DESIGN OF ROBUST FLIGHT CONTROL SYSTEM FOR FIXED-WING UAV

Takahiro Tenno* and Kenji Uchiyama*
*Department of Aerospace Engineering, Nihon University
csta11018@.nihon-u.ac.jp; uchiyama@aero.cst.nihon-u.ac.jp

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Abstract

This paper describes the design of flight control system for small UAVs to have robustness against disturbances. In particular the effect of gust of wind on a small fixed-wing UAV whose span is less than 1 meter cannot be neglected when designing a control system. Additionally, nonlinearity of the dynamics would become strong due to disturbances. We apply $H_{\infty}$ control method to a small fixed-wing UAV. The dynamic inversion is also employed to linearize the dynamic behavior that is separated into slow and fast dynamics. The validity of the proposed flight control system is verified by numerical simulations.

1 Introduction

Unmanned Aerial Vehicles (UAVs) have been developed for using in disaster monitoring, pesticide application and atmospheric observation from the point of view of human element. The progress in sensor technologies has especially leaded to the development of small UAV and research activities in the flight control system of UAV [1-6]. We have conducted a study on autonomous flight control of a small fixed-wing UAV [7-9]. The UAV was developed [7, 8] to realize formation flight. Small UAVs have several advantages such as easy to operation, short term development and low cost. On the other hand, the UAV is readily disturbed by wind disturbance and tends to be unstable because of small inertial mass. It is difficult to obtain experimentally accurate data in terms of the stability and control derivatives of the UAV, which flies in low Reynolds number, because measurement signals of force and moment are very small. Therefore uncertainty due to the modeling error should be taken into account for the controller design.

Flight dynamics of a small UAV is expressed as MIMO (Multi-Input Multi-Output) system and has highly nonlinearity. It is required to maintain adequate stability and performance even if the dynamic characteristic and any disturbance exist during control. Gain scheduling is a very successful and the most popular method for modern flight control system. However, it is time-consuming to adjust lots of controller parameters by experience and the switch of different controller parameters is not always smooth. The dynamic inversion method which has grown popular in recent years is also used when designing flight control system. The dynamic inversion method make inversion model for canceling nonlinear term. Therefore, if UAV dynamics is inaccurate, the inversion error results in deterioration of control performance.

Various ways relating to control technique are proposed to overcome these problems. Qui et al. [10] designed flight control system using dynamic inversion and $H_{\infty}$ controller. Yuan et al. [11] combined dynamic inversion and $\mu$-synthesis. Various challenges have been made to improve the robustness of this flight control system [12-13] against sensor noise and wind disturbance. However, UAVs treated in these studies can never be small. A robust controller was designed for a small UAV [1]. The restricted use of the controller would be imposed because the system did not consider the nonlinearity of its dynamics.

We attempt to design flight control system for a small fixed-wing UAV whose wingspan is supposed less than 1m. The proposed flight
control system minimizes influence from wind disturbance. The time scale separation technique with DI method is used to solve the control problem. The dynamic behavior of a vehicle is separated into slow and fast dynamics by using time scale which can reduce inversion error of nonlinear dynamic characteristics. In this way, it is designed to consider wind gust which is verified by means of numerical simulation.

2 Nonlinear Aircraft Modeling

2.1 Developed UAV

We have studied the autonomous flight control of a small fixed-wing UAV. Figure 1 shows the developed UAV with the wingspan 0.94m, the length 0.96m, wing area 0.195 m², and the weight 0.58kg [7, 8]. The UAV is controlled by using aileron, elevator, rudder, and throttle. The UAV is made by expanded polypropylene.

The avionics consists of a GPS module, which provides the longitude and latitude as position of the UAV, ground speed, altitude and heading angle, a 32-bit microcomputer, a radio communication unit an inertial measurement unit (IMU) that measures acceleration, angular rates and angles around three axes of body fixed coordinate system, air data sensor that measures angle of attack, side slip angle and airspeed. We will apply a flight control system proposed here to the mathematical model of the UAV in the numerical simulation. Non-dimensional stability and control derivatives of the UAV are calculated by the estimation formula [14] shown in Table 1.

