is proposed to calculate a near-optimal noise abatement departure route (ground track) in real time, assuming a given (optimal) vertical flight profile.

The accuracy of the developed virtual force snake algorithm is demonstrated in numerical examples, in which simulated results are compared with known exact trajectory optimization solutions that were generated offline. It is shown that using the snake method, a ground track can be produced for a realistic departure scenario with an accuracy margin of less than 4% in terms of the defined noise performance criterion, relative to the exact optimal solution.

1 Introduction
The noise resulting from flight operations at major airports is a continuing source of annoyance in nearby residential communities. One option to reduce the noise impact is to reshape the arrival and departure trajectories into and out of an airport. For this purpose, noise-abatement routes and procedures have been designed and implemented. Most current noise abatement procedures are local adaptations of generic procedures, aimed at optimizing aircraft noise footprints [1]. It is readily clear though that a trajectory that is optimized with respect to a generic criterion, such as the noise footprint size, provides a poor indication of the true noise impact, which evidently depends on the actual population distribution in the communities surrounding an airport [2].

To enable the calculation of individually customized environmentally friendly approach and departure trajectories, a number of trajectory optimization frameworks have been developed, including a tool called NOISHHH [3-5]. This innovative tool combines a noise model, a dose-response relationship, an emission inventory model, a geographic information system, and a dynamic trajectory optimization algorithm. The NOISHHH tool generates routings and flight-paths that minimize the environmental impact in the residential communities surrounding the airport, while satisfying all imposed operational and safety constraints. As the NOISHHH tool is based on optimal control theory, it provides optimal control solutions in open-loop form using an iterative numerical procedure. Solving a trajectory optimization problem using an optimal control approach typically requires a huge computational effort due to the large number of trajectory integrations necessary to obtain a single optimal solution. It is readily clear that an optimal control theory based tool such as NOISHHH does not lend itself to real-time on-board applications.

In this study we propose the development of a noise abatement path planning algorithm based on so-called virtual force techniques that provides a near-optimal solution in real time, suitable for on-board application in a trajectory-based operations environment. The employed force field method originates from the field of robotics where it is widely used for autonomous mobile robot path planning, primarily owing to its elegant mathematical analysis and simplicity.
[6]. More specifically, we seek to develop a so-called “snake method” to calculate a near-optimal noise abatement departure route (ground track), assuming a given procedure (i.e., speed/altitude/thrust profiles).

2 Snake Concept Description

One of the most elegant virtual force techniques for real-time suboptimal path planning is the “snake method” [6]. In a mechanical analogy, the snake can be thought of as an elastic band, represented by a chain of point masses connected to one another by springs and dampers (see Figure 1). One end of the chain is attached to the initial point of the departure route, while the other end is attached to the terminal (exit) point of the route. Without any additional external forces acting on the masses, the shape of the snake in steady state is defined by the shortest path between the initial and terminal points, i.e., a straight line track. By having noise exposed communities exert a virtual external repulsive force on the snake control points (the masses), the snake will expand and start to move away from these noise exposed sites. In order to maintain the shortest possible path, whilst circumventing the communities, the internal forces will then try to contract the snake, causing an oscillatory motion until an equilibrium between external and internal forces is obtained. Once a new potential energy minimum (or equilibrium) is reached a new curved “optimal” path is obtained. By altering the spring stiffness or damping ratio the curvature of the snake as well as the settling time of the interconnected spring-mass-damper system can be influenced.

The governing set of equations of motion for the basic snake model is given by:

\[ m \ddot{x}_i = m_i \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} = \bar{F}_i - \bar{F}_{i-1} + \bar{F}_{i+1} + \sum_{k=1}^{N} \bar{F}_{k,i}, \quad (1) \]

where \( \bar{F}_i \) represents the spring force exerted on control point \( i \) and is given by Eq. (2):

\[ \bar{F}_i - \bar{F}_{i-1} = k_i \left( (q_{i+1} - q_i) - (q_i - q_{i-1}) \right), \quad (2) \]

where \( k_i \) is the spring constant. Note that \( q_i \) represents the position coordinates of control point \( i \) (mass \( m_i \)). In Eq.(1), \( \bar{F}_i \) represents the damper force exerted on control point \( i \) and is given by Eq. (3):

\[ \bar{F}_i - \bar{F}_{i-1} = c \cdot \left( (\dot{q}_{i+1} - \dot{q}_{i}) - (\dot{q}_i - \dot{q}_{i-1}) \right), \quad (3) \]

where \( c \) is the damper constant. In Eq.(1), \( \bar{F}_{k,i} \) is the virtual repelling force exerted on control point \( i \) by the noise-exposed community (obstacle) \( k \).

