Abstract

The objective of this paper is to test the feasibility of an adaptive automatic control technique, for UCAVs. In this respect, the unfalsified adaptive switching supervisory control method is reviewed and specific aspects for implementation in UCAV control are provided. A case study using the ADMIRE model is included.

1 Introduction

The usage of Unmanned Air Vehicles (UAV) has increased exponentially during the last decade, accompanied by a significant R&D effort, as new types of missions are entrusted to them. The domain extended from simple reconnaissance missions to complex active missions, both for civil (such as border supervision, forest fire mapping, search and rescue, etc.) and military (search and destroy, close air support, strike missions etc.) accomplished by Unmanned Combat Air Vehicles (UCAV). As a consequence, most of the missions currently flown by manned fighters are to be flown, in 30 years from now, by UAVs and UCAVs, according to several articles in different journals (as for example, Aviation Week, June 8, 2009 p.46, August 10, 2009 p.56). However, the same articles also show the difficulties faced in the case of remotely piloted UAV/UCAVs, when two operators (a pilot and system operator for sensors and weapons) are required. More than this, when long lasting missions are involved (as is the case of Predator/Reaper or Global Hawk that usually fly more than 12 hours), at least two crews are needed to remotely control the UAV/UCAV. This has led to the peculiar situation that Unmanned Aircraft actually require more pilots and crew members then manned aircraft. Another difficulty is that UAV operations put a great strain on satellite communications. Each UAV requires its own dedicated channel (Aviation Week, December 7, 2008), and, considering the bandwidth limitations involved in satellite communications, the number of UAV mission to be flown simultaneous is limited. These situations impose to continue the effort for increasing the performance of automatic flight control. Three directions of interest are mentioned in this context:

- Control at high angles of attack: this would allow the aircraft to perform extreme maneuvers that would take full advantage of the lack of a pilot.
- Complete autonomous flight: developing an automatic pilot, capable to fly the aircraft without a remote human pilot, would greatly reduce manning and satellite bandwidth requirements.
Formation flight: it would allow a single operator to control several UAVs. Moreover, UAVs can be integrated with manned fighters, such that the command and control of the UAVs is transferred to the manned fighter(s), thus eliminating the problems involved in long range communications.

Over the last two decades a lot research has been put in Adaptive Switching Supervisory Control (ASSC) (see [1], [2], [4], and [8]). ASSC is in fact an adaptive variant of classical gain scheduling, turned, by the use of a supervisory logic based on plant input/output recorded data, from an open loop switching mechanism to a closed loop one. A typical ASSC is depicted in Figure 1. Where a data driven “high-level unit” S, called supervisor, which controls each plant G belonging to the given set G of plant models by connecting an appropriate controller K from the set K of candidate controllers The supervisor decides if the currently switched-on controller works properly, and, in the negative case, it replaces it by another candidate controller. The scheduling task (when to substitute the acting controller) and the routing task (which controller to switch on) are carried out in real time by monitoring purely data-driven test functional [1].

The main current approaches to ASSC can be subdivided into two different groups: the first consists of the so called Multi-Model ASSC (MASSC), wherein a dynamic nominal model is associated with every candidate controller, the second called Unfalsified ASSC (UASSC) [1], [8] wherein a switching logic that dispenses with the need for a-priori knowledge of the dynamic model is used. Both these methods have their advantages and disadvantages. MASSC schemes work by comparing norms of sequences of estimation errors based on the various nominal models, as the candidate controller associated to the nominal model yielding the prediction norm of minimum magnitude is believed to be the most suitable one. The main advantage is the fact that transient times before finding a stabilizing controller tend to be small. However this can be achieved only by using a very dense model distribution. If this condition is not enforced neither convergence to a final controller, nor boundness can be guaranteed. In contrast UASSC schemes as described by [8], can select in finite time a final controller yielding a finite affine gain from the reference to the data, under the minimal conceivable requirement regarding the existence of a stabilizing candidate controller. This along with the fact that the plant need not be linear makes this schemes from the robustness point of view much better suited to aerospace applications than MASSC, the asymptotic stability properties of the latter being typically only guaranteed if the unknown plant is tightly approximated by at least one nominal model. However the main disadvantage of UASSC schemes used so far, as noted in [1] and [4], stems from the fact that they do not provide protection against the temporary insertion in the loop of destabilizing controllers, which might lead to long transient times and temporary trends to divergence before the final stabilizing controller is switched on. In the examples provided in [8] the supervisor needs about 70 seconds before finding the stabilizing controller, which wouldn’t be convenient when trying to stabilize the short period longitudinal dynamics of an aircraft. For this reasons in [1] a scheme called Multi Model Unfalsified Adaptive Switching Supervisory Control, that combines the advantages of both methods (low transient times for MASSC and asymptotic stability for UASSC), was proposed.

