MODELING OF ATMOSPHERIC CONDITION INFLUENCE ON SONIC BOOM

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Abstract

The state of the art in research of atmospheric conditions influence on sonic boom is discussed. The influence of atmospheric turbulence on the characteristic ray trajectories and amplitude of propagating plane shock wave is investigated with the simplified model approach. Isotropic character of atmosphere turbulence is assumed. It is also supposed that the atmospheric turbulence is "frozen", i.e. the time of acoustic wave passing through the turbulent media is considerably less, than the time of evolution of atmospheric turbulence structures. The turbulence field is simulated as a series of independent realizations of random scalar and vector fields that are not related one with the other. Propagation of the sonic boom wave is described on the base of classical Hayes - Zhilin theory equations. The using of Hopf's transformation allows proving the unique existence of their solution. It is shown that under given arbitrary initial overpressure distribution p_0 *the sought distribution* p *in the whole area of the flow is continuously.*

1 Introduction

Indubitably tomorrow is with supersonic passenger aviation. For the further progress in this area the adequate estimation of sonic boom level is extremely important. The clear international requirements imposing restrictions to sonic boom characteristics are not elaborated for the present, however sonic boom level exceeding 50 Pa is considered to be objectionable. Sonic booms constraints may essentially affect on aircraft configuration of supersonic transports (SST) are elaborated just as routing and flight regimes selection [1]. For the present-day for small-sized aircraft, such as supersonic business jet, the level of 15 Pa overpressure in head shock wave and loudness volume about 65 dBA, in principle, are accessible [2, 3].

The theoretical analysis of the phenomenon under consideration has reached enough high level. Particularly, the quite efficient algorithms and the computer codes of sonic boom calculations have been developed in TsAGI.

The tools available today for sonic boom prediction allow obtaining various sonic boom characteristics with rather good accuracy at state-determined (nominal) atmosphere conditions and they can be used also for optimizing aircraft configuration and flight regimes to minimize shock wave impact [4-6].

At the same time, atmospheric conditions such as wind, cloudiness, atmospheric turbulence, etc. notably effect upon shock wave overpressure signature and noise spectrum, as well as sonic boom carpet. The issue of more accurate calculation taking into account the meteorological condition effect on the sonic boom remains less evident [7-9].

Inhomogeneity of atmosphere is characterized by various parameters. Nevertheless, as a matter of fact, it may be described by superposition of two factors: slow variations of state due to stratification, and more rapid variations due to random fluctuations of wind velocity and air temperature or atmospheric turbulence.

The analysis of the results of measurements of sonic boom characteristics performed in flight experiments has shown [9] that the sonic boom intensity perceived on ground surface can be greatly more (or, on the contrary, more less), than nominal computed value, and that this phenomena - sooner rule, than exception.

The fundamentals for theoretical analysis of the propagating the acoustic waves of the finite amplitude in dissipative turbulent medium are stated by D.I. Blokhintcev and V.I. Tatarskii in their monographs [10, 11]. State of the art in research on the problem of modeling of the atmospheric turbulence influence on propagation of sonic boom wave was considered in the paper [12]. We shall notice that the expansion of the investigations in this area is observed at last years [13-15]. However using existing models is limited for present-day by the simplest cases of the shock waves propagation and for their practical application will take else much efforts.

In present work the influence of atmospheric turbulence on the characteristic ray paths and amplitude of two-dimensional propagating acoustic wave is investigated in simplified model problem definition. Isotropic character of atmosphere turbulence is assumed. It is also supposed that the atmospheric turbulence is "frozen", i.e. the time of acoustic wave passing through the turbulent media is considerably less, than the time of evolution of atmospheric turbulence structures. The turbulence field is simulated as a series of independent realizations of random scalar and vector fields that are not related one with the other [16].

Propagation of the sonic boom wave is described on the base of classical Hayes - Zhilin theory equations. The using of Hopf's transformation allows proving the unique existence of their solution. It is shown that under given arbitrary initial overpressure distribution p_0 the sought distribution p in the whole area of the flow is continuously.

2 Modeling of the atmospheric turbulent boundary layer influence on sonic boom wave propagation

To evaluate the ground level parameters of the sonic boom wave passing through the atmospheric turbulent boundary layer the isotropic character of the atmosphere turbulence was assumed (this seems to be the most reasonable way).