Table 1. Non-Dimensional Derivatives of UAV

<table>
<thead>
<tr>
<th>C_{Du}</th>
<th>C_{Dv}</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_{D\dot{\alpha}}</td>
<td>0</td>
<td>C_{Du}</td>
</tr>
<tr>
<td>C_{\dot{v}\beta}</td>
<td>-0.287</td>
<td>C_{\dot{v}\beta}</td>
</tr>
<tr>
<td>C_{\dot{v}y}</td>
<td>0.288</td>
<td>C_{\dot{v}y}</td>
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<tr>
<td>C_{\dot{v}\dot{\alpha}}</td>
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<tr>
<td>C_{\dot{\alpha}Q}</td>
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<td>C_{\dot{\alpha}Q}</td>
</tr>
<tr>
<td>C_{\dot{\alpha}P}</td>
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<td>C_{\dot{\alpha}P}</td>
</tr>
<tr>
<td>C_{\dot{\beta}P}</td>
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<td>C_{\dot{\beta}P}</td>
</tr>
<tr>
<td>C_{\dot{\beta}}</td>
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</tr>
<tr>
<td>C_{\dot{m}q}</td>
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<td>C_{\dot{m}q}</td>
</tr>
<tr>
<td>C_{\dot{m}\dot{\alpha}}</td>
<td>-1.608</td>
<td>C_{\dot{m}\dot{\alpha}}</td>
</tr>
<tr>
<td>C_{\dot{m}P}</td>
<td>-0.013</td>
<td>C_{\dot{m}P}</td>
</tr>
<tr>
<td>C_{\dot{n}\dot{\alpha}}</td>
<td>0</td>
<td>C_{\dot{n}\dot{\alpha}}</td>
</tr>
</tbody>
</table>


2.2 Equations of Motion

The design process of a flight control system needs mathematical model of an aircraft. The 6-DOF rigid-body dynamics with respect to translational and rotational motion of an UAV can be described by the following differential equations:

\[ \dot{U} = RV - WQ - g\sin\theta + \frac{D\cos\alpha - L\sin\alpha}{m} \]  

(1)

\[ \dot{V} = -UR + WP + g \sin\phi \cos\theta + \frac{Y}{m} \]  

(2)

\[ \dot{W} = UQ - VP + g \cos\phi \cos\theta + \frac{D\sin\alpha + L\cos\alpha}{m} \]  

(3)

\[ I_x \dot{\alpha} = I_x \dot{\alpha} + (I_y - I_z)QR + I_{xz} PQ + I \]  

(4)

\[ I_y \dot{\beta} = (I_x - I_z)RP + I_{xz}(P^2 - R^2) + M \]  

(5)

\[ I_z \dot{\gamma} = I_{xz} \dot{\alpha} + (I_y - I_z)PQ - I_{xz} QR + N \]  

(6)
where $U$, $V$, $W$ and $P$, $Q$, $R$ are standard notations for velocities and angular rates, respectively. $I_x$, $I_y$, and $I_z$ denote moment of inertia. $I_{zc}$ is only considered in this mathematical model. The vehicle mass is expressed by $m$, $D$, $Y$, and $L$ are forces, and $l$, $M$, and $N$ moments of force that are accessible from the models of aerodynamics and thrust. The aerodynamic forces $L$, $D$, $Y$ and moments $l$, $M$, $N$ are assumed to be functions of the state variables and control inputs as the following equations.

\[
D = \bar{q}S_{D} \tag{7}
\]

\[
Y = \bar{q}S_{Y} \tag{8}
\]

\[
L = \bar{q}S_{L} \tag{9}
\]

\[
l = \bar{q}S_{b}C_{l} \tag{10}
\]

\[
M = \bar{q}S_{c}C_{m} \tag{11}
\]

\[
N = \bar{q}S_{b}C_{n} \tag{12}
\]

where $S$ is the wing reference area, $c$ the wing mean aerodynamic chord, $b$ the wing span, and $\bar{q}$ the dynamic pressure. With the forces and moments expressed as linearized functions of the state and control, the nonlinear inverse can be constructed based on these relationship. The stability and control derivatives are used to define the linearized functions. Thus the aerodynamic coefficients are assumed to be functions of these parameters. These equations are used in the next section by nonlinear dynamic inversion technique.

