ELLIPTICALLY CONICAL TWIST AND CAMBER OF DELTA WINGS

S.A. Takovitskii*
***Central Aerohydrodynamic Institute (TsAGI)**
c.a.t@tsagi.ru

Keywords: *supersonic wing, lift-dependent drag, optimization*

Abstract

Minimization of drag due to lift in supersonic flow is considered. Optimal profiling of delta wing median surface is developed on the base of simplified optimization method coupled with theoretical analysis. It is founded that the median surface providing reduced aerodynamic drag consists of elliptical cones and flat elements passing through the wing apex. The results of theoretical analysis and direct numerical optimization are compared for different flight conditions. Flow modeling and aerodynamic forces calculation are conducted within the framework of Euler's equations.

1 General Introduction

Increasing the lift-to-drag ratio under a lift capability constraint is a traditional problem in the high speed aerodynamics. At supersonic flight conditions the lift dependent drag contains not only a vortex component, but also a wave component. In the case of a delta wing, the drag depends weakly on the sweep angle in regimes that correspond to supersonic leading edges, and it increases abruptly as the sweep increases in regimes with subsonic leading edges. The smaller the aspect ratio of the wing is, the relatively smaller the wave part of drag due to lift is and the more significant the benefit regarding the drag at a specified lift that can be achieved by means of the median surface warping is [1]. Well known design result is the wings with no leading edge loading. Wing deformation removes the subsonic leading edge pressure singularity and replaces the localized suction by the distributed suction on a forward facing wing surface. The main effect is increasing effective aspect ratio of the wing due to decreasing overflow from the lower surface to the upper surface. The simplest and reliable means of optimization is a conical twist and camber. The transition to spatial deformation of the wing mean surface does not provide any appreciable improvement in the aerodynamic characteristics [2, 3].

The optimization research is aimed on studying characteristic features of optimal configurations and foundation of analytical solutions for the problem. For flying vehicles design it is very important to determine the simplest shape deformations that ensure aerodynamic performance improvement. A reliable tool for analytical solving optimization problems at supersonic flight conditions by means of small variations of the shapes of bodies with a given distribution of aerodynamic loading is developed in [4, 5]. This method combines Newton-based optimization algorithm and quadratic approximation of the objective function on the base of local aerodynamic loading analysis.

The method turned out to be effective for studying various design problems. Near to optimal configurations of two-dimensional airfoils, axisymmetrical forebodies realizing minimum of wave drag and axisymmetrical nozzles with maximum thrust were founded. Wedges, rhombus and cones were used as the initial geometric shapes for which surface pressure distributions are stated theoretically. Assuming a linear relationship between a change in the orientation of small elements of the surface and the corresponding increment of the pressure, the aerodynamic functions were approximated by quadratic forms. The conditions for the minimum of the objective function led to a system of linear equations for determining the optimum geometric parameters. In the case of lifting wings, the solution can be constructed as an improving shape variation to the flat delta wing.

2 Problem Statement

Consider a delta wing at supersonic flow (figure 1). It is required to minimize the aerodynamic drag and to determine the corresponding wing warping for given lift. The free stream is characterized by Mach number M_∞. The wing is assumed to be infinitely thin. The wing surface is deformed in the way that the wing projection onto the base plane remains constant. Distributions of twist and camber are such that the surface slope in the stream wise direction is constant along rays through the apex. So flow field near the wing is conical. The leading edge sweep angle is χ. The angle of attack α is defined as angle between free stream velocity V_∞ and centre chord of the wing.

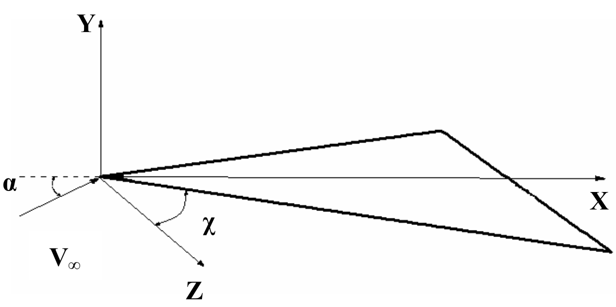

Fig. 1 Delta wing

Aerodynamic lift and drag coefficients of the wings are calculated by the pressure integration along the wing span

$$c_L = \frac{2}{\gamma p_\infty M_\infty^2} \int_0^1 \left(p_d(z) - p_u(z)\right) dz$$

$$c_D = \frac{2}{\gamma p_\infty M_\infty^2} \int_0^1 \left(p_d(z) - p_u(z)\right)\left(\alpha_{fl} + \delta(z)\right) dz$$

Here, γ – ratio of specific heats, p_∞ – free stream pressure, p_d и p_u – windward and leeward surface pressure, α_{fl} – angle of attack of the flat wing providing desired lift coefficient, δ – local angle of attack increment due to surface deformation, coordinate z is normalized by half wing span. In that statement, lift dependent drag is only considered. Surface friction drag and leading edge suction are ignored.