Within the frame of this work the elaboration of the empirical method for analysis of relationships between the sonic boom parameters and the turbulence characteristics is based on Yu.L. Zhilin's method of the sonic boom calculation based on the geometric acoustics laws [17, 18], and on the Ph. Blanc-Benon et al. stochastic model of the acoustic waves propagating through the random scalar and vector fields [19, 16].

Hereinafter, we limit consideration of the model problem of the passing of the plane acoustic wave through two-dimensional stochastic field.

The velocity $\mathbf{V}$ of 2D stochastic isotropic vector field at any given point x has a fluctuating component that may be presented as a sum of n random Fourier-modes:

$$\mathbf{v}'(\mathbf{r}) = \sum_{i=1}^{n} \mathbf{u}_i(\mathbf{K}_i)\cos(\mathbf{K}_i\mathbf{r} + \varphi_i),$$
$$\mathbf{u}_i(\mathbf{K}_i)\cdot\mathbf{K}_i = 0,$$

where $\mathbf{r}$ is the radius-vector of the point with the orthogonal coordinates (x_1, x_2), the x_1–axis coincides with the initial direction of the wave front.

The direction of the wave vector $\mathbf{K}_i$ of each mode is random; within two-dimensional case, it is characterized by the random angle θ_i. The homogeneity of turbulence field is ensured by the randomness of the phase shift φ_i. The angle θ_i and phase shift φ_i are independent random variables with uniform distributions. The amplitude of the velocity fluctuations $|\mathbf{u}\,(\mathbf{K}_i)|$ is a deterministic variable, its value is set according to the energy spectrum $E(K)$, with K=$|\mathbf{K}_i|$:

$$\left|\mathbf{u}_i(\mathbf{K}_i)\right| = \sqrt{E(K)\cdot \Delta K}\ ,$$

where ΔK is a K increment.

In this case we consider the fields with a Gaussian correlation function $f(r) = \exp\left(-\dfrac{r^2}{L^2}\right)$. The length scale L is related to the longitudinal integral length scale L_f by

$L_f = \frac{\sqrt{\pi}}{2} L$, and r is a distance between two arbitrary chosen points between which the correlation of velocity fluctuations is evaluated.

For two-dimensional Gaussian random velocity fields the energy spectrum is determined by equation:

$$E(K) = \frac{\overline{v'^2}}{8} \cdot K^3 L^4 \exp\left(-\frac{K^2 L^2}{4}\right),$$

with $\overline{v'^2} = \overline{v_1'^2} = \overline{v_2'^2}$ is the mean value of the square of the velocity fluctuations. We note that changing the kind of the energy spectrum constitutes no any difficulties.

It is assumed, that $K_{\min} = 0.1/L$, and $K_{\max} = 10/L$. We are restricted by 50 random Fourier-modes in our simulations. The averaging was performed over the ensembles of order of 100 realizations of the stochastic field.

Two-dimensional random temperature fluctuations field is defined by the same way as the random velocity fluctuations field:

$$T'(\mathbf{r}) = \sum_{j=1}^{n} \Theta_j(\mathbf{K}_j) \cos(\mathbf{K}_j \mathbf{r} + \psi_j),$$

As previously, the direction of the wave vector $\mathbf{K}_i$ and the phase shift ψ_j are the independent random variables with the uniform distributions, but $\Theta_j(\mathbf{K}_j)$ is defined by the temperature fluctuations energy spectrum

$$G(K) = \frac{\overline{T'^2}}{2} \cdot K \cdot L^2 \exp\left(-\frac{K^2 L^2}{4}\right),$$

where $\overline{T'^2}$ is the mean value of the square of the temperature fluctuations.

The linear geometric acoustics forms the basis for the determination of the ray paths in each of turbulent layer realizations. As far as it is known the rays constitute the line tangent to the group velocity $\mathbf{c}_g = c\cdot\mathbf{n} + \mathbf{V}$. Here c is the local sound velocity, $\mathbf{n} = \mathbf{P}/P$ is the unit vector along the direction of the wave front propagation, $\mathbf{V}$ is a medium velocity vector, $\mathbf{P}$ is a dimensionless wave vector, $P = N/(1 + \mathbf{M}\cdot\mathbf{n})$, $N = c_0/c$ is the refraction index and $\mathbf{M} = \mathbf{V}/c$ is the Mach number, c_0 is the sound velocities in undisturbed medium.

