Multi-disciplinary Design Optimization of Supersonic Transport Wing Using Surrogate Model

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Key words: *Silent Supersonic Transport, Design Exploration, Data Mining, Kriging model*

Abstract

In this study, high efficient design tool is developed with several informatics approaches for multi-disciplinary design optimization and knowledge discovery of supersonic wing geometry. In this tool, multi-objective genetic algorithm (MOGA) is applied as an optimizer, while Kriging model is also used to reduce computational cost. To obtain the information of the design space, functional Analysis of Variance (ANOVA) and parallel coordinate plot (PCP) is applied. For Kriging model construction, 107 sample points are evaluated. This tool is applied to the multidisciplinary design problem of supersonic wing. The objective functions are to maximize lift to drag ratio and to minimize sonic boom intensity at supersonic cruise, and to minimize wing weight. According to their results, there is trade-off relationship among three objective functions. ANOVA results say that the cambers of the wing section at the root and the kink are effective to lift to drag ratio, the inner wing sweep back angle is in sonic boom intensity, camber of wing section at kink and aspect ratio are in the wing weight. According to PCP result, the design space information from sampling results could be visualized quantitatively. The present design space exploration process using MOGA is carried out based on Kriging surrogate models, therefore proposed MDO is effective in terms of computational cost.

1 Introduction

In this paper, to acquire the design knowledge, multi-disciplinary design optimization (MDO) tool is constructed for the computer aided design. Design target is silent supersonic transport (SST) demonstrator (S^3TD) developed by JAXA [1, 2] (Fig.1 (a)). S^3TD should be designed with low drag, low sonic boom at the supersonic cruise, low structural weight, low noise at take-off/landing, and so on. Then, this design problem has multi-objective.

To carried out the design with little computational cost, the CAD-based automatic panel analysis system (CAPAS) developed by JAXA [1] is applied. In CAPAS, the pressure on the aircraft can be evaluated using a full potential flow solver with little computational cost. After the pressure distribution is obtained, CAPAS estimate the shock wave using Thomas's waveform parameter method, and a waveform of a sonic boom can be obtained by Whitham's theory. To construct the high-efficient design tool, Kriging model is applied to reduce the computation time. Kriging surrogate model is well-known to predict function values at unknown points [3].

This paper focuses on the S^3TD wing designs that can help achieve a high lift over drag ratio (L/D) and low sonic boom intensity (ΔP), and low structural weight of the wing (W_{wing}). Therefore, the design problem in this paper has three objectives; to maximize L/D, to minimize ΔP and W_{wing}. 107 individuals in design space are sampled to create the surrogate model for every objective functions. Multi-objective genetic algorithm (MOGA) [4] is used to obtain the non-dominated solutions on Kriging surrogate model and additional sample points. MOGA is heuristic search method and can solve multi-objective problem well. After the sampling process, analysis of variance (ANOVA), which is one of the multivariate analysis methods, is used to acquire quantitative information about the contributions of the design variables to every objective functions. Parallel coordinate plot (PCP) [5], which is one of the statistic data mining techniques, is also applied to the sampling result to visualize the design space information. Through these analyses, design knowledge about the multi-disciplinary design problem of the supersonic wing can be obtained.

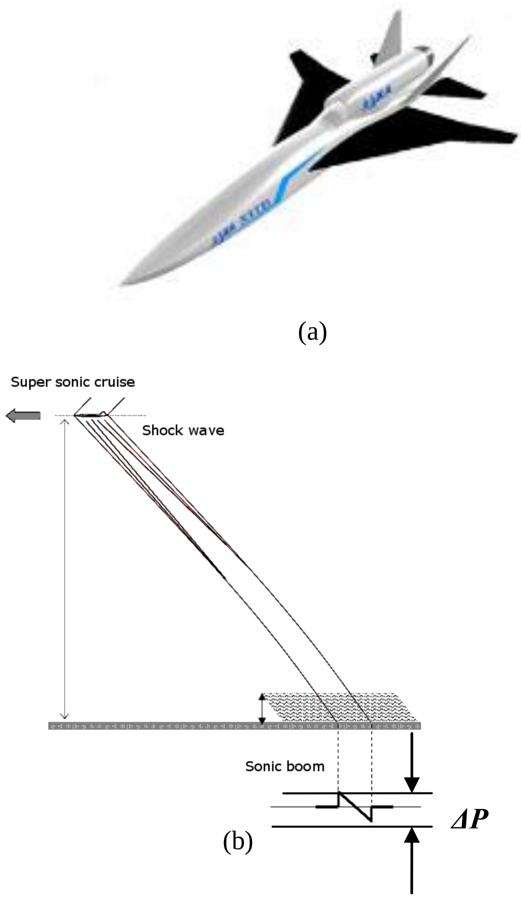


Figure 1. Silent Supersonic Transport Demonstrator (S^3TD) Designed at JAXA. (a)Conceptual design, and (b) definition of sonic boom intensity.

2 Design Suggestion

This study focused on creating the MDO tool with efficient global design methods, Kriging model based MOGA. The proposed design procedure is illustrated in Fig. 2. The detail of the exploration process as follows.