The problem is simplified by assumptions that the pressure distribution on the flat wing is known and the deformation of the wing is small, i.e. the plane tangent to the wing at an arbitrary point is located at small angle to the base plane of the wing. Pressure distribution on the flat wing is supposed to be a linear function of the angle of attack. Increment of pressure due to wing deformation is evaluated on the base of Ackeret's formula for linearized supersonic flow in assumption that there is no aerodynamic interference between surface elements

$$p_d(z) - p_u(z) = 2p_\infty\left(R(z)\alpha_{fl} + k\delta(z)\right)$$

$$k = \gamma M_\infty^2 \Big/ \sqrt{M_\infty^2 - 1}$$

Here, $R(z)$ describes pressure distribution on the flat wing.

3 Elliptically Conical Twist and Camber

The task is to find $\delta(z)$ that provides $c_D = min$ at given c_L. The primary function of the variation problem is

$$F = (R(z)\alpha_{fl} + k\delta(z))(\alpha_{fl} + \delta(z) + \lambda)$$

where λ – Lagrange multiplier.

In this case the Euler equation is simply

$$F'_\delta = 0$$

It gives the extreme of the angle of attack increment

$$\delta(z) = -0.5(R(z) + k)\alpha_{fl} - 0.5\lambda$$

Value of the Lagrange multiplier is in keeping with condition on lift coefficient. It leads to the final expression for $\delta(z)$ that may therefore be written as

$$\delta(z) = \frac{\alpha_{fl}}{2k}\left(R_{av} - R(z)\right)$$

$$R_{av} = \frac{\gamma M_\infty^2 c_L}{4\alpha_{fl}}$$

Optimal deformation of the wing depends on aerodynamic loading on the flat wing. The local angle of attack increases on absolute value as a linear function of difference between the average loading R_{av} on the flat wing and the loading *R(z)* at the corresponding wing section. The more aerodynamic loading on the surface element differs from the mean value, the stronger deformation. Thus optimal shape deformation leads to pressure alignment in the wing span direction.

Wing geometry analysis connects the increment to local angle of attack with the expression of the guide line *y(z)* representing the trailing edge of the wing (coordinates y and z are normalized by half wing span)

$$\delta(z) = \left(zy' - y\right)/tg\chi$$

So the guide line of the optimal wing is represented by differential equation

$$zy' - y = \frac{\alpha_{fl} tg\chi}{2k}\left(R_{av} - R(z)\right)$$

The equation differentiation gives information about the guide line curvature

$$zy'' = -\frac{\alpha_{fl} tg\chi}{2k} R'$$

The optimum wing is leeward convex ($y'' \le 0$) because of aerodynamic loading on the flat wing increases in direction towards the leading edges (R'>0).

To ascertain essential features of wings with low lift dependent drag well known results of the linear theory are used. Pressure distributions on flat wings have a different representation depending on whether the leading edges are ahead of or behind the Mach cone from wing vertex. If μ_∞ is Mach angle then flow field conditions are determined by the relative sweepback n

$$n = \frac{tg\mu_\infty}{ctq\chi} = \frac{tg\chi}{\beta}$$

$$\beta = \sqrt{M_\infty^2 - 1}$$

For wings with supersonic leading edges the relative sweepback is less than unity, and in the case of sonic or subsonic leading edges – $n \ge 1$. Of course, the flow field is essentially conical, that is, the pressure is constant on rays through the vertex of the wing.