The purpose of the current paper is to investigate the feasibility of UASSC for aerospace applications, namely the stabilization of the short period dynamics of a fighter aircraft. The paper is organized as follows. In Section 2 the UASSC method is summarized, in Section 3 the different requirements pertaining to implementing UASSC for an UAV autopilot are discussed. Section 4 outlines a simulation example run using the ADMIRE model. Section 5 discusses the results of the previous section and draws a number of conclusions and future research areas are highlighted.
Consider the following closed loop control system:

\[
\begin{align*}
\dot{y}(s) &= G(s)u(s) \\
u(s) &= K\left(r(s) - y(s)\right)
\end{align*}
\]  

(1)

where \(G(s)\) denotes the transfer function of the controlled system, \(K(s)\) stands for the controller, \(r\) is the reference signal, \(u\) and \(y\) are the control variable and the system output respectively.

Though UASSC methods can be used on non-linear plants, linearity will be assumed in throughout the paper for simplicity and also the later case study is conducted using robust controllers design based on linearization of aircraft dynamics.

It is assumed \(G\) belongs to a plant uncertainty set \(\mathcal{G}\). The controller \(K\) belongs to a finite family \(\mathcal{K}\) of linear time invariant (LTI) controllers.

**Definition 1** Given a signal \(x(t), t \geq 0\) is said that \(x_\tau(t) = \begin{cases} x(t), & t \in [0, \tau] \\ 0, & \text{otherwise} \end{cases}\) represents a truncation of \(x(t)\) with the truncated norm

\[
\|x\|_\tau = \left( \int_0^\tau x^2(t)dt \right)^{\frac{1}{2}}.
\]

With the above definition the following slight generalization of input-output stability will be adopted throughout the paper [1], [4], [10]

**Definition 2** A dynamic system with the input \(r\) and the output \(y\) is called stable, or the stability is unfalsified by the data \((u, y)\), if there exist \(\alpha, \beta \geq 0\) such that:

\[
\|y\|_\tau \leq \alpha\|r\|_\tau + \beta,
\]

\(\forall \tau \geq 0\) and for all \(r \in L_2\), \(L_2\) denoting the space of all functions with finite energy on any finite interval.

Otherwise if \(\sup_{\tau \geq 0; \|r\|_\tau \neq 0} \|y\|_\tau \to \infty\), it is said that the stability of the system is falsified by the data \((u, y)\)

The presence of the term \(\beta \geq 0\) in the above definition is related to the situation where non-zero initial conditions, of the system, are taken into account. The next definition will be used in the following developments (see also [1], [4])

**Definition 3** The adaptive control problem is feasible if, for every \(G \in \mathcal{G}\), there exists at least one controller \(K \in \mathcal{K}\) such that the resulting system obtained by coupling \(K\) to \(G\) is stable and it accomplishes the performance objectives.

The unfalsified adaptive control techniques are essentially based on the so-called fictitious reference signal and on an associated performance index which allows choosing appropriate controllers, \(K \in \mathcal{K}\), for which the problem is feasible [8]

**Definition 4** Let the data \((u, y)\) be the input and output measurements of a plant \(G\) over the time interval \([0, \tau]\). Then the fictitious reference signal \(\bar{r}_K\) associated to a controller \(K \in \mathcal{K}\) is the signal defined over \([0, \tau]\) that produces the same set of data \((u, y)\) if \(K\) would be connected to \(G\).