2. 1 Kriging model

Kriging model [4, 6] expresses the value, $y(x_i)$, at the unknown design point x_i. $y(x_i)$ is calculated as below.

$$y(x_i) = \mu + \varepsilon(x_i) \qquad (i = 1, 2, ..., m) \tag{1}$$

where, m is the number of design variables, μ is a constant global model and $\varepsilon(x_i)$ represents a local deviation from the global model. The correlation between $\varepsilon(x_i)$ and $\varepsilon(x_j)$ is strongly related to the distance between x_i and x_j. In the model, the local deviation as an unknown point x is expressed using stochastic processes. Some design points are calculated as sample points and interpolated with Gaussian random function as the correlation function to estimate the trend of the stochastic process.

2. 2 MOGA based on Kriging Models

Once the each objective function's model is made, the optimum points can be explored using an arbitrary optimizer on the model. For example, GA (Fig. 3) can explore the global optimum. However, it is possible to miss the global optimum, because the surrogate model includes uncertainty at the predicted point. Therefore, this study used *EI* values as the criterion.

EI for maximization problem can be calculated as bellow.

$$E[I(x)] = (\hat{y} - f_{\max})\Phi\left(\frac{\hat{y} - f_{\max}}{s}\right) + s\phi\left(\frac{\hat{y} - f_{\max}}{s}\right) \tag{2}$$

EI for minimization problem can be calculated as bellow.

$$E[I(x)] = (f_{\min} - \hat{y})\Phi\left(\frac{f_{\max} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\max} - \hat{y}}{s}\right) \tag{3}$$

where, f_{max} and f_{min} are the maximum/minimum values among sample points. $\hat{y}$ is the value predicted by Eq. (1) at unknown point. Φ and ϕ are the standard distribution and normal density, respectively. *EI* considers the predicted function value and its uncertainty, simultaneously. Thus, the solution that has a large function value and a large uncertainty may be a promising solution. Therefore, by selecting the point where *EI* takes the maximum value, as the additional sample point, robust exploration of the global optimum and improvement of the model can be achieved simultaneously because this point has a somewhat large probability to become the global optimum. To apply multi-objective problem, this study considers three *EI* values based on three Kriging models; $EI_{L/D}$, $EI_{\Delta P}$,and EI_{weight}.

To select additional samples, non-dominated solutions can be obtained solving *EI* maximization problem by MOGA. Obtained non-dominated solutions are divided into *N* clusters base on k-means clustering method [7] as shown in Fig. 4, and the mean values in each cluster is selected as additional samples.

2. 3 Analysis of Variance; ANOVA

An ANOVA [3, 4, 6] which is one of the multivariate analyses is carried out to differentiate the contributions to the variance of the response from the model. To evaluate the effect of each design variables, the total variance of the model is decomposed into that of design variable model $\hat{y}$. The main effect of design variable x_i is as bellow.

$$\mu_i(x_i) \equiv \int \cdots \int \hat{y}(x_1,.....,x_n)dx_1,...,dx_{i-1},dx_{i+1},..,dx_n - \mu \quad (3)$$

Two-way interaction effect x_i and x_j is written as bellow.

$$\mu_{i,j}(x_{i,j}) \equiv \int \cdots \int \hat{y}(x_1,.....,x_n)dx_1,...,dx_{i-1},dx_{i+1}...,dx_{j-1},dx_{j+1},..,dx_n - \mu_i(x_i) - \mu_j(x_j) - \mu \quad (4)$$

where, total mean μ is as bellow.

$$\mu \equiv \int \cdots \int \hat{y}(x_1,.....,x_n)dx_1,.....,dx_n \quad (5)$$

The variance due to the design variable x_i is

$$\varepsilon \equiv \int [\mu_i(x_i)]^2 dx_i \quad (6)$$

The proportion of the variance due to design variable x_i to total variance of model can be expressed.

$$p \equiv \frac{\varepsilon}{\int \cdots \int [\hat{y}(x_1,....,x_n) - \mu]^2 dx_1 ... dx_n} \quad (7)$$

The denominator of Eq. (7) means variance of the model. The value obtained by Eq. (7) indicates the sensitivity of the objective function to the variation of the design variable.

2. 4 Parallel Coordinate Plot (PCP)

Parallel coordinate plot (PCP) is one of the statistical visualization techniques for a high-dimensional data at a glance in the two-dimensional graph. To create PCP, the attribute values in the design problem have to be normalized to compare in the same axis. After the normalization, axes are arranged in consisting parallel line. Generally, the distances between a line and the next are equivalent. In this study, the normalization value p_i from design variable x_i is given the below.

$$p_i = \frac{x_i - x\min_i}{x\max_i - x\min_i} \quad (8)$$

where, $x\min_i$ represents the lower bound of i^{th} design variable and $x\max_i$ does the upper bound of i^{th} design variable.