On the wing with supersonic leading edges two types of surface elements must be distinguished. The first element is located inside the Mach cone from vertex ($0 \le z \le n$). The second element is lying between the leading edge and the Mach cone ($n \le z \le 1$). In the last case pressure is constant and corresponds to two dimensional flow conditions. Pressure distribution along wing span is expressed in the following way

$$R(z) = \frac{\gamma M_\infty^2 \cos\chi}{\sqrt{M_\infty^2 \cos^2\chi - 1}}\left(1 - \xi\frac{2}{\pi}\arcsin\frac{\sqrt{n^2 - z^2}}{\sqrt{1 - z^2}}\right)$$

$$\xi = \begin{cases} 1, & 0 \le z \le n \\ 0, & n \le z \le 1 \end{cases}$$

On the wing with subsonic leading edges aerodynamic loading is increased infinitely on the leading edges

$$R(z) = \frac{\gamma M_\infty^2}{tg\chi\sqrt{1 - z^2}\, E\left(\sqrt{1 - n^{-2}}\right)}$$

$$0 \le z \le 1$$

Here E expresses the complete elliptic integral of the second kind. If $n = 1$ then $E(0) = \pi/2$.

On the base of these regularities the guide lines of the optimal wings are constructed. It is drawn a conclusion that wings of low drag due to lift consist of elliptical cones and planes elements. These elements have triangular forms on plan view with apexes coinciding with the wing apex. The flat elements are bordered on the wing leading edges.

It should be noted that the theoretically stated optimal wings with subsonic leading edges are represented by elements of elliptical cones surfaces. This leads to result which is inconsistent with linear theory since it predicts infinite surface slope at leading edges. Due to pressure distribution singularity the approximate solution is locally in serious error, while the general behavior of the guide line is predicted correctly.

The optimal wing can be described by two geometrical parameters A and m which specify concavity and relative sizes of the wing elements. The elliptical and rectilinear portions of the guide line are expressed as

$$y(z) = A\begin{cases} \left(\sqrt{1 - z^2/m} - 1\right) & 0 \le z \le m \\ \left(\dfrac{1-z}{\sqrt{1-m}} - 1\right) & m \le z \le 1 \end{cases}$$

4 Numerical Studying

The numerical studying is carried out within the framework of Euler's equations. The inviscid flow near the wing is computed by a marching method with respect to the longitudinal coordinate. The equations of motion are written in conservative form, which made it possible to treat shocks and other flow discontinuities correctly without any special procedures for tracking their spatial locations. The explicit finite-difference scheme is implemented on multizone grids. The flow region under investigation consists of two zones located under and above the wing. The zones are bounded by the wing surface, the plane of symmetry and the bow shock wave. In each cross section the computational grid consists of about 10 000 points.

The free stream conditions are characterized by Mach number M_∞=1.5÷6. Delta wings with supersonic and subsonic leading edges are considered. The lift coefficient is c_L=0.1. The flat wing base area is adopted as reference value. The leading edge sweep angle corresponds to the relative sweepback which varies in the range n=0.58÷2.2.

The optimal wings are found by the direct variation of two geometric parameters A and m. The flat wing is taken as a starting one. The coordinate wise descent method is used. The method has been fairly effective in the case under consideration. Satisfactory convergence to the optimum has been achieved in not more than three cycles of coordinate wise descent. The problem is complicated due to need of definition of the angle of attack corresponding to the given lift coefficient.

The wings with sonic leading edges (n=1) are studied at Mach numbers M_∞=1.5÷6. Cross sections (x=*const*) of the optimal at c_L=0.1 wings are compared in figure 2. The wings designed at greater Mach numbers have bigger values of the geometrical parameter A than the wings at lower Mach numbers. In addition the flat elements near the leading edges have bigger relative dimensions (defined by the parameter m). The joint points of the elliptical and rectilinear

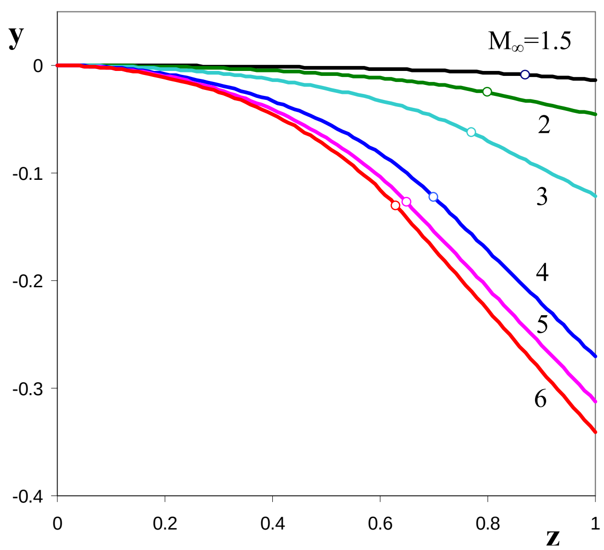

Fig. 2 Optimal wings (n=1)

elements of the guide line are marked by circles ($z=m$).