\[
C_D = C_{D0} + C_{Da}\alpha + C_{Dq}Q\frac{c}{2V} + C_{D\theta}\frac{c}{2V} + C_{Du}\frac{u}{V} \tag{13}
\]

\[
C_y = C_{y\beta}\beta + C_{yp}P\frac{b}{2V} + C_{yr}R\frac{b}{2V} + C_{y\alpha}\delta_{a} + C_{y\delta}\delta_{r} \tag{14}
\]

\[
C_L = C_{L0} + C_{La}\alpha + C_{Lq}\frac{c}{2V} + C_{L\alpha}\frac{c}{2V} + C_{L\alpha}\delta_{e} \tag{15}
\]

\[
C_m = C_{m0} + C_{ma}\alpha + C_{mq}\frac{c}{2V} + C_{m\alpha}\frac{c}{2V} + C_{m\alpha}\delta_{e} \tag{17}
\]

\[
C_n = C_{n\beta}\beta + C_{np}P\frac{b}{2V} + C_{n\alpha}\frac{b}{2V} + C_{n\alpha}\delta_{a} + C_{n\delta}\delta_{r} \tag{18}
\]

### 3 Flight Control System

#### 3.1 Dynamic Inversion

The dynamics of the air vehicle is modeled using six nonlinear dynamic differential equations as mentioned previous section. The nonlinear dynamic model of an UAV can be linearized by using dynamic inversion. The dynamic model of the UAV is reformulated as two lower-order systems by invoking time-scale separation between fast dynamics and slow dynamics. The six nonlinear differential equations of motion are rewritten as t

\[
\dot{y} = f_y + g_y\omega \tag{19}
\]

\[
\dot{\omega} = f_\omega + g_\omega u \tag{20}
\]

where

\[
y = \begin{bmatrix} \alpha & \beta & \phi \end{bmatrix}^T, \omega = \begin{bmatrix} P & Q & R \end{bmatrix}^T.
\]

\[
u = \begin{bmatrix} \delta_{a} & \delta_{e} & \delta_{r} \end{bmatrix}^T.
\]
\[
\mathbf{f}_s = \begin{bmatrix}
-L - T \sin \alpha + mg (\sin \alpha \sin \theta + \cos \alpha \cos \phi \cos \theta) \\
Y - T \cos \alpha \sin \beta + mg (\cos \alpha \sin \beta \sin \theta + \cos \beta \cos \theta \sin \phi - \sin \alpha \sin \beta \cos \phi \cos \theta) \\
0
\end{bmatrix}
\]
\[
mV \cos \beta
\]
\[
mV
\]
\[
\mathbf{g}_s = \begin{bmatrix}
-\cos \alpha \tan \beta & 1 & -\sin \alpha \tan \beta \\
\sin \alpha & 0 & -\cos \alpha \\
1 & \sin \phi \tan \theta & \cos \phi \tan \theta
\end{bmatrix}
\]
\[
\mathbf{f}_F = \begin{bmatrix}
I_x & 0 & -I_{xz} \\
0 & I_y & 0 \\
-I_{xz} & 0 & I_z
\end{bmatrix}^{-1}
\begin{bmatrix}
(I_y - I_z)QR + I_{xz}PQ + l_0 \\
(I_z - I_x)PR + I_{xz}(R^2 - P^2) + M_0 \\
(I_x - I_y)PQ - I_{xz}QR + N_0
\end{bmatrix}
\]
\[
\mathbf{g}_F = \bar{q}Sb\begin{bmatrix}
I_x & 0 & -I_{xz} \\
0 & I_y & 0 \\
-I_{xz} & 0 & I_z
\end{bmatrix}^{-1}\begin{bmatrix}
bC_{\beta \alpha} & 0 & bC_{l\beta} \\
0 & cC_{m\alpha} & 0 \\
bC_{n\alpha} & 0 & bC_{n\alpha}
\end{bmatrix}
\]
\[
l_0 = \bar{q}Sb\left(C_{l\beta} + C_{lP} \frac{b}{2V} + C_{lr} \frac{b}{2V}\right)
\]
\[
M_0 = \bar{q}Sb\left(C_{m0} + C_{ma} \alpha + C_{ma} \dot{\alpha} \frac{c}{2V} + C_{mq} \frac{c}{2V}\right)
\]
\[
N_0 = \bar{q}Sb\left(C_{n\beta} + C_{np} \frac{b}{2V} + C_{nr} \frac{b}{2V}\right)
\]
\[
\omega_d = \mathbf{g}_s^{-1}(k_1 \mathbf{I}_{3x3}(y_c - y) - \mathbf{f}_s)
\]
\[
\mathbf{u} = \mathbf{g}_f^{-1}(k_2 \mathbf{I}_{3x3}(\omega_d - \omega) - \mathbf{f}_s)
\]