For the plane wave propagating through the random field the ray trace calculation consists in the solution of the system of eight ordinary differential equations:

1) four equations to determine the coordinates of radius-vector $\mathbf{r}$ and components of the wave vector $\mathbf{P}$ with the initial conditions

$$\mathbf{r}_{t=0} = \begin{pmatrix} 0 \\ x_2^0 \end{pmatrix}; \ \mathbf{P}_{t=0} = \frac{N}{(1 + \mathbf{M}\cdot\mathbf{n})} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

2) four equations to determine the components of the geodesic elements $\mathbf{R}^{\alpha} = \left(\frac{\partial \mathbf{r}}{\partial x_2^0}\right)$ and $\mathbf{Q}^{\alpha} = \left(\frac{\partial \mathbf{P}}{\partial x_2^0}\right)$ with the initial conditions

$$\mathbf{R}^{\alpha}{}_{t=0} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \ \mathbf{Q}^{\alpha}{}_{t=0} = \frac{\partial \mathbf{P}_{t=0}}{\partial x_2^0} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

The determination of the geodesic elements $\mathbf{R}^{\alpha}$ and $\mathbf{Q}^{\alpha}$ describing the wave front evolution along each ray is necessary to calculate the elementary ray tube area.

Two limit cases are considered:

1) The turbulence is caused by the temperature fluctuations only, i.e. Mach number $\mathbf{M} = 0$, and the refraction index $N = 1 - T'/2T_0$, where T_0 is the temperature of undisturbed medium. It is possible to represent the refraction index with a good accuracy as: $N = \exp(-T'/2T_0)$. Such a representation allows simplifying the process of the spatial derivatives calculation and, thereby, permits to accelerate the solution of the differential equations system.
2) The turbulence is caused by the velocity fluctuations only. In this case $N = 1$.

The nonlinear transport equation for the propagation of the wave along the eigenrays to obtain the solution for the acoustic pressure p is used. This transport equation taking into account terms of the second order in momentum equation was derived by Robinson [20].

This equation is of the form of

$$\frac{\partial}{\partial s}\left[\frac{|\mathbf{A}|}{\rho_0 c}\cdot|\mathbf{n}+\mathbf{M}|\cdot(1+\mathbf{M}\cdot\mathbf{n})\cdot p^2\right]- \\ -\frac{2\gamma|\mathbf{A}|}{\rho_0^{\,2} c_0^{\,4}}p^2\frac{\partial p}{\partial t'}=0\,. \quad (1)$$

Here s is the arc length along the single ray trace; t' is the retarded time coordinate. For the plane wave the elementary ray tube area $|\mathbf{A}|=\left|\mathbf{i}R_2^{\alpha}-\mathbf{j}R_1^{\alpha}\right|\cdot\cos\lambda$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors along axes x_1 and x_2, λ is the angle between phase and group velocity vectors, γ is the nonlinearity coefficient (γ depends on the refraction index, when $N=1$, $\gamma=1.2$), ρ_0 is the density of undisturbed medium.

Unlike the approach stated in the paper [16], the equation (1) is solved in time-domain (i.e. in s and t' coordinates). If the eigenray passes through a caustic, then $|A|\to 0$ in the neighborhood of this point and the equation (1) has a singularity.

In order to avoid troubles in the numerical solution of the equation (1) the regularizing known as the method of artificial viscosity is applied [21]: the term $\varepsilon\dfrac{\partial^2 p}{\partial t'^2}$ is added to the transport equation (the ε is a small parameter). Then the modified equation (1) is solved with the use of algorithm described in Ref. 22. To some extent this artificial technique may be considered as the effects of absorption modeling [23].

The described method of evaluation of atmosphere turbulent boundary layer influence on the sonic boom wave parameters was applied for the solution of model task, namely the propagation of the plane N-wave through the random temperature or the velocity fluctuation field. The parameters of initial N-wave are the same as in Ref. 19. The peak overpressure is 500 Pa, the duration is 15 μs, and the rise time τ_0 (time portion between 10% and 90% of peak pressure) is 1 μs. The linear scale of turbulence L is assumed to be equal to 0.1 m.