The twist angle $\varphi(z)$ of the optimal wings is defined on the base of the guide line $y(z)$ expression

$$\varphi(z) = \left(\frac{y(1) - y(z)}{1 - z} - y(1) \right) \Big/ tg\chi$$

So the flat elements bordered on the wing leading edges are characterized by the constant twist angle ($\frac{y(1) - y(z)}{1 - z} = -\frac{A}{\sqrt{1 - m}} = const$ for $z>m$).

In accordance with supersonic theory the angle of attack appropriate to $c_L=0.1$ increases with increase of Mach number. In considered Mach number range the angle of attack changes by a factor of five. This is why the optimum wings differ markedly from the flat wings at high supersonic speeds.

It is more convenient to compare the optimal wings by means of the span wise distribution of the increment $\delta(z)$ to the local angle of attack. Figure 3 represents comparison of the increment $\delta(z)$ with respect to the angle of attack α for the optimal at Mach numbers $M_\infty=2$, 4 and 6 wings, and for the wing which optimal within the framework of the linear theory. The linear theory proposes the wing with the aerodynamic load vanishing at the leading edges – the local angle of attack equals zero at $z=1$.

The results of numerical studying are in a good concordance with the theoretical data at the central section of the wing up to $z=0.4\div0.5$. In the immediate vicinity of the leading edges the numerically constructed wings have smaller angles of surface deformation. It should be noted that the increment δ to the local angle of attack is equal to the twist angle φ at the side vertex of the delta wing. The bigger Mach number is, the lower the relative value of the twist angle with respect to the angle of attack is.

Aerodynamic shape optimization results in redistribution of pressure both on the wing surface and in the shock layer. Cross section flow fields ($x=const$) at $M_\infty=4$ and $n=1$ are shown in figure 4. The left half of the figure represents the flat wing and the right half – the optimal wing. The pressure level lines $p/p_\infty=const$ are plotted with step 0.1.

The shock wave is detached from the leading edge of the flat wing. In the neighborhood of the leading edge it is observed a fan of rarefaction waves. The flow over the leeward side of the wing is accelerated in the cross direction up to a supersonic speed. The supersonic region extends to the plane of symmetry and is bounded by the hanging shock wave. The spatial locations of the inner shock

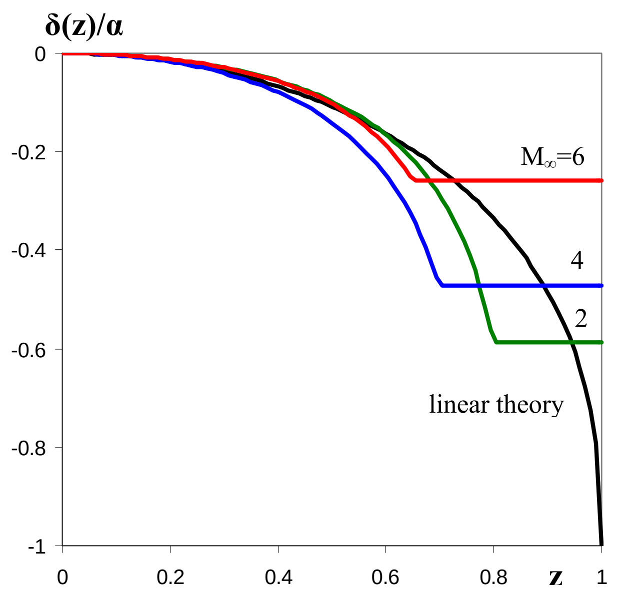

Fig. 3 The increment to the local angle of attack ($n=1$)

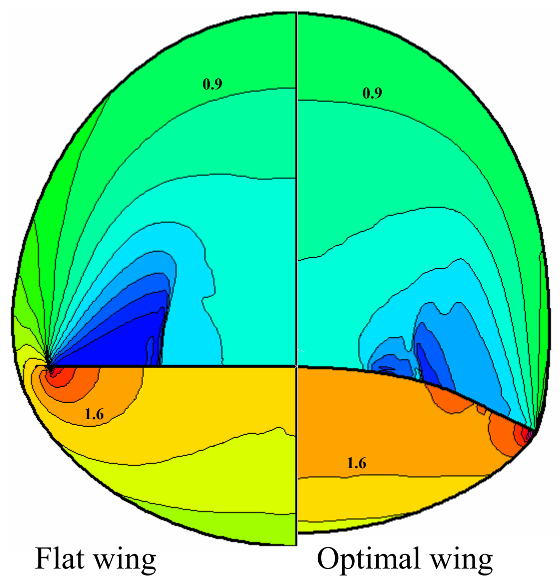

Fig. 4 Cross section flow fields $p/p_\infty=const$ ($M_\infty=4$, $n=1$)

waves were not determined exactly, they spread out over several adjacent mesh points.

