CONTROL ALLOCATION FOR AIRCRAFTS WITH MULTIPLE CONTROL EFFECTORS DURING TAKEOFF AND LANDING PHASES

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Abstract

This paper discusses the control allocation problem for aircrafts with multiple control effectors during takeoff and landing phases. Based on the analysis results of performance indices during takeoff and landing phases, a control effector superiority evaluation method is proposed, and further an optimized control allocation model is formalized for increased flight performance.

As a case study, the control effector superiority and takeoff/landing performance parameters of a typical canard-delta wing aircraft are evaluated. The simulation results show that the control allocation model formalized in this paper can efficiently make use of the takeoff and landing performance potentials of the target aircraft, which indicates that the control effector superiority evaluation method and control allocation model we proposed are sound and effective.

1 Introduction

Modern aircrafts are usually equipped with multiple control effectors in order to achieve desired flight performances. As a consequence, the number of control effectors tends to be greater than the number of control parameters, which results in an infinite number of solutions. Therefore, there needs to be an effective method for solving the control allocation problem for aircrafts with multiple control effectors. [1]

When the control effectors deflect, not only the moments but also the lift and drag, i.e. the flight performance will be affected. [2] Besides, multiple control effectors also means that the control effectiveness of a single control effector is relatively limited which will easily cause the actuators to saturate and hinder the flying qualities.

The flight performance requirements and the corresponding control allocation problem vary from different flight phases. Takeoff and landing are two critical phases of aircraft operation. [3] The flight performances during these two phases play an important role in determining the size of the airport and have a significant effect on flight safety as well as landing gear load. Compared with free-flight condition, the operating process and the forces acting on the aircraft during takeoff and landing are more complicated, which leads to more complex control allocation problems.

In order to make full use of the takeoff and landing performance potentials, the control allocation problems during takeoff and landing phases will be discussed in this paper to achieve increased flight performances.

2 Control Allocation Problem

The goal of control allocation is to find a set of permissible control effector deflections to achieve desired control effects. The input is the total control effect to be produced, i.e., the virtual control input $\boldsymbol{v}(t)\in\boldsymbol{R}^k$. The output is the true control input $\boldsymbol{u}(t)\in\boldsymbol{R}^m$, where $m>k$ [4].

For linear systems,

$$\boldsymbol{Bu}(t)=\boldsymbol{v}(t) \tag{1}$$

where the control effectiveness matrix $\boldsymbol{B}$ is a $k \times m$ matrix with a rank of k.

To incorporate actuator position and rate constraints, it is required that:

$$\boldsymbol{u}_{\min} \le \boldsymbol{u}(t) \le \boldsymbol{u}_{\max} \tag{2}$$

$$\boldsymbol{\rho}_{\min} \le \dot{\boldsymbol{u}}(t) \le \boldsymbol{\rho}_{\max} \tag{3}$$

Since the control allocator is usually implemented as part of a time-discrete control system, it is reasonable to approximate the time derivative as

$$\dot{\boldsymbol{u}}(t) \approx \frac{\boldsymbol{u}(t)-\boldsymbol{u}(t-T)}{T} \tag{4}$$

where T is the sampling time. Combining (2) ~ (4) yields

$$\underline{\boldsymbol{u}}(t) \le \boldsymbol{u}(t) \le \overline{\boldsymbol{u}}(t) \tag{5}$$

$$\begin{aligned} \underline{\boldsymbol{u}}(t) &= \max\{\boldsymbol{u}_{\min}, \boldsymbol{u}(t-T)+T\boldsymbol{\rho}_{\min}\} \\ \overline{\boldsymbol{u}}(t) &= \min\{\boldsymbol{u}_{\max}, \boldsymbol{u}(t-T)+T\boldsymbol{\rho}_{\max}\} \end{aligned} \tag{6}$$

Equation (1) constrained by (5) constitutes the standard formulation of the linear control allocation problem.

$$\begin{aligned} \boldsymbol{B}\boldsymbol{u} &= \boldsymbol{v} \\ \underline{\boldsymbol{u}} \le \boldsymbol{u} &\le \overline{\boldsymbol{u}} \end{aligned} \tag{7}$$

There are three possible outcomes for control allocation problems: [4]

1. No solution exists
2. One unique solution
3. An infinite number of solutions

This paper focuses on the third case which is typical in control allocation for aircrafts with multiple control effectors.