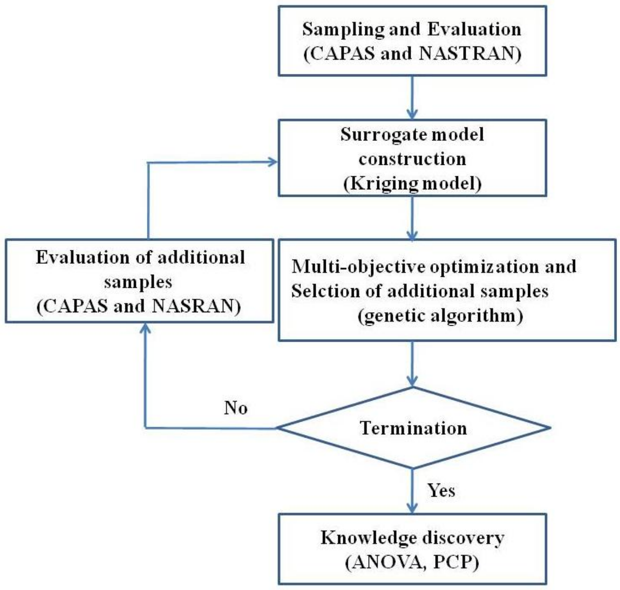


Figure 2. Procedure of multi-disciplinary design optimization for the wing

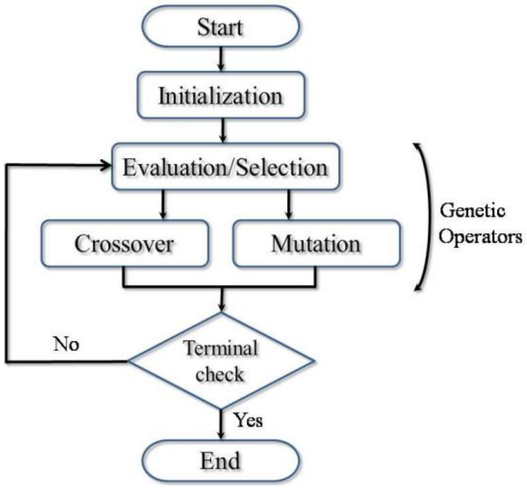


Figure 3. The calculation flowchart of genetic algorithm

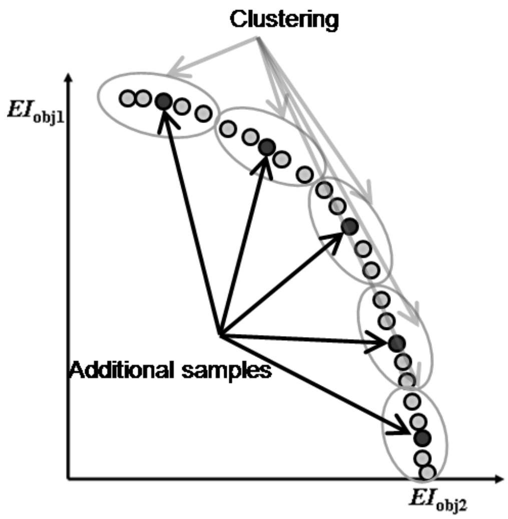


Figure 4. The decision to pick up additional samples with k-means method.

3 Evaluations of the Objective Functions

3. 1 Aerodynamics Performance and Sonic Boom

L/D and *ΔP* are evaluated by CAPAS, which is developed by JAXA. The pressure distribution is obtained by CAPAS, as shown in Fig. 5. CAPAS is developed as a conceptual design tool; this tool includes a CATIA® v4/v5 application programming interface (API) and a full potential solver with panel method. The thickness in this study is small enough to be regarded as shown below.

$$(M_\infty^2 - 1)\phi_{xx} - \phi_{yy} - \phi_{zz} = 0 \tag{9}$$

L/D and C_L are directly estimated from the pressure distribution.

To evaluate the sonic boom, the shockwave form is simulated from the pressure distribution around the aircraft based on Whitham's theory [1]. Thomas's waveform parameter method [1] is used to estimate *ΔP* from the pressure distribution near the aircraft, which is estimated using Whitham's theory.

3. 2 Wing Structural Weight

In this study, structural optimization of thickness of each multi-frame for inboard wing and wing stacking sequences of laminated composites for outboard wing was performed to realize the minimum weight with constrain of the strength requirement. Given the wing outer mold line for each individual, finite element model (FEM) was automatically generated from aerodynamic evaluation results at supersonic cruise, such as, coordinates, pressure coefficient, and normal vectors. The strength characteristic was evaluated by MSC.NASTRAN™, commercial software.

The design variables were four, stacking sequences (fiber angle of a ply θ and number of symmetrical stacking n) of the skin in outboard wing. θ was defined as symmetrical stacking [-θ/ 0/ θ/ 90]. Note that θ was set on 15, 30, 45, 60, and 75 degrees. As the stacking sequences are optimized for the skin in outboard wing, each thickness is minimized for the skin and multi-frame in inboard wing. NASTRAN evaluation is repeated for every individual until the strength requirement is met. This process is shown Fig. 6.

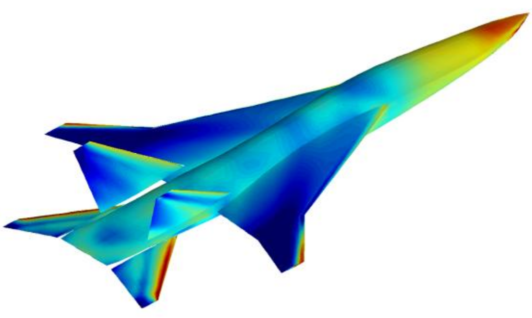


Figure 5. Pressure coefficient distribution obtained by CAPAS.