Note that the above definition requires the invertibility of \(K\) in which case the fictitious reference signal is given by \(\bar{r}_K = K^{-1}u + y\). This expression of \(\bar{r}_K\) reveals another major constraint for \(K\), namely it must be minimum phase since otherwise the fictitious reference \(\bar{r}_K\) can be unbounded for \(t \to \infty\). Some aspects concerning these constraints will be discussed in the next section,
The performance index $J(K, u, y, \tau)$ is a positive defined function defined on $\mathcal{K} \times \mathcal{U} \times \mathcal{Y} \times \mathbb{R}_+$ where $u$ and $y$ are truncated on the interval $[0, \tau]$. It is defined according to the design specification of the controller $K$ and it represents a measure of the performance provided by $K$ on the time interval $[0, \tau]$.

**Definition 5** A controller $K \in \mathcal{K}$ is called falsified at the time $\tau$ with respect to a given cost level $\gamma > 0$ by the data $(u, y)$ measured on the time interval $[0, \tau]$ if $J(K, u, y, \tau) > \gamma$. Otherwise the controller $K$ is called unfalsified by the measurements $(u, y)$ on $[0, \tau]$.

According to the terminology used in [10] the set of all unfalsified controllers with the unfalsified cost level $\gamma > 0$ at time $t$ stands for the unfalsified controller set.

The unfalsified adaptive controllers are not always safe, in the sense that some unfalsified destabilizing controllers inserted in the closed-loop can produce large signals for long intervals of time.

**Definition 6** Consider the reference signal $r$ and the measured set of data $(u, y)$ obtained by a finite number of switches of controllers $K \in \mathcal{K}$, mapping $[r(t), y(t)]$ to $u(t)$ and denote by $t_f$ the final switching time and by $K_f$ the final controller. Then the pair $(J, \mathcal{K})$ is called cost-detectable if the following assertions are equivalent:

- **a)** $J(K_f, u, y, \tau)$ is monotone increasing and bounded for $\tau \to \infty$;
- **b)** The closed loop system in Figure 1 with $K_f$ is unfalsified by the data $(u, y)$ when $\tau \to \infty$.

In [4], [8], [10] the following performance index is considered:

$$J(K, u, y, \tau) = \frac{\|w_1 * (y - r)|^2_{L^2[0, \tau]} + \|w_2 * u|^2_{L^2[0, \tau]} }{\|r|^2_{L^2[0, \tau]}}$$

(2)

where $*$ denotes convolution and $w_1(t)$ and $w_2(t)$ denote dynamic weighting functions used for determining controllers $K$ as solutions of the mixed sensitivity problem

$$\left\| \begin{bmatrix} W_1 S & \gamma \\ W_2 KS \end{bmatrix} \right\|_{\sigma} \leq \gamma$$

$S := (I + GK)^{-1}$ denoting the sensitivity function and $\| \cdot \|_{\sigma}$ representing the $H_\infty$ norm of the system $\cdot$ (for more details see [3])

In [4] it is shown that the constrains mentioned above concerning the invertibility and minimum phase properties of $K \in \mathcal{K}$ can be removed considering the coprime factorization of the controllers $K \in \mathcal{K}$. A similar idea is used in a discrete-time version in [1].

**Definition 7** The ordered pair $(U, V)$, with $U, V \in RH_\infty$, where $RH_\infty$ denotes the space of all asymptotically stable transfer function matrices, is called a left-coprime factorization of the transfer function $G$ if:

- 1) $V$ is square and invertible;
- 2) $G = V^{-1}U$.

Moreover if $VV^* + UU^* = I$ where $V^*(s) = V^T(-s)$ and $U^*(s) = U^T(-s)$ denote the adjuncts of $V$ and $U$ respectively, then the pair $(U, V)$ is said to be a normalized left-coprime factorization of $G(s)$.

A computation method to determine left coprime factorization of a given $G(s)$ may be found for instance in [6].

The closed loop system with $K = V^{-1}U$ is shown in Figure 2.