3 Superiority of Control Effectors

For aircrafts with multiple control effectors, all the control effectors have the ability to control the aircraft. In other words, they all have a certain influence on the flight performance when activated. However, it does not necessarily mean that they have to be involved in control for all flight phases. If the control effectors are used inappropriately, the flight performances potential will not be fully utilized. Besides, from the angle of reliability and complexity of the flight control system, the number of control effectors simultaneously involved in flight control should be as few as possible.

To satisfy the performance requirements of aircrafts with multiple control effectors, control effectors which are participated in control should be selected on the basis of evaluating the superiority of control effectors.

3.1 Performance Requirements

The most important performance indices of takeoff and landing phases include takeoff ground run distance, landing ground roll distance, liftoff speed and touchdown speed [5]. The first two parameters are directly related to the range of the airport. Additionally, higher liftoff velocity requires a longer takeoff distance. Touchdown velocity will affect the landing safety as well as the landing gear load [6].

3.2 Superiority of Control Effectors

During the takeoff phase, the aircraft needs high lift and noseup pitching moment [6]. Because of the low dynamic pressure, the aerodynamic drag is negligible relative to the takeoff thrust.

During the landing phase, before touchdown, the aircraft also needs high lift and pitching up moment. The aerodynamic drag can not be ignored because of the low landing thrust. After touchdown, to minimize the ground roll distance, the aircraft needs low lift and high drag.

In addition, due of the low dynamic pressure, the control effectiveness of aerodynamic control effectors is relatively low, and the actuator positions and rates of the control effectors are easy to saturate. Specifically, for takeoff and landing before touchdown, the superiority of control effectors can be formalized as

$$P_{\delta i} = a_L R_{L\delta i} + a_m R_{m\delta i} + a_{rl} R_{rl\delta i} \tag{8}$$

For landing after touchdown, the superiority of control effectors can be given by

$$P_{\delta i} = -a_L R_{L\delta i} + a_D R_{D\delta i} \tag{9}$$

where $P_{\delta i}$ is the superiority parameter of the *i-th* control effector.

$$R_{m\delta i} = \frac{\Delta C_{m\delta i}}{\left|\Delta C_{m\delta 0}\right|}$$

$$R_{L\delta i} = \frac{\Delta C_{L\delta i}}{|\Delta C_{L\delta 0}|}$$

$$R_{D\delta i} = \frac{\Delta C_{D\delta i}}{|\Delta C_{D\delta 0}|}$$

$\Delta C_{L\delta 0}$ and $\Delta C_{D\delta 0}$ are the lift and drag coefficient increment per unit deflection of the reference control effector. Reference control effector could be anly control effector of the aircraft. $\Delta C_{L\delta i}$ and $\Delta C_{D\delta i}$ are the lift and drag coefficient increment per unit deflection of the *i-th* control effector.

a_L, a_m, a_{rl} and a_D are the weighting parameters of the lift, pitching up control effectiveness, rate limit and drag characteristics respectively. The weighting parameters depend on the performance requirements, aerodynamic characteristics of the aircraft and each control effector, and the actuator performance. For a given equation, the weighting parameters satisfy that $a_L, a_m, a_{rl}, a_D \in [0,1]$, and their sum equals to 1.

In the following section, we will formalize the control allocation model for the takeoff and landing phases respectively.

4 Control Allocation Model

After specifying the performance requirements and control effectors participated in control, a proper allocation method should be selected to build the control allocation model.

4.1 Takeoff

A typical takeoff phase consists of three steps: 1) with the engine producing maximum thrust, the aircraft is accelerated to the takeoff speed; 2) after reaching the takeoff speed, the aircraft is rotated noseup so that the angle of attack (AOA) increases to generate sufficient lift for liftoff; 3) the aircraft starts climbing to the obstacle height (11.5m). [3]

The force acting on aircraft with thrust vectoring (TV) during two gear takeoff ground run is shown in Fig. 1. [7]

The vertical forces are

$$G\cos\alpha = L + L_{\delta_e}\cdot\delta_e + L_{\delta_n}\cdot\delta_n + 2N_2\cos\alpha + P\sin(\delta_T + \alpha) \tag{10}$$

where L_{δ_n} is the lift increment (per unit) produced by deflection of control effectors ahead of the center of gravity (CG), such as canard; L_{δ_e} is the lift increment (per unit) produced by deflection of control effectors aft of CG, such as elevator and/or elevon.