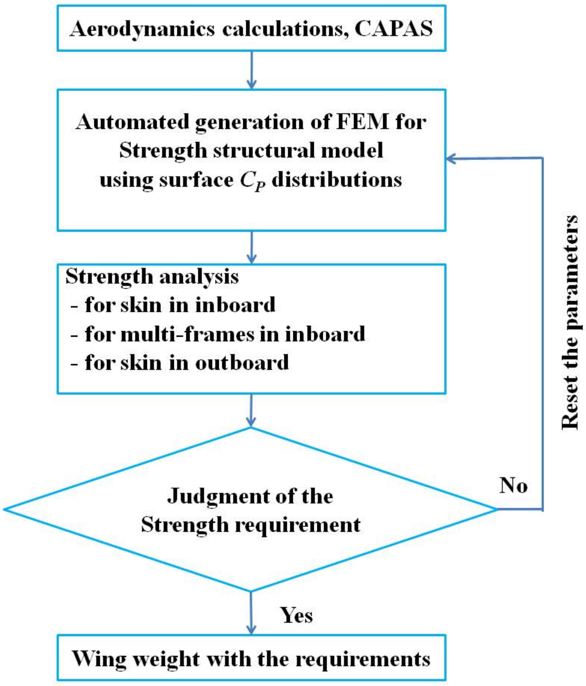


Figure 6. Procedure of wing weight calculation

4 Formulation

The three-dimensional main wing design of 2.5th S^3TD composed by main wing, fuselage, vertical tail and stabilizer is considered. Fuselage length is 13.8m and maximum take-off weight is 3500Kg. The main wing is formed with three airfoils at root, kink, and tip (Fig. 7).

4. 1 Objective Functions

The objective functions are the maximization of lift to drag ratio (L/D) at a target C_L and minimization of sonic boom intensity (ΔP) which has the largest peak across boom carpet, and minimization of wing structural weight (W_{wing}). ΔP is defined as the peak differences between the highest and lowest pressure (Fig. 1(b)). Thus, the design problem is given bellow.

$$\begin{cases} \text{maximize} & L/D\,(\text{target} \quad C_L = 0.105) \\ \text{minimize} & \Delta P \\ \text{minimize} & W_{wing} \end{cases} \tag{10}$$

Cruise Mach number is 1.6 and the altitude is 14.0Km.

4. 2 Design Variables

Design variables for a three-dimensional supersonic wing are defined, as shown in Fig. 2. The design variables and their values are summarized in table1. In the supersonic cruise, the inboard wing has supersonic leading edge to acquire the high aerodynamic performance at supersonic cruise, and the outboard wing has subsonic leading edge to maintain the subsonic aerodynamic performance for the landing and take-off.

4. 3 Wing Structural Model

Wing structure model was based on reference [2]. Inboard wing is made from aluminum material and outboard wing is made from composite material. Inboard wing is composed of multi-frame structure, such as frame, rib, and spar. Outboard wing is compounded from full-depth honeycomb sandwich structure.

The computational condition was set on the symmetrical maneuver +6G and the margin of safety was on 1.25. The speed of sound and the air density was set on the altitude of 14Km.

The strength requirement in inboard has less than 200Mpa in every element while that in outboard meets the criteria of Tsai-Wu [2].

4. 4 Constrains

To maintain the trim balance of the aircraft, the deflection angle of the horizontal tail wing is changed in every wing design. To decide this angle the aerodynamic evaluations are executed for each sample with two different deflection angle of the horizontal tail wing. Thus, evaluation cost becomes expensive. Besides, the attachment location of main wing is subsidiary design variable to maintain MAC at 25% chord.

Not to deteriorate subsonic aerodynamics performance, wing area is fixed to 21 square meters. Then, this study has three constrains.

4. 5 Selection of Additional Samples

EIs maximization problem is solved by MOGA as discussed in Sec. 2.2. In this study, this process is iterated three times. For MOGA search, Eq. (10) are written for the present design problem as bellow.

$$
\begin{cases}
\text{maximize} \\
\bullet\ EI_{L/D} = (\hat{y} - L/D_{\max})\Phi(\frac{\hat{y} - L/D_{\max}}{s}) + s\phi(\frac{\hat{y} - L/D_{\max}}{s}) \\
\text{minimize} \\
\bullet\ EI_{\Delta P} = (\Delta P_{\min} - \hat{y})\Phi(\frac{\Delta P_{\min} - \hat{y}}{s}) + s\phi(\frac{\Delta P_{\min} - \hat{y}}{s}) \\
\text{minimize} \\
\bullet\ EI_{W_{wing}} = (W_{wing_{\min}} - \hat{y})\Phi(\frac{W_{wing_{\min}} - \hat{y}}{s}) + s\phi(\frac{W_{wing_{\min}} - \hat{y}}{s})
\end{cases} \quad (11)
$$

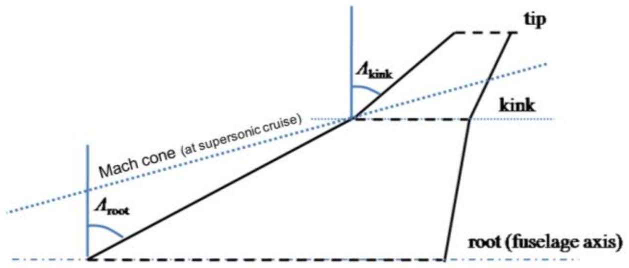


Figure 7. Illustration of wing geometry to be designed.