Note that the optimal wing is flowed under bigger angle of attack than the flat wing. So the shock layer has smaller proportions. There is a pressure increase near the plane of symmetry. On the whole, the pressure on the optimal wing is distributed more uniformly over the wing span. On the flat wing the leading edges are under heavier aerodynamic loading.

The theoretical and numerical results on integral aerodynamic characteristics (lift c_L and drag c_D coefficients) of the wings are in general agreement. Within the framework of the linear theory it was stated the optimal conical wing with the sonic leading edges has the lift dependent drag smaller on 8.3% in comparison with the flat wing at design lift and Mach number. Numerical optimization reveals a dependence of aerodynamic drag reduction on Mach number. When Mach number increases from M_∞=1.5 to M_∞=6 the relative diminution of the drag due to lift increases from 7.5% to 10%. The main reason is an increment of the angle of attack corresponding to the design lift and enhancing nonlinear effects.

At Mach number M_∞=2 the wings with sweep angles χ=45º÷75º are investigated. The study covers the wings with subsonic (n>1) and supersonic (n<1) leading edges.

The linear theory revealed the nonplanar wings with leading edge attachment that do not yield to the flat wings with the leading edge thrust. At the supersonic edges there are not suction. As the relative sweepback n of the wing with subsonic leading edges increases the suction force changes from zero up to 50% of the lift dependent drag.

The theoretical and numerical investigation results are compared in figure 5. The minimal value of the lift dependent drag c_D with respect to the drag $c_{D\ FL}$ of the flat wing without the leading edge suction is shown as a function of inverse of n. The smaller the aspect ratio of the wing is, the more significant the benefit regarding the drag achieved by means of the median surface warping is. At $n \leq 1$ the elliptically conical wings provide the drag due to lift diminution prescribed by the linearized theory. For the wing with sweep angle χ=45º

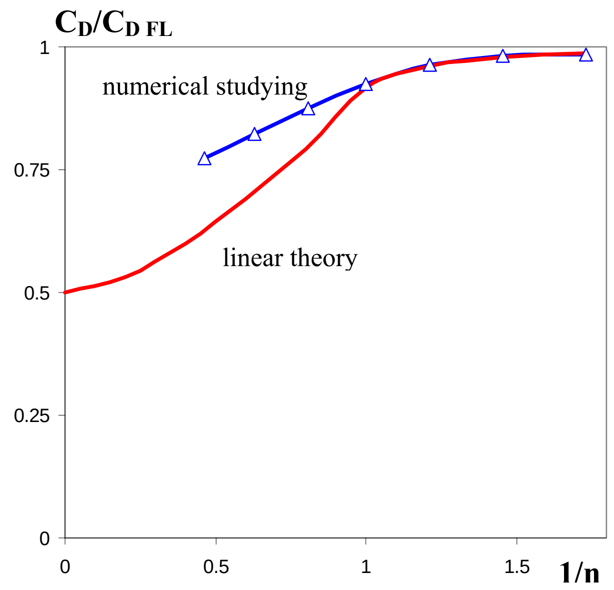

Fig. 5 Lift dependent drag (M_∞=2)

the drag decrease is not more than 2%. For wings with subsonic leading edges (n>1) the theory predicts an appreciable reduction of the lift dependent drag. Numerical investigation of the wings formed by elliptical cones and planes elements leads to more modest result.

The proposed aerodynamic shape in the form of elliptical cones and planes elements is used to construct the median surface of a complex plane form wing. The considered wing is a version of the wing of the aircraft Tupolev 144. The wing is treated as a delta wing with a low sweep extension. The central section of the wing is represented by the elliptical cone and the wing end is flat. The wing elements are joined along lines passing trough the wing apex. The optimal shape of the wing is defined by way of the direct variation of two geometric parameters A and m.