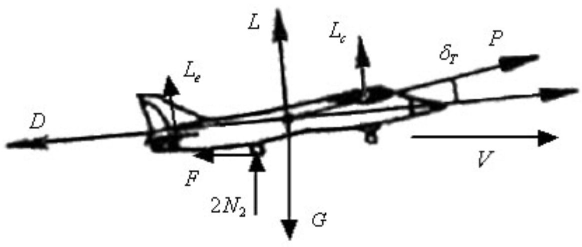

Fig. 1 Force acting on aircraft with TV during takeoff ground run

The net pitching moment:

$$M_0 + M(\alpha) + M(\delta_e) + M(\delta_n) = P\cdot\sin(\delta_T + \alpha)\cdot x_{tv} - 2N_2(x_{mg} + \mu\cdot H_{cg}) \tag{11}$$

where

$M(\delta_n)$	pitching moment increment due to the deflection of control effectors ahead of CG
$M(\delta_e)$	pitching moment increment due to the deflection of control effectors aft of CG
M_0	pitching moment when $\alpha = 0$ and no clean configuration
$M(\alpha)$	pitching moment increment due to AOA
x_{tv}	horizontal distance between the TV nozzle and CG
$P\sin\delta_T\cdot x_{tv}$	pitching moment generated by TV
x_{mg}	horizontal distance between main gear and CG
H_{cg}	height of CG
$2N_2\cdot x_{mg}$	pitching moment generated by reaction of main gear
$\mu\cdot 2N_2\cdot H_{cg}$	pitching moment generated by friction of main gear

According to the takeoff performance requirements specified in Section 3, maximum lift can be formalized as the object function, and linear programming is selected as the allocation method [8][9]. Therefore, the control allocation model for takeoff can be formalized as follows:

$$\max\,(L_{\delta_n}\cdot\delta_n + L_{\delta_e}\cdot\delta_e + P\cdot\sin\delta_T)$$

$$\begin{cases} M(\delta_e)+M(\delta_n)-P\cdot\sin(\delta_T+\alpha)x_{tv} \\ \quad =-M_0-M(\alpha)+2N_2(x_{mg}+\mu H_{cg}) \\ \sin(\bar{\delta}_{T\min})\le\sin\delta_T\le\sin(\underline{\delta}_{T\max}) \\ \bar{\delta}_{e\min}\le\delta_e\le\underline{\delta}_{e\max} \\ \bar{\delta}_{n\min}\le\delta_n\le\underline{\delta}_{n\max} \end{cases} \quad (12)$$

All the deflections are positive if they are deflected downward.

4.2 Landing

4.2.1 Before Touchdown

Before touchdown, an increase in lift and drag is required. Hence, the lift and drag increments generated by control effector deflections can be selected as the object function for control allocation using the linear programming method. Therefore, the control allocation model for landing before touchdown can be formalized as:

$$\max(L_{\delta_e}\delta_e + L_{\delta_n}\delta_n + P\sin\delta_T \\ \quad + D_{\delta_e}\delta_e + D_{\delta_n}\delta_n - P\cos\delta_T)$$

$$\begin{cases} M_0+M(\alpha)+M(\delta_n)+M(\delta_e) \\ \quad \ge P\cdot\sin(\delta_T+\alpha)\cdot x_{tv} \\ \sin(\bar{\delta}_{T\min})\le\sin\delta_T\le\sin(\underline{\delta}_{T\max}) \\ \bar{\delta}_{e\min}\le\delta_e\le\underline{\delta}_{e\max} \\ \bar{\delta}_{n\min}\le\delta_n\le\underline{\delta}_{n\max} \end{cases} \quad (13)$$

4.2.2 After Touchdown

After touchdown, to minimize the ground roll distance, the lift should be minimized, while the drag and friction forces should be maximized. Decreasing the lift can lead to an increase in the reaction and eventually the friction force; In other words, minimum lift is coincident with maximum friction force. Besides, during ground roll, the moment problem can be ignored. Hence, the control allocation model for landing after touchdown can be built as

$$\max(-L_{\delta_e}\delta_e - L_{\delta_n}\delta_n - P\sin\delta_T \\ \quad + D_{\delta_e}\delta_e + D_{\delta_n}\delta_n - P\cos\delta_T)$$

$$\begin{cases} \sin(\bar{\delta}_{T\min})\le\sin\delta_T\le\sin(\underline{\delta}_{T\max}) \\ \bar{\delta}_{e\min}\le\delta_e\le\underline{\delta}_{e\max} \\ \bar{\delta}_{n\min}\le\delta_n\le\underline{\delta}_{n\max} \end{cases} \quad (14)$$

5 Simulations

We use the ADMIRE (Aero-Data Model in Research Environment) developed by FOI, [10] as the simulation platform. Control allocation for takeoff and landing phases are realized and evaluated on the basis of control allocation models built in this paper.