The repelling force exerted by a noise-exposed community should obviously depend on the size of the community, the population density and the level of the exposure. The four main parameters that affect the sound exposure level in affected communities are the aircraft’s thrust setting, its true airspeed, its slant range and elevation angle with respect to a given community. The influence of thrust level and slant range are evident. When the airspeed increases, the time during which a community is exposed to the noise is reduced and, consequently, so is the sound exposure level SEL (the noise metric employed in this study). The elevation angle of the aircraft relative to the observer location in a community has significant influence on the attenuation of sound due to over-ground propagation [1].

To assess the true noise impact in a community it is useful to have a set of relationships that show which annoyance level is associated with a given noise exposure level. In essence, such so-called dose-response
relationships combine the physical measurement of sound with a scientific assessment of the community perception of sound. With the implementation of a dose-response relationship, the noise impact in noise-sensitive areas can be directly assessed. In the present study use is made of a sleep disturbance relationship as proposed by the FICAN [1]. This dose-response relationship, which is derived from laboratory experiments and field studies, gives an upper bound on the number of expected awakenings due to a single night-time flyover. The resulting curve can be found in Figure 2.

In order to calculate the number of awakenings at a particular geographic location, the percentage of awakenings as predicted by the FICAN dose-response relationship is coupled to a Geographic Information System containing population density data. To facilitate acoustic calculations the well-known Integrated Noise Model INM has been employed in this study [1].

When modeling the virtual force exerted by an exposed community, the number of people or rather, the population density in a certain community matters as much as the community size. For this reason a novel repulsive force model has been conceived that captures these essential characteristics. The proposed repulsive model is given by the following expression:

\[ \hat{F}_{k,i} = -\frac{1}{2} k_i \rho_i \left( \frac{1}{\| q_i - p_k \|} - \frac{q_i - p_k}{\| q_i - p_k \|} \right), \]

\[ \text{if} \quad \| q_i - p_k \| \neq 0, \]

where \( k_k \) is the repulsive coefficient, \( p_k \) the location and \( r_k \) the radius of community \( k \). In the model described by Eq.(4), the repulsive force of the snake is inversely proportional to the distance from the considered control point to the noise-affected community. For the repulsive coefficient, the following relationship has been empirically established:

\[ k_k = k_{\text{tuning}} \cdot \rho_k \cdot 10^{-4}, \]  

where \( k_{\text{tuning}} \) is the tuning parameter and \( \rho_k \) is the population density per square kilometer. The best value for the tuning parameter has been established experimentally on the basis of a variety of departure scenarios, for which exact optimal solutions were obtained using the NOISHHH tool. It turns out that a parameter value \( k_k = 0.25 \) offers a good overall performance. For this reason, this particular value has been adopted in the numerical examples presented in this study.

When employing the repulsive force model specified by Eq. (4), the repulsive force will, in principle, always act on a control point, even at a distance at which no noise pollution is discernable. To avoid this form occurring, an artificial barrier has been created around a community that limits the influence range of the virtual force. On control points residing outside the barrier, no virtual force is exerted by the considered community. Based on numerical experimentation, a fixed barrier size (for any community) of 2000 m was decided upon in this study.

3 Initial Results for the Snake Path Planner

The trajectory of an aircraft can be decomposed into two components, the horizontal trajectory, i.e. the ground track (route) and the vertical trajectory, i.e. the profile, which describes the altitude and corresponding thrust setting. In this study, the primary goal is to develop a virtual
force model to shape the ground track, for a given vertical profile. The vertical profile considered in the simulation of the snake path planner is extracted from the optimal 3D trajectory, calculated using the NOISHHH tool for a given scenario. This has the advantage that the validation of the 3D trajectory produced by the snake method remains essentially restricted to the geometry of the ground track.