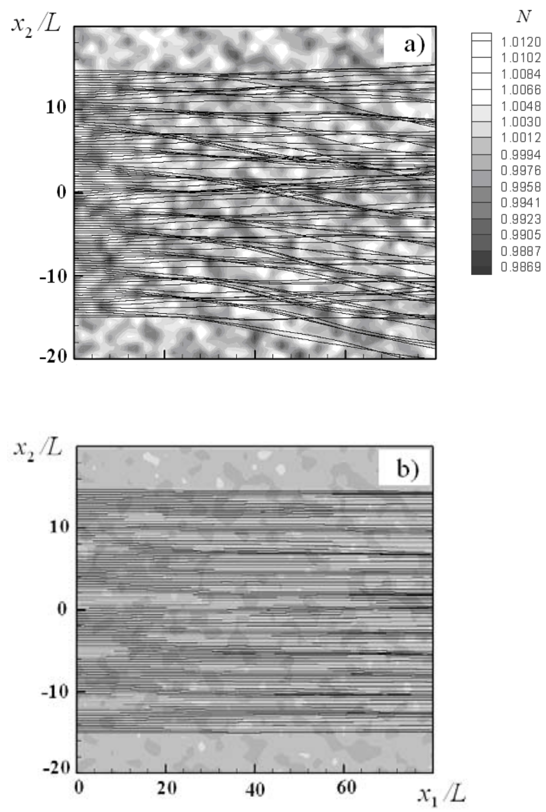

Fig. 1. Propagation of the acoustic rays through various realizations of the random temperature field:
a) $T'_{rms}/T_0=1.172\cdot10^{-2}$; b) $T'_{rms}/T_0=2.344\cdot10^{-3}$.

In Fig. 1 the ray traces of acoustic wave propagating through the various realizations of random temperature field are shown. The distribution of the N refraction index in calculation domain is shown as shades of gray map. It is demonstrated in the figure the focusing and defocusing phenomena of the acoustic wave on the local heterogeneities as well as the influence of the temperature stochastic field parameters on this process.

The first caustics appear at $T'_{rms}=\sqrt{\overline{T'^2}}=1.172\cdot10^{-2}\cdot T_0$ in the range of $15< x_1/L <25$. It is clear from the concentration and the crossings of the ray traces. We note that the pattern of the wave propagation essentially depends on the particular realization of the stochastic field even at the same average parameters.

Fig. 1b demonstrates the decrease of temperature turbulence influence on the acoustic wave as far as the parameter T'_{rms}/T_0 is reduced.

The trajectories of the three eigenrays propagating through stochastic field and distribution of the parameter S along them are shown in Fig. 2. This parameter is of the form of

$$S = \begin{cases} +\sqrt{\left(R_1^{\alpha}\right)^2 + \left(R_2^{\alpha}\right)^2}, & \text{at} \quad R_2^{\alpha} > 0\,; \\ -\sqrt{\left(R_1^{\alpha}\right)^2 + \left(R_2^{\alpha}\right)^2}, & \text{at} \quad R_2^{\alpha} \le 0\,, \end{cases},$$

and characterizes the elementary ray tube area. It is clear that $|S| \to 0$ when the ray traces are intersected.

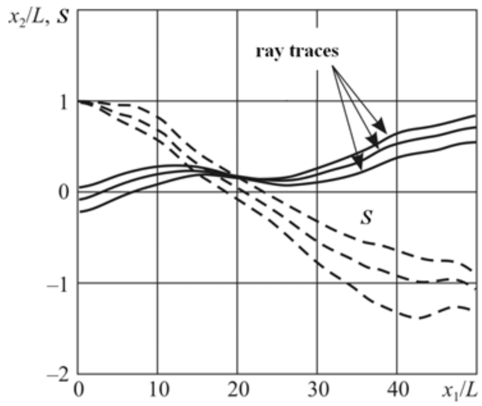

Fig. 2. Acoustic rays focusing in the random temperature field and distributions of parameter *S* along the rays traces.

The ray traces of acoustic wave propagating through the random velocity field with zero mean component (without wind) are shown in Fig. 3, $v'_{rms} = \sqrt{\overline{v'^2}}$. The field of vector $\mathbf{M}_T = \mathbf{v}'/c$ is shown by arrows.

The values of T'_{rms}/T_0 and v'_{rms}/c_0 are specified in such a way that fluctuating part $\mu = -\dfrac{T'_{rms}}{2T_0} - \dfrac{v'_{rms}}{c_0}$ of the refraction index $n = 1 + \mu$ was the same both for the temperature fluctuation field and for the velocity fluctuation field.