![Fig.2 Closed loop control system with $K=V^{-1}U$](image)

The above configuration can be alternatively implemented using the so-called “observer-form” configuration (see also [4] and their references) in Figure 3, where direct algebraic computations show that the new
reference signal \( \tilde{z} \), which generates the data set \((u, y)\), is given by:

\[
\tilde{z} = V u + U y
\]  

(3)

Therefore the computation of the new fictitious reference \( \tilde{z} \) does not require an invertibility condition; moreover when \( u \) and \( y \) are bounded, \( \tilde{z} \) is bounded to since \( U \) and \( V \) are stable. The performance index will be determined as in (2) replacing the fictitious reference signal \( \tilde{r} \) by \( \tilde{z} \).

3 Using UASSC for UAV autopilots

For the application considered in the next section the following polytopic representation of the plant family \( \mathcal{G} \) is considered

\[
\mathcal{G} := \left\{ G(s) | G(s) = \lambda_1 G_1(s) + \cdots + \lambda_n G_n(s), \quad \sum_{i=1}^{n} \lambda_i = 1; \lambda_i \geq 0, \ i = 1 \ldots n \right\}
\]

where \( G_i(s), i = 1, 2 \ldots n \) are known transfer matrices corresponding to “n” nominal flight conditions.

For each \( G_i(s) \) one determines via the mixed-sensitivity design method mentioned in the previous section, the controller \( K_i(s) = V^{-1}(s)U_i(s) \) where the coprime factors \( V_i(s) \) and \( U_i(s) \) are stable. Then the following parameterization of the controller set is defined:

\[
\mathcal{K} := \left\{ K(s) | K(s) = \sum_{i=1}^{n} \lambda_i K_i, \sum_{i=1}^{n} \lambda_i = 1; \lambda_i \geq 0 \right\}
\]

Based on the left coprime factorization of \( K_i(s), i = 1, \ldots n \) one obtains:

\[
K_{\lambda}(s) = \sum_{i=1}^{n} \lambda_i V^{-1}_i(s)U_i(s) =
\]

\[
= V^{-1}_1 \left( \lambda_1 U_1 + \sum_{i=2}^{n} \lambda_i V_i V^{-1}_i U_i \right) =
\]

\[
= V^{-1}_1 U_{\lambda} \quad \text{were the following notation has been introduced:}
\]

\[
V_{\lambda} = \lambda_1 U_1 + \sum_{i=2}^{n} \lambda_i V_i V^{-1}_i U_i
\]

(4)

From (3) it follows that the fictitious reference \( \tilde{z} \) is in fact a function of \( \lambda \) and so is the performance index (3). Using the collected data set \((u, y)\) on the interval [0, \( \tau \)] one can determine the optimal unfalsified controller \( K_{\lambda}^*(s) \) of form (4) with

\[
\lambda^* = \arg \min_{\lambda} j(K, u, y, \tau),
\]

subjected to \( \sum_{i=1}^{n} \lambda_i = 1; \lambda_i \geq 0 \). The stability of the coprime factors \( V_i \) and \( U_i, i = 1 \ldots n \) ensures the cost-detectability property of the pair \((J, \mathcal{K})\).

4 Case study

For the case study the ADMIRE model, provided as freeware by the Swedish Research Administration, has been used.

The aim of the case study is to investigate the feasibility of unfalsified control for the control of airplane short-period dynamics. Therefore only the equations containing the angle of attack and pitch rate were used. Also to simplify the study just one of the control inputs was considered out of the three available for maneuvers in the longitudinal plane.

First a series of controllers were designed as vertices of the polytope. The design was carried out on linearizations of the short period dynamics of the ADMIRE model, at four flight conditions:

- Flight Condition 1: Mach 0.4, altitude 4500m;
- Flight Condition 2: Mach 0.6, altitude 1500m;
- Flight Condition 3: Mach 0.85, altitude 5500m;
- Flight Condition 4: Mach 0.9, altitude 2000m.

The controllers for these flight conditions were designed in the Matlab software package using the Weighted Mixed Sensitivity Criteria. As
weighting functions we have used \( W_1(s) = \frac{5(0.3s+3)}{s+0.15} \) and \( W_2(s) = \frac{1}{0.01s+1} \).