The flight control system with two time scale controller is shown in Fig.2. The slow time scale controller is designed to control the slower dynamics given by Eq.(19) using fast states \(\mathbf{\omega}\) to linearize the slower dynamics. The faster dynamics given by Eq.(20) is controlled by fast time scale controller using control input to linearize faster dynamics. In the outer-loop, the control input for the desired angular rate \(\omega_d\) is designed as the following equation to follow the command \(y_c\).

\[
\mathbf{\omega}_d = \mathbf{g}_s^{-1}(k_1 \mathbf{I}_{3x3}(y_c - y) - \mathbf{f}_s)
\]

The control input for the aircraft in the inner-loop is defined to follow the desired angular rate \(\mathbf{\omega}_d\) that is the output of the outer-loop expressed by Eq.(28).

\[
\mathbf{u} = \mathbf{g}_f^{-1}(k_2 \mathbf{I}_{3x3}(\omega_d - \omega) - \mathbf{f}_s)
\]

where \(k_1\) and \(k_1\) are control gains of tracking error, \(\mathbf{I}_{3x3}\) is unit matrix in three rows and three columns.

### 3.2 \(H_{\infty}\) Controller

The \(H_{\infty}\) controller is applied to slow dynamics in order to satisfy against wind disturbance. In applying the \(H_{\infty}\) control approach, different weighting functions are used for each signal as shown in Fig.3.
There are weighting functions for disturbance $W_d$, output $W_1$, control input $W_2$, for tracking error $W_3$, and for sensor noise $W_n$. Disturbance, noise, and control weighting functions are chosen as constants. Output weighting function and error weighting function are defined as low pass filter and high pass filter, respectively. These weighting functions, disturbance inputs and evaluated outputs are used for structuring generalization plant. $H_\infty$ controller is derived on the basis of generalization plant by means of Riccati equations.

4 Numerical Simulation

Numerical simulation is performed to verify the proposed flight control system. Wind disturbance model is used simple low pass filter and its time history is shown in Fig.4. The command values of state variables as defined in Eqs.(19) and (20) are set to be zero except the command of angle of attack $\alpha_c=5$ degrees.
Figures 5 to 7 show the time histories of the angle of attack, sideslip angle, and roll angle of the UAV, respectively. The blue line denotes the result when using the proposed flight control system. The control system, which employs constant gains, is also applied to the UAV model to compare with the proposed system. It is clear from the figure that both of the control systems work well. The response of the UAV using the proposed system is better than the system which uses constant gains. The controller using constant gains cannot sufficiently suppress the sideslip as shown in Fig. 6.

Figures 8 to 10 show the time histories of the control inputs. The control surfaces of the UAV are restricted as $\delta_a, \delta_e, \delta_r = \pm 30 \text{ degs}$. Aileron angle never exceed the limit in both cases. There are no distinct differences with respect to other control surfaces between both controllers. It is shown from these results that the proposed method works effectively against wind disturbance.

5 Conclusions

In this paper, flight control system is designed for small fixed-wing UAV to reduce the effect of wind disturbance on its dynamics. Dynamic inversion is used for UAV dynamics, which is separated into fast and slow motion by using timescale properties, to apply the $H_\infty$ controller. The validity of the proposed method was verified through the numerical simulation. We will apply the flight control system to the developed small UAV.

References


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