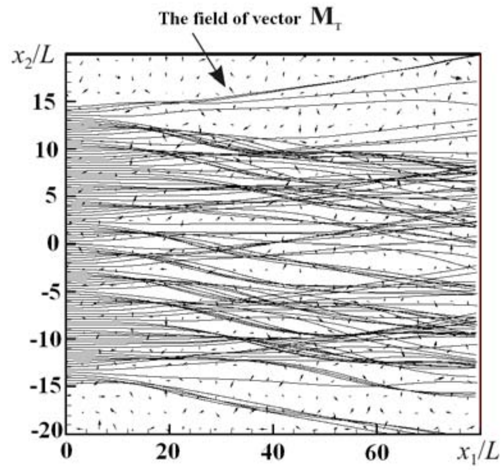

Fig. 3. Propagation of the acoustic rays through the random velocity fluctuations field. $v'_{rms}/c_0 = 0.586\cdot10^{-2}$.

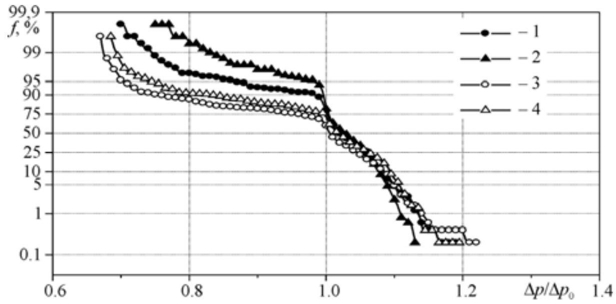

Fig. 4. Cumulative probability *f* of $\Delta p/\Delta p_0$ in the sonic boom wave propagating through stochastic field.

$x_1 = 25L$: 1 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0.586\cdot10^{-2}$;

2 – $T'_{rms}/T_0 = 1.172\cdot10^{-2}$, $v'_{rms}/c_0 = 0$;

$x_1 = 50L$: 3 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0.586\cdot10^{-2}$;

4 – $T'_{rms}/T_0 = 1.172\cdot10^{-2}$, $v'_{rms}/c_0 = 0$

The form of received overpressure signature in sonic boom wave propagating in stochastic temperature or velocities fields depends essentially on the specific realization of the random field as well as on the receiver location. In order to get the information on the turbulence influence on this or that characteristic of the sonic boom wave the statistical ensemble averaging is necessary.

In Fig. 4 the results of calculations of cumulative probability *f* of $\Delta p/\Delta p_0$ in the sonic boom wave propagating through the stochastic temperature and velocity fluctuations fields at the distances of x_1=25L and x_1=50L from source are represented. The Δp is the maximum overpressure and Δp_0 is the calculated value of Δp without turbulence. The results are generalized on the base of the calculation about 50 eigenrays and 100 various realizations for each curve.

As a whole it is possible note that described model allows qualitatively predicting the basic tendencies observed in the experimental investigations of the atmospheric turbulence influence on the sonic boom propagation.

3 Using the Hopf's transformation for the solution of nonlinear transport equation

The nonlinear transport equation (1) may be transformed to Burgers type equation

$$\frac{\partial \Pi}{\partial Z} = \alpha \Pi \frac{\partial \Pi}{\partial t'}, \qquad (2)$$

where

$$\Pi = K \cdot p,$$

$$K = \sqrt{\frac{|\mathbf{A}|}{|\mathbf{A}_s|} \frac{\rho_{0s} c_{0s}}{\rho_0 c} \cdot |\mathbf{n} + \mathbf{M}| \cdot (1 + \mathbf{M} \cdot \mathbf{n})},$$

$$\alpha = \frac{\kappa + 1}{2} \cdot \frac{1}{\rho_{0s} c_{0s}^{\;3}},$$

ρ_{0s}, $\mathbf{A}_s$ and c_{os} are the density, ray tube area and sound speed near the source, κ - specific heats ratio. The distortion distance variable Z is given by equation

$$\frac{dZ}{ds} = \sqrt{\frac{|\mathbf{A}_s|}{|\mathbf{A}|} \frac{\rho_0 c_0^{\;5}}{\rho\, c^5} \cdot |\mathbf{n} + \mathbf{M}|^{-3} \cdot (1 + \mathbf{M} \cdot \mathbf{n})^{-3}}.$$

The dissipation effects may be included in equation (2) if we rewrite it as

$$\frac{\partial \Pi}{\partial Z} = \alpha \Pi \frac{\partial \Pi}{\partial t'} + \beta \frac{\partial^2 \Pi}{\partial t'^2}, \qquad (3)$$

where β is the effective coefficient of dissipation that includes the temperature and viscous dissipation so as molecular relaxation. For the homogeneous atmosphere parameter $\beta = \frac{2}{3} \cdot \frac{\kappa^2}{c_0^{\;3}} \cdot \nu$, where ν is the kinematic viscous coefficient.