The mixed sensitivity problem was slightly altered to require the following minimization

\[
\left\| \begin{bmatrix} W_1 S & \lambda \end{bmatrix} W_2 GKS \right\|_\infty \leq \gamma
\]

as such, and also because of the implementation of the modification from [4], the performance specification was changed from (2) to the following:

\[
J(K, u, y, \tau) = \frac{\|w_1(y-z)(y-z)\|_{L_2([0,\tau])}^2 + \|w_2(y\tau)\|_{L_2([0,\tau])}^2}{\|z\|_{L_2([0,\tau])}^2}
\]  (5)

where \( w_1 \) and \( w_2 \) are the impulse responses of the weighting transfer functions \( W_1(s) \) and \( W_2(s) \) used in the design of the four controllers \( (K_1 \ldots K_4) \), corresponding to the four flight conditions. Thus we impose on the falsification algorithm that only those controllers meeting the same design criterions as the pre-designed controllers be unfalsified. The “*” symbol means convolution, \( \tilde{z} \) represents the fictitious reference signal as defined by (3), \( y \) is the plant output signal, and \( u \) is the control signal.

This translates into the following cost function.

\[
J = \sum_{k} \left[ \| \tilde{w}_1 \|_{L_2([j-1)k,jk]}^2 + \| \tilde{w}_2 \|_{L_2([j-1)k,jk]}^2 \right]
\]

where \( k \) is the dwell interval (the minimum time for which a controller is in the loop, and during which measurements of \( u \) and \( y \) are performed), \( j \) represents the index of the current dwell interval, \( \tilde{w}_1 \) is a vector containing the responses of the \( W_1 \) transfer function to the inputs contained in the vector \( (y-z) \), \( \tilde{w}_2 \) is a vector containing the responses of the \( W_2 \) transfer function to the inputs contained in the vector \( y \) and \( z \) is the vector containing the fictitious reference signals calculated over the \([0,(j-1)k]\) using (5).

The vertical black lines in the four figures represent the points at which the controller was switched by the algorithm. As can be seen with both plants the algorithm discarded the initial controller, which was unfit, after the first dwell interval. In the simulation that was ran at flight condition 4 the algorithm ultimately converged to the controller that was pre-designed for it. Also it is to be noted that while a final controller was selected after at least 10 seconds, the output signal of the plant never achieved high values.

### 5 Conclusions

The case study illustrated above confirmed the predictions of [8]. Before using the left-coprime factorization of the controllers, the authors tried using the original algorithm from [8] by
calculating invertible controllers. Despite meeting this requirement the computed virtual reference signal was unbounded and thus the algorithm did not provide conclusive results.

Using the left-coprime factorization, as one can see above in section 4, the algorithm performed as specified, further more it never produced unbounded virtual references.

Still despite the promising results several problems were noticed:

- In some instances when run at the flight conditions for which the vertex controllers were calculated, the algorithm tended to choose another vertex controller. This indicates a necessity to modify the cost function to emphasis the qualities used in the design of the vertex controllers.

- The values of the cost functions calculated for the different controllers tended to be very close together which may cause the algorithm to choose controllers which are not to appropriate for the considered flight condition.

Still even with the problems specified above the Unfalsified Adaptive Switching Supervisory Control approach shows a lot of promise for the control of Unmanned Aerial Vehicles, offering a capacity to learn and adapt to quickly changing flying conditions.

Further research will be conducted by the authors to address the problems illustrated above. Namely modifications of the cost function by implementing signal analysis tools will be considered so that the algorithm can better discriminate between the validity of different controllers. Also a generalized method of minimizing the cost function will be considered, so that the falsification algorithm becomes faster.

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References


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Fig. 4 Response of the closed-loop system with the plant linearized at Mach 0.75, altitude 5500 and an angle of attack of 12 degrees. Dwell interval 2 seconds.

Fig. 5 Response of the closed-loop system with the plant linearized at Mach 0.75, altitude 5500 and an angle of attack of 12 degrees. Dwell interval 1 second.
Fig. 6 Response of the closed-loop system with the plant linearized at flight condition 4 (Mach 0.9, altitude 2000 and angle of attack of 0 degrees). Dwell interval 2 seconds.

Fig. 7 Response of the closed-loop system with the plant linearized at flight condition 4 (Mach 0.9, altitude 2000 and angle of attack of 0 degrees). Dwell interval 1 second.