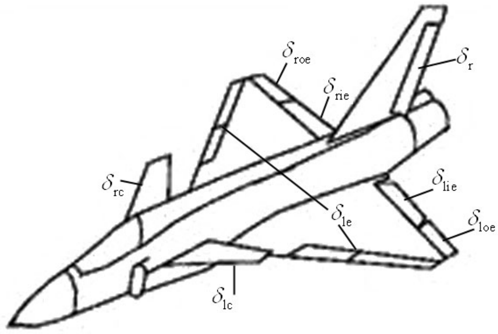

Fig. 2 Layout of the example aircraft

As shown in Fig. 2, the aerodynamic control surfaces of ADMIRE include: two close-coupled canards, four elevons, a leading-edge flap (LEF) and a rudder. The maximal allowed deflections and angular rate of the control surfaces are given in Table 1.

Table 1 Control surface deflection limits

Control Surface	Min.	Max.	Angular Rate
Canard	-55°	25°	±50°/s
Rudder	-30°	30°	±50°/s
Elevon	-25°	25°	±50°/s
LEF	-10°	30°	±20°/s
TV	-25°	25°	±25°/s

This paper focuses on the longitudinal problem. The canards/elevons are considered as one canard/elevon deflecting respectively.

$$\begin{aligned} \delta_n &= (\delta_{lc}+\delta_{rc})/2 \\ \delta_e &= (\delta_{lie}+\delta_{loe}+\delta_{rie}+\delta_{roe})/4 \end{aligned} \quad (15)$$

5.1 Choosing Control Effectors

The weighting parameters a_L, a_m, a_{rl} for takeoff and landing before touchdown are selected as 0.4, 0.4 and 0.2 respectively. The weighting parameters a_L, a_D for landing after touchdown can be selected based on the braking friction, and are set to 0.4 and 0.6 in our simulation. Since the forces and moments generated by thrust vectoring do not vary with the velocity of the aircraft, $V = 60$ m/s is taken as the rence speed in our simulation.

Choosing elevon as the reference control effector, the superiority parameters at takeoff speed are given in Tables 2 to 4.

Table 2 Superiority parameters for takeoff

Control effector	$R_{L\delta i}$ $a_L = 0.4$	$R_{m\delta i}$ $a_m = 0.4$	$R_{rl\delta i}$ $a_{rl} = 0.2$	$P_{\delta i}$
Canard	0.032	0.35	1	0.35
Elevon	1	-1	1	0.2
LEF	-0.024	-0.022	0.4	0.06
TV(down)	0.43	-6.53	0.5	-2.34
TV(up)	-0.43	6.53	0.5	2.54

Table 3 Superiority parameters for landing before touchdown

Control effector	$R_{L\delta i}$ $a_L = 0.4$	$R_{m\delta i}$ $a_m = 0.4$	$R_{rl\delta i}$ $a_{rl} = 0.2$	$P_{\delta i}$
Canard	0.032	0.35	1	0.35
Elevon	1	-1	1	0.2
LEF	-0.024	-0.022	0.4	0.06
TV(down) Idle	0.04	-0.61	0.5	-0.13
TV(up) Idle	-0.04	0.61	0.5	0.33
TV(down) Maximum	0.43	-6.53	0.5	-2.34
TV(up) Maximum	-0.43	6.53	0.5	2.54

Table 4 Superiority parameters for landing after touchdown

Control effector	$R_{L\delta i}$ $a_L = 0.4$	$R_{D\delta i}$ $a_D = 0.6$	$P_{\delta i}$
Canard (down)	-0.056	0.57	0.319
Canard (up)	0.0323	0.6	0.373
Elevon (down)	-1	1	0.2
Elevon (up)	1	1	1
TV (down)	-0.04	0.02	-0.004
TV (up)	0.04	0.02	0.028