Table 1. Design variables and their values.

	Design variable	Lower bound	Upper bound
*dv*1	Sweepback angle at inboard section	57 (°)	69 (°)
*dv*2	Sweepback angle at outboard section	40 (°)	50 (°)
*dv*3	Twist angle at wing root	0 (°)	2(°)
*dv*4	Twist angle at wing kink	–1 (°)	0 (°)
*dv*5	Twist angle at wing tip	–2 (°)	–1 (°)
*dv*6	Maximum thickness at wing root	3%*c*	5%*c*
*dv*7	Maximum thickness at wing kink	3%*c*	5%*c*
*dv*8	Maximum thickness at wing tip	3%*c*	5%*c*
*dv*9	Aspect ratio	2	3
*dv*10	Wing root camber at 25%*c*	–1%*c*	2%*c*
*dv*11	Wing root camber at 75%*c*	–2%*c*	1%*c*
*dv*12	Wing kink camber at 25%*c*	–1%*c*	2%*c*
*dv*13	Wing kink camber at 75%*c*	–2%*c*	1%*c*
*dv*14	Wing tip camber at 25%*c*	–2%*c*	2%*c*

5 Results

5. 1 Solutions Space

First, 75 initial samples were calculated to construct initial Krigng models. Then additional samples were picked up by solving the multi-objective problem as discussed in Sec. 4.5. Fig. 8(a)-(c) shows the solution space (*L/D* - *ΔP*, *L/D* - W_{wing}, *ΔP* - W_{wing}, respectively). Note that the W_{wing} in these figures is for the half span of the wing. Diamonds and squares represent initial samples and additional samples, respectively. Most of additional samples formed around optimum direction.

Fig. 8(a) shows the weak trade-off between *L/D* and *ΔP*. It has been reported that *ΔP* should less than 1 [8] if the aircraft fly over the land. In this study, two samples achieve lower *ΔP* than 1. On the other hand, they achieve approximately *L/D*, 6. This quantitative information suggests remarkable of the main wing design for the low boom supersonic aircraft.

Fig. 8(b) shows sever trade-off between *L/D* and W_{wing}. There are some samples whose weight around 250kg in the sampling results. On the other hand, they achieve lower *L/D* than 6. If the wing achieves more *L/D* than 7, these weights increase more than 400kg. This difference is remarkable for the present demonstrator whose total weight is 4000kg.

Fig. 8(c) shows scatter plots between *ΔP* and W_{wing}. Two samples which achieve lower *ΔP* less than 1 can be found as already discussed. If the W_{wing} becomes lower than about 100kg, *ΔP* would become higher than around 1.1. This result suggests that these two objective functions also have strong trade-off.

5. 2 ANOVA

Fig. 9(a)-(c) shows the result of ANOVA about each objective functions. Fig. 9(a) says *dv10-13* have big influences on *L/D*. This result suggests that the inboard which has a subsonic leading edge almost gains the lift with the cambers at supersonic cruise.

Fig. 9(b) shows *dv1* has a great influence on *ΔP*. Sweep back angle change in inboard section brings the change of MAC location. It also brings the different attachment location of main wing to satisfy constrain dependently. It would have A_e distribution.

Fig. 9(c) shows that *dv6, dv9, dv12, dv13* have influences on W_{wing}. The wing weight should be increased when the aerodynamic load is increased. The wing kink camber (*dv12, and dv13*) which influences on aerodynamic load around the main wing at supersonic cruise, thus *dv12*, and *dv13* show the large contribution to W_{wing}. Besides, small wing weights are intend to have small aspect ratio (*dv9*) and it corresponds to typical wing structure theory.

5. 3 PCP visualizations

Fig. 10(a)-(c) shows PCP results which are created from the champion data about each objective functions in the design space.

According to Fig. 10(a), there are four desirable values, *dv6, 10-13,* to achieve higher *L/D*. From described above, *dv6* should be set to 3%*c*, *dv10-13* should be set to -0.5%*c*, -0.64%*c*, 0.75%*c*, and 0.22%*c*, respectively. (*c* represents the chord length at the designed cross sections.) , aerodynamic This result suggests that wings which achieves better *L/D* tend to have negative camber around the wing root, and positive camber around the wing kink. Thus, the aerodynamic lift becomes large around the kink, and the drag around the root which have relative thick airfoil becomes small.

According to Fig. 10(b), *dv4, 6, 7, 9, 10, 14* would have ideal values to achieve low *ΔP*, that is, around 3%*c*, 3%*c*, 2.3, -0.46%*c*, and 1.45%*c*, respectively. ANOVA result discussed in Sec. 5.2 suggests that the sweep back angle of the inboard wing (*dv1*) would have great impact to *ΔP*. PCP shows the trend to have relative large *dv1* for low *ΔP*.

Fig. 10(c) shows that required design variables for low W_{wing} designs are almost different trend from the design variables for higher *L/D* and lower *ΔP*. This result suggests that there are sever trade-off between the aerodynamic performance and the structural weight of the supersonic wing. Additionally, low weight wing achieves low *L/D*. Therefore, aerodynamic load on such wings becomes small. As this result, the airframe (fuselage, tail wing) would gain the lift. This knowledge could be obtained in consideration of the complex aircraft geometry with efficient design process.