The Hopf's transformation [24] is concluded in presentation of the functions Π in the form $\Pi = \frac{n}{u} \cdot \frac{\partial u}{\partial t'}$, where $n = \frac{2\beta}{\alpha}$.

As a result transport equation (3) is converted to the form:

$$\frac{\partial u}{\partial Z} = a^2 \frac{\partial^2 u}{\partial t'^2}, \quad a = \sqrt{\beta}. \qquad (4)$$

Got equation is equation of heat transfer. It is proved that its solution exists, single and continuously depends on initial distribution of u_0 at Z=0 [25].

It is possible to write latter as

$$u_0 = \exp\left(\frac{1}{n} \int_{-\infty}^{t'} \Pi dt' \right),$$

where t'_0 is the initial value of retarded time coordinate t'.

The Hopf's transformation was earlier used for solution of the transport equation [26, 27], but for gaining of approximate solutions.

If $u_0 = f(t')$, then the solution of equation (4) is known:

$$u = \frac{1}{2a\sqrt{\pi Z}} \int_{-\infty}^{\infty} f(\xi) \cdot \exp\left(-\frac{(\xi - t')^2}{4a^2 Z} \right) \cdot d\xi,$$

If we accept a piecewise-linear approximation for overpressure signature then the solution of equation (4) will be of the form of finite sum, which each member is expressed through error functions $erf(\eta)$ or $erfi(\eta)$.

For example, for N-wave with initial overpressure in head pressure jump Δp_0, with pulse duration of T and zero rise time τ the solution may be written as:

$$u = 1 + \frac{1}{2} \left\{ \frac{r_3}{r_2} \cdot I_1 \cdot \exp\left(\frac{r_4^{\;2}}{r_3^{\;2}} - \frac{\bar{t}^{\,2}}{r_2^{\;2}} \right) - I_2 \right\},$$

$$I_1 = sign(\eta_1)\cdot erf|\eta_1| - sign(\eta_2)\cdot erf|\eta_2|,$$

$$I_2 = sign(\eta_3)\cdot erf|\eta_3| - sign(\eta_4)\cdot erf|\eta_4|,$$

where $\eta_1 = \dfrac{1-r_4}{r_3}$, $\eta_2 = -\dfrac{r_4}{r_3}$, $\eta_3 = \dfrac{1-\bar{t}}{r_2}$, $\eta_4 = -\dfrac{\bar{t}}{r_2}$.

Expression for sought function derivative on retarded time t' will be of the form of

$$\frac{\partial u}{\partial t'} = \frac{2}{\sqrt{\pi}}\cdot\frac{1}{r_2^3}\cdot\frac{1}{T}\left\{I_3\cdot\exp\left(\frac{r_4^2}{r_3^2}-\frac{\bar{t}^2}{r_2^2}\right)+I_4\right\},$$

$$I_3 = \frac{r_3^2}{2}\left[\exp\left(-\frac{r_4^2}{r_3^2}\right)-\exp\left(-\frac{(1-r_4)^2}{r_3^2}\right)\right]+$$
$$+\frac{\sqrt{\pi}}{2}I_1(r_4-\bar{t})\cdot r_3,$$

$$I_4 = \frac{r_2^2}{2}\left[\exp\left(-\frac{\bar{t}^2}{r_2^2}\right)-\exp\left(-\frac{(1-\bar{t})^2}{r_2^2}\right)\right].$$

Here $\bar{t} = \dfrac{t'}{T}$, r_1, r_2, r_3, r_4 - parameters of similarity:

$$r_1^2 = \frac{n}{\Delta p_0 T},\quad r_2^2 = \frac{4a^2 Z}{T^2},$$

$$r_3^2 = \frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}},\quad r_4^2 = \frac{\dfrac{r_2^2}{2}+\bar{t}r_1^2}{r_1^2+r_2^2}.$$

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