As shown in Tables 2 to 4, if the TV nozzle deflects upward during takeoff and landing before touchdown, it will generate significant noseup pitching moment, and elevon will be allowed to deflect a larger downward angle to increase the lift coefficient. If TV nozzle deflects upward during landing after touchdown, it will increase the landing gear reaction and friction forces. From the highest superiority to the lowest, the control effector sequence for takeoff is TV (upward), canard, elevon, and leading edge flap. The sequence for landing before touchdown is the same as takeoff, but the superiority parameters are different because of variations in thrusts. The sequence for landing after touchdown is elevon (upward), canard (upward) and TV (upward).

5.2 Takeoff

Table 5 gives the liftoff velocity and ground run distance under different control allocation configurations.

As indicated in Table 5, for a given elevon deflection, the aircraft can obtain high lift coefficients under larger elevon deflects.s Meanwhile, large downward elevon deflection will generate significant nosedown pitching moment, which will make the aircraft hard to pitch up, and increase the ground run distance. Upward TV nozzle deflection allows the elevon to have a larger downward deflection, which will increase the lift coefficient and decrease the liftoff speed and ground run distance.

Table 5 Simulation results of takeoff performance

Elevon (deg)	TV (deg)	Liftoff Speed (m/s)	Ground Run Distance (m)
0	0	93.15	491
4	0	94.48	505.27
8	0	95.16	513.71
optimal	0	82.91	391.75
optimal	optimal	74	301

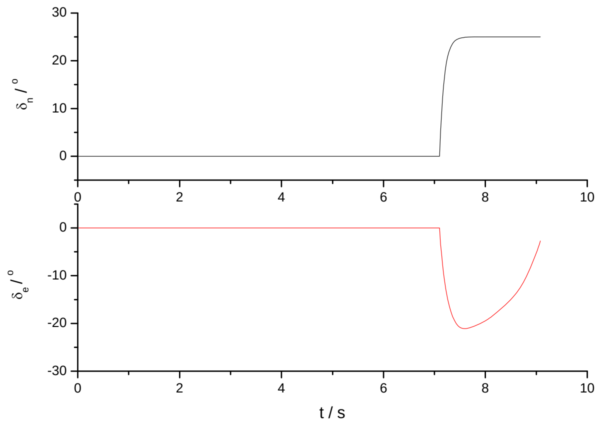

Fig. 3 Control effector deflections without TV during ground run

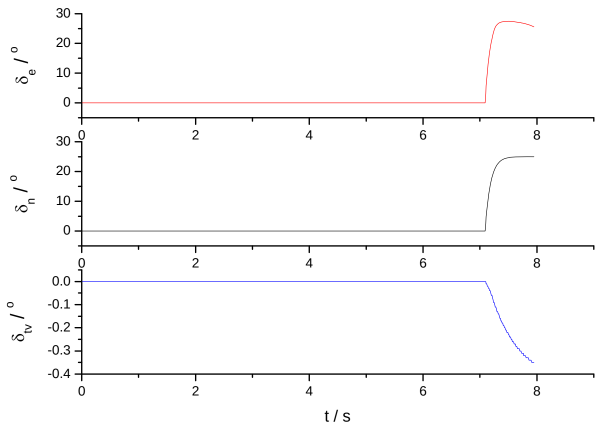

Fig. 4 Control effector deflections with TV during ground run

Figure 3 and 4 give the takeoff simulation results without and with TV using the control allocation model built in Section 4.

As shown in Fig. 3, after reaching the takeoff speed, the canard deflects to the maximum downward angle to increase lift and noseup pitching moment; the elevon deflects upward to increase noseup pitching moment. With the increasing velocity and AOA, the lift will increase while the main gear reaction and nosedown pitching moment will decrease. As a result, the upward elevon deflection decreases to get higher lift coefficient.

As shown in Fig. 4, after reaching the takeoff speed, canard deflects to the maximum downward angle to increase lift and noseup pitching moment; TV nozzle deflects upward to generate noseup pitching moment; elevon deflects downward to increase lift coefficient. As the velocity increases, the aerodynamic lift and pitching moment will grow accordingly. Because the control effectiveness of TV does not vary with the velocity, the required TV nozzle deflection will increase.