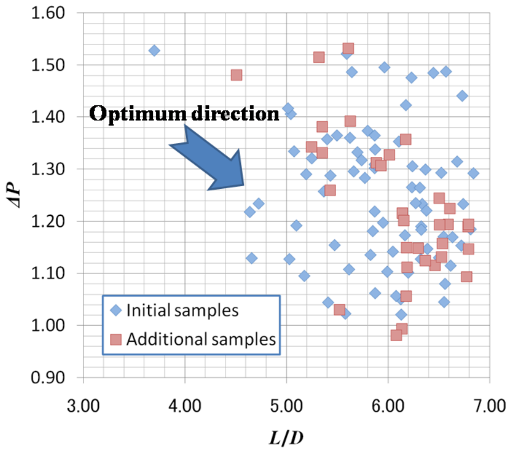
5. 4 Comparison of geometries from non-dominated solutions

In Fig. 11(a)-(c) and Fig. 12, the designed layout, the pressure distribution, and the waveform of sonic boom are shown and compared among the extreme solutions from sapling results and 2.5th S^3TD designed in JAXA [1, 9]. *Design A* has the maximum *L/D, Design B* does the minimum *ΔP*, and *Design C* shows the minimum wing weight. Note that the dot-lines in Fig. 11(a)-(c) show the 2.5th design. Table 3 shows the value of each objective function and angle of attack. Fig. 13 shows the distributions of the equivalent area A_e about *Design A-C* with *Darden's* distributions.

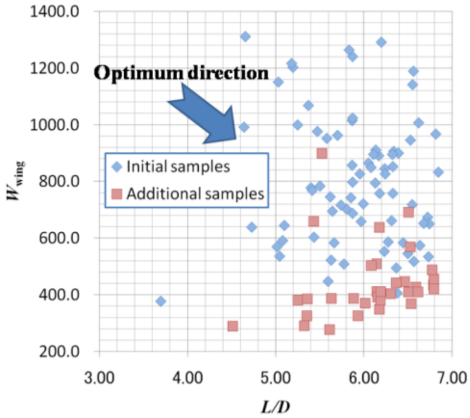
Design A has thinner wing compared with the others. The thinner wing would be achieves higher *L/D*, according to the CAPAS result. Because wave drag could have larger influence than induced drag [10], this result is reasonable.

Design B has the most similar differences A_e distributions compared with *Darden* distributions. According to ANOVA results, sweepback angle at inboard has a great influence to *ΔP*. This result suggests that the sweepback is very effective to A_e. Thus, the sweepback angle is one of the key parameter for *ΔP* minimization.

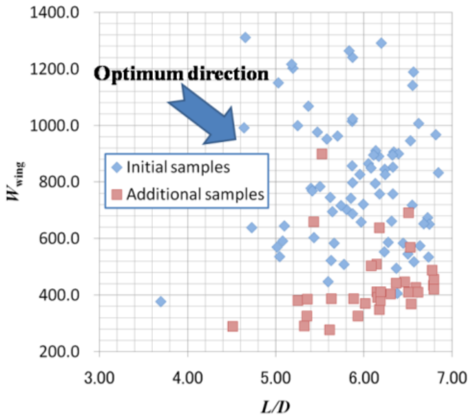
Design C has a negative camber at kink, tip. Typically, the main wing has to get the sufficient lift with positive cambers. However, this design has negative pressure area on the lower surface, because it has negative camber around the kink. This illustration means that the horizontal tail has to gain the rest of lift instead. Therefore, this configuration would make low aerodynamic load, and the wing weight is reduced. Such kind of noble and remarkable knowledge can be obtained because this design problem considers the trim balance as a constraint.