The objective function is the lift dependent aerodynamic drag. Optimization studying is carried out at Mach number M=2.1 and lift coefficient c_L=0.1. The flat wing area is adopted as the reference value. The leading edge sweep angles are 75º (before the leading edge break) and 61º (after the break). Thus at flying conditions under consideration the wing has subsonic-supersonic leading edges.

Optimum deformation of the wing leads to more uniform distribution of the aerodynamic loading on the wing along span direction. Lower and upper surface pressure difference in a vicinity of the leading edges decreases (figure 6). The pressure level lines $p/p_\infty=const$ are plotted with step 0.05. A reduction of the vortex-type flow regions on the upper surface of the wing is observed. Shock waves pressures on the outboard wing lower surface are significantly diminished. At the central section of the wing there is a pressure growth.

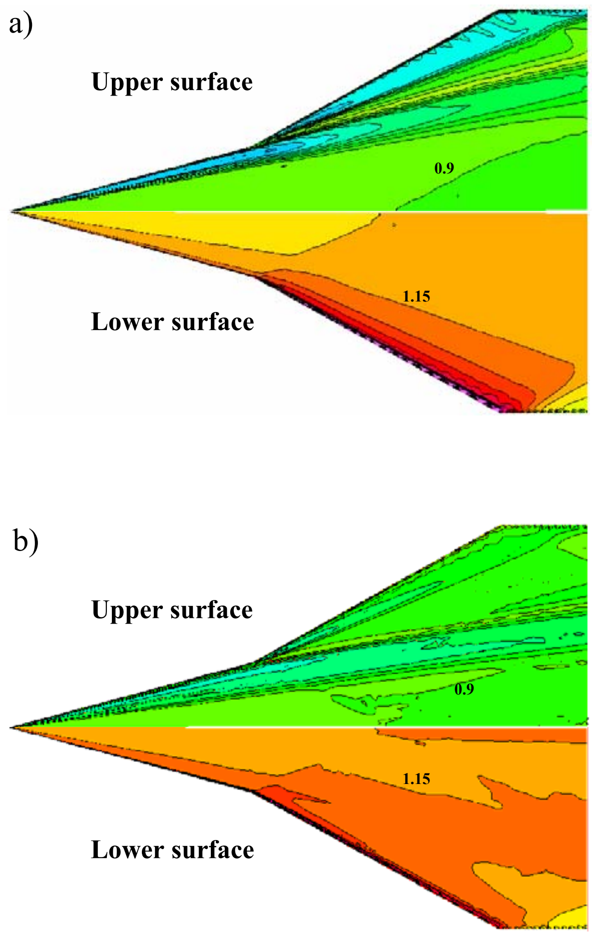

Fig. 6 Surface pressure distribution, $p/p_\infty=const$
a) flat wing; b) optimal wing

The flat and optimum wings have similar lift and moment characteristics. The aerodynamic centre is located on a distance of about 70% of the wing length from the wing apex. As a result of the wing deformation the derivative of the lift coefficient on the angle of attack increases and the aerodynamic centre moves slightly forward. Comparison of the aerodynamic drag polar shows the superiority of the wing with a nonplanar median surface. A relative reduction of the lift dependent drag achieves 11%. It confirms importance of researches on definition of the simple deformations in problems of aerodynamic shapes optimization.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research, project No. 10-01-00208a.

References

[1] Roper G.M. Drag reduction of thin wings at supersonic speeds by the use of camber and twist. *Aeronautical Research Council Reports and Memoranda,* No. 3132, 1957.

[2] Kogan M.N. On bodies of minimum drag in a supersonic gas flow. *Prikl. Mat. Mekh.*, Vol. 21, No. 2, pp 207-212, 1957.

[3] Smith J.H.B. and Mangler K.W. The use of conical camber to produce flow attachment at the leading edge of a delta wing and to minimize lift-dependent drag at sonic and supersonic speeds. *Aeronautical Research Council Reports and Memoranda,* No. 3289, 1957.

[4] Takovitskii S.A. Analytic solution in the problem of constructing an airfoil with minimum wave drag. *Fluid Dynamics*, Vol. 38, No. 6, pp 933-941, 2003.

[5] Takovitskii S.A. Direct optimization method and aerodynamic shape design at supersonic flight conditions. *Proc. 27th Intern. Congr. of Aeronaut. Sci. (ICAS),* Nice, France, pp 110.1–110.8, 2010.

[6] Takovitskii S.A. The choice of the system of geometric parameters of an optimized wing. *J. Appl. Maths Mechs,* Vol. 62, No 5, pp 763-770, 1998.

Copyright Statement