As indicated by Table 5, the optimized control allocation model improves takeoff performance significantly.

Table 6 Performance of landing before touchdown with idle thrust

Without TV			With TV		
AOA (deg)	Elevon (deg)	Touchdown Speed (m/s)	Elevon (deg)	TV (deg)	Touchdown Speed (m/s)
5	9.95	81.59	11.49	-25	79.81
8	11.14	69.12	13.30	-25	67.60
12	10.93	58.48	13.98	-25	57.18
15	12.30	52.76	16.04	-25	51.59

Table 7 Performance of landing before touchdown with maximum thrust

Without TV			With TV		
AOA (deg)	Elevon (deg)	Touchdown Speed (m/s)	Elevon (deg)	TV (deg)	Touchdown Speed (m/s)
5	9.95	81.59	28.38	-25	67.5
8	11.14	69.12	30	-17.25	59.11
12	10.93	58.48	30	-12.13	51.68
15	12.30	52.76	30	-9.44	47.88

5.3 Landing

5.3.1 Before Touchdown

Table 6 and 7 show the landing simulation results using the control allocation model built in Section 4 with idle and maximum thrust respectively.

As shown in Tables 6 to 7, when TV is not used, the throttle has no effect on the touchdown speed. Lift coefficient will increase while the touchdown speed will decrease as AOA increases regardless of the use of TV.

At idle thrust, although the thrust is low, the control effectiveness of TV is sufficient as compared to aerodynamic control effectors. The nozzle can deflect upward to generate noseup pitching moment, and the elevon is allowed to deflect downward to increase lift coefficient. This indicates that using TV can reduce the touchdown speed, which is reflected from the simulation results shown in Table 6.

At maximum thrust, as a result of higher control effectiveness, TV can generate higher noseup pitching moment, and elevon is allowed to deflect a larger downward angle to get a higher lift coefficient. Hence, using TV can reduce the touchdown speed significantly. Besides, because of the high control effectiveness, the required nozzle angle is smaller than idle thrust.

5.3.2 After Touchdown

Table 8 gives the simulation results of landing after touchdown. The reference touchdown speed is 48m/s; the braking friction factor is 0.4, and rolling friction factor is 0.02.

Both the upward deflection of elevon and canard can reduce landing ground roll distance by decreasing lift coefficient while increasing drag coefficient. Elevon is relatively more effective as indicated by the simulation results. Because of the low thrust, the effect of TV is limited.

Table 8 Performance of Landing after touchdown

Control effector	Landing Ground Roll (m)
No Deflection	330.5
Elevon (up to max)	277.6
Canard (up to max)	301.2
TV (up to max)	316.9
Elevon + Canard + TV	249.9

The simulation results of landing ground roll in Table 8 agree well with the trends of the superiority parameters in Table 4.

5. Conclusion

This paper discussed the control allocation problem during takeoff and landing phases. The control allocation models were formalized and an aircraft was used for verification. The simulation results indicate that he control allocation model built in this paper can fully utilize the takeoff and landing performance potentials.

References

[1] Ken Bordignon. Constrained control allocation for systems with redundant control effectors. PhD thesis, Virginia Polytechnic Institute and State University, 1996

[2] Fang Baorui. The aerodynamic configuration design of aircraft. 1997

[3] Bandu N. Pamadi. Performance, stability, dynamics, and control of airplanes. AIAA Education Series. 2004

[4] Ola Harkegard. Backstepping and control allocation with application to flight control. PhD thesis, Linkoping University, 2003

[5] Department of Defence. Glossary of Definitions, ground rules, and mission profiles to define air vehicle performance capability. MIL-STD-3013, 2003

[6] Jin Changjiang, Fan Liqin. Flight dynamics – flight performance calculation. 1990

[7] Shui Qingcai, Wang Xinhua. A mathematical model used to calculate the takeoff performance for an aircraft. Flight Dynamics. Vol.19, No.4, 2001

[8] Zhang Jianzhong, Xu Shaoji. Linear Programming. 1990

[9] Susan A. Frost, Marc Bodson, Diana M. Acosta. Sensitivity analysis of linear programming and quadratic programming algorithms for control allocation. AIAA 2009-1936

[10] Lars Frossell, Ulrik Nilsson. ADMIRE the aero-data model in a research environment 4.0. Swedish Defence Research Agency (FOI), FOI-R-1624-SE, 2005

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