The accuracy of the ground track produced by the snake path planner is demonstrated in a numerical example related to an existing departure procedure currently in use at Schiphol airport in the Netherlands, viz. the Spijkerboor departure [7]. The accuracy will be assessed by comparing the performance of the snake solution to the corresponding NOISHHH solution in terms of the performance criterion, viz., the expected number of awakenings due to a single flyover event. It needs to be noted that the Spijkerboor departure was not used in the tuning process of the snake model parameters.

In the numerical example, performance data (including fuel flow characteristics) pertaining to a Boeing 737-300 aircraft is used. The data set comprises separate drag polars for each flap setting and for the configurations with the undercarriage extended or retracted. All calculations are performed using the assumption of standard atmospheric conditions, and no wind present.

Figure 3 shows the noise-optimized solution in a 3D space for a Spijkerboor departure (overlaying a map showing population densities), as produced by NOISHHH. Although the expected number of awakenings has been the primary optimization criterion used to establish this particular noise-optimized solution, a small fuel consumption penalty has been included as a secondary criterion such as to avoid excessive path extensions to circumnavigate noise exposed communities. In Figure 4, the corresponding SEL level contours (a) and local awakenings (b) are shown. The expected number of awakenings for this noise-optimized NOISHHH solution is 2,805.

The NOISHHH solution is based on a population density grid comprising 6400 (80x80) cells of 1x1 km². This population density map now needs to be converted into something the snake algorithm can use. Although each cell could in principle be used as a single “community”, it makes sense to cluster cells into larger communities and select only the relevant ones.

Figure 5 presents a “noise obstacle” map that has been inferred from the population density map in the vicinity of the Spijkerboor departure route, using essentially an ad-hoc approach. It is readily clear that the seven modelled communities are widely varying in terms of size as well as population density.
In [8] a more structured approach to clustering a population grid to generate a set of virtual noise obstacles is presented. In this approach, the population data is pre-processed using a K-means clustering algorithm to aggregate the distributed population cells into a given number of population clusters. In future research, this particular approach will be likely adopted in building noise abatement scenarios for the snake algorithm.

In the simulation of the snake, the following (spring-mass-damper) system parameters have been adopted: mass $m_i = 1$; spring constant $k_s = 0.5$; damping ratio $\zeta = 1$. In Figure 6, the resulting ground track for the Spijkerboor departure obtained by using the modified snake method, is compared to the one obtained using NOISHHH. An inspection of the results reveals a very close correspondence for the two ground tracks. Only towards the end of the trajectory, near the community of Stede Broec, a slight difference between the two ground tracks can be observed. As the population density in this region is rather low, it is expected that this will result in only a minor increase in the total number of expected awakenings.

In Figure 7, the SEL contours and geographic awakenings distribution associated to the snake solution are illustrated. A comparison of the snake results with the corresponding NOISHHH results shown earlier in Figure 4 reveals that the two solutions appear to be in close agreement. Nevertheless, the number of awakenings has increased in the snake solution to 2,912, which is about 4% in relative terms.

In addition to the Spijkerboor departure presented herein, several more Standard Instrument Departure scenarios pertaining to Schiphol airport have been examined [9]. It turns out that with the proposed snake technique, using exactly the same system model and settings, even better results can be obtained than in the Spijkerboor scenario, frequently resulting in an accuracy level of better than 1%.

4 Conclusions

This paper has presented a study on the development of real-time path planning algorithms for the synthesis of noise abatement routings based on virtual force methods. In particular, concept descriptions and initial results related to the snake methods were presented.

To render the snake method - which originates from the field of robotics – applicable to noise abatement route planning, a novel repulsive force model has been developed which captures the essential characteristics of the noise-exposed communities.
The accuracy of the developed virtual force snake algorithm is demonstrated in numerical examples, in which simulated results are compared with known exact trajectory optimization solutions that were generated offline. The snake performance has been assessed based on realistic night-times scenario’s pertaining to Standard Instrument Departures currently in place at Schiphol airport in the Netherlands, using expected number of awakenings due to a single flyover as the primary optimization criterion.

The research bears out that the snake method holds out great promise for noise abatement path planning applications. Using the snake method, a ground track can be produced for a realistic departure scenario with an accuracy margin of less than 4% in terms of the expected number of awakenings relative to the corresponding NOISHHH result.

Future research will focus on improved techniques for clustering population into noise obstacles, and into improved repulsive force models. In addition, the introduction of torsion springs will be considered to improve trajectory shaping in tight turns.

References

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