(a) *L/D vs. ΔP*

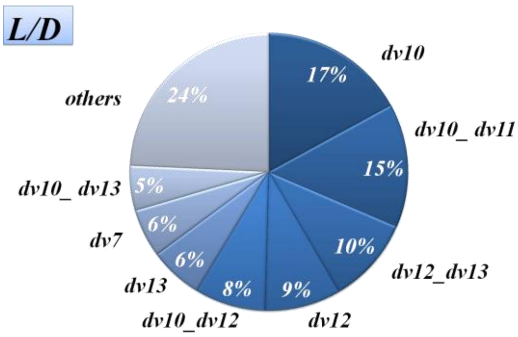


(b) *L/D vs.* W_{wing}

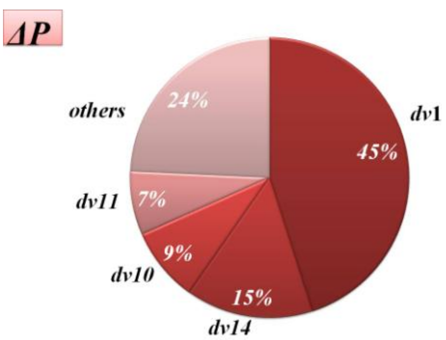


(c) *ΔP vs.* W_{wing}

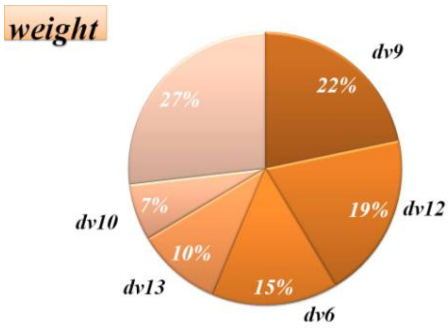
Figure 8. Solutions spaces



(a) *L/D*

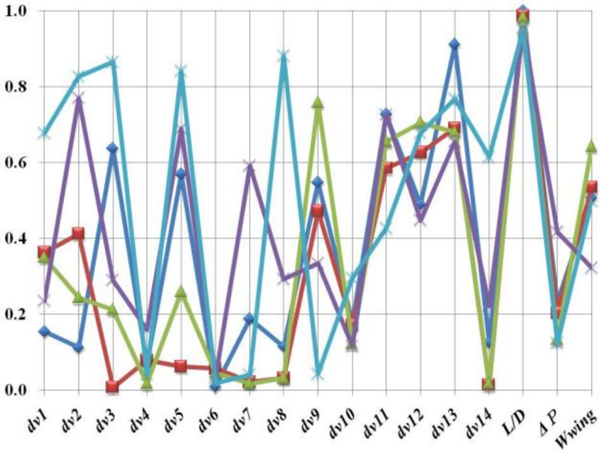


(b) *ΔP*

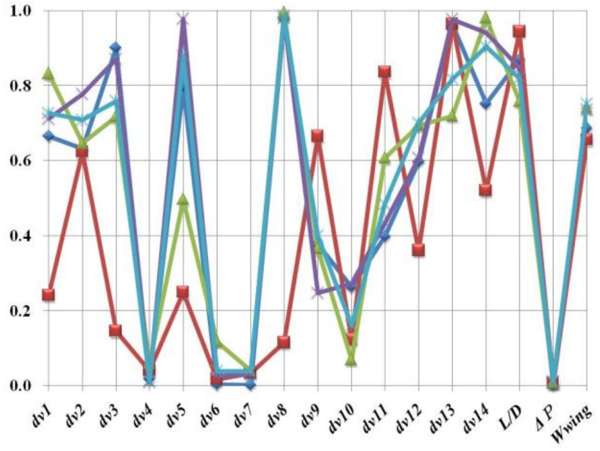


(c) W_{wing}

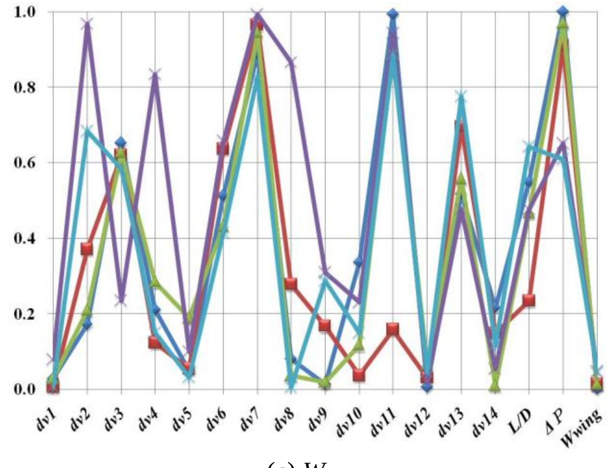
Figure 9. The results of ANOVA



(a) *L/D*

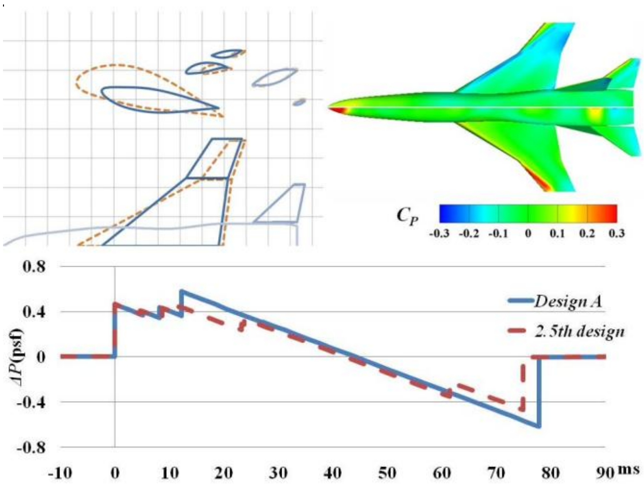


(b) *ΔP*

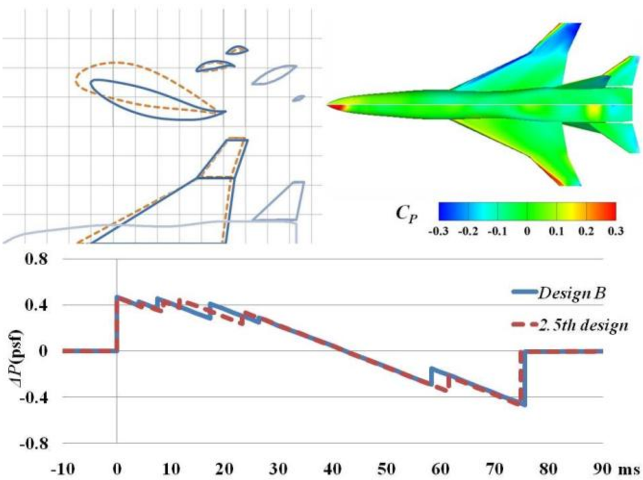


(c) W_{wing}

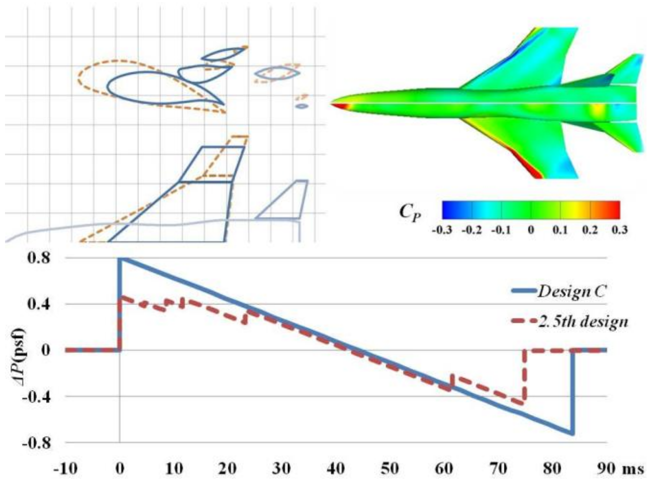
Figure 10. The result of PCP for the best 5 individuals regarding to every objective functions



(a)*Design A* (maximum *L/D*)



(b)*Design B* (minimum *ΔP*)



(c)*Design C* (minimum W_{wing})

Figure 11. The configuration and C_P distributions and sonic boom wave of extreme solutions about every objective function

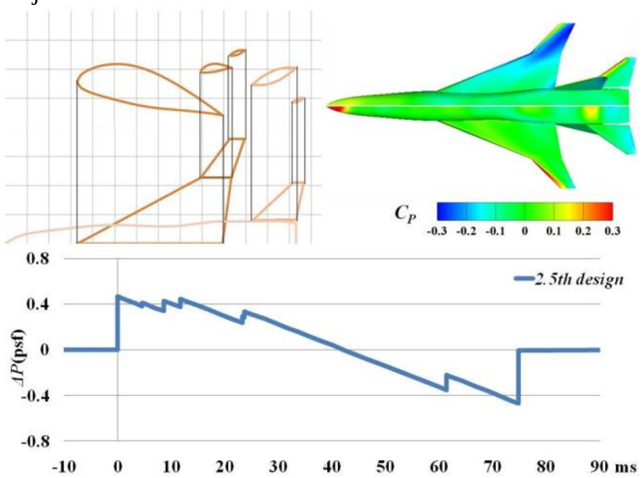


Figure 12. The configuration and C_P distributions and sonic boom wave of 2.5th design

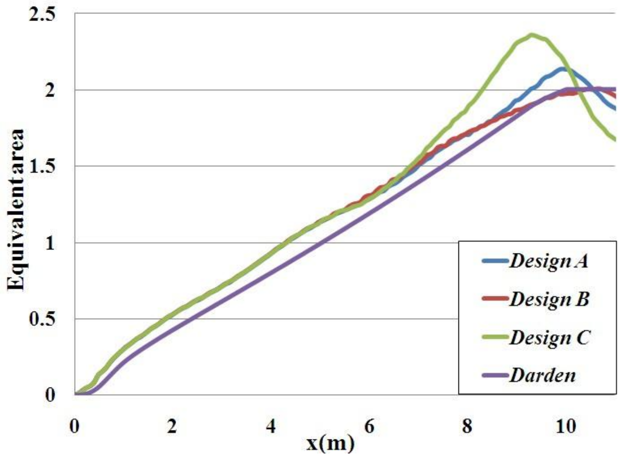


Figure 13. Equivalent Area of extreme solutions

Table2 Comparison of evaluated values among the *Designs A, B, C* and 2.5th Design

	L/D	*ΔP*(psf)	*Wwing*(kg)	AoA
Design A	7.02	1.19	612	2.5
Design B	6.08	0.97	502	2.7
Design C	5.60	1.53	276	2.6
2.5 design	6.90	0.98	691	2.1

6 Conclusions

This study carried out the construction of high efficient MDO tool with several informatics approaches. For the design example, the design of S^{3}TD developed by JAXA was also demonstrated. Proposed MDO tool with Kriging model based MOGA was able to obtain many non-dominated solutions. Remarkable design knowledge can be also visualized and discovered by ANOVA and PCP.

Discovered knowledge from ANOVA and PCP for the S^{3}TD design are follows.

- Not only the planform of the wing but also the camber geometry at the wing kink influences on every objective functions.
- The sweepback angle at inboard is mainly decided the equivalent area, and the sonic boom intensity.

These knowledge can be obtained in several weeks, thus the efficiency of the proposed design tool can also be demonstrated for the conceptual design.

Acknowledgement

I would like to thank my paper adviser, Dr. Masahiro Kanazaki, for his guidance, and support. I also wish to thank Dr. Yoshikazu Makino, and Dr. Ken Takatoya, researchers in Aviation Program Group/Japan Aerospace Exploration Agency, for their useful advice and support.

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