ASYMPTOTIC MODELS OF BOUNDARY LAYER FLOW CONTROL

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Keywords:*boundary layer, control, mathematical models*

Abstract

Investigated are local flows in the laminar boundary layers in the vicinity of heating elements. On the basis of asymptotical analysis mathematical models are formulated and similarity parameters are found. Determined are flow parameters providing flow control (separation, transition). Presented are results of numerical and analytical analysis.
Another method of the flow control is associated with the technology of new materials development. For example, porous metals allow use of passive control methods to influence boundary layer separation or laminar-turbulent transition

1 Introduction

New materials development particularly porous metals technology lead to the opportunity to create new passive methods of the boundary layer flow control. It may be used to delay boundary layer separation as well as to delay laminar-turbulent transition.

Porous metal structure usually is associated with the surface flow suction (injection) due to pressure difference between external and internal surfaces of porous metal plate. In many cases it may be supposed that distributed mass transfer will exist which will obey Darsi law (or linear dependence between vertical velocity distribution on the wall and pressure change distribution).

From mathematical point of view this condition allows to reconsider many early obtained classical results describing self-induced boundary layer separation for the case of passive control. This model includes boundary layer equations with an additional relation determining induced pressure distribution.
Corresponding mathematical problem was formulated. It was found that for unsteady self-induced boundary layer separation it is needed to take into account time delay in Darcy law.
Presented are numerical results describing self-induced laminar boundary layer separation in the flow near porous wall. Obtained are pressure coefficient and longitudinal length of preseparated region dependencies as a functions of porosity coefficient. It was found that in limiting cases we will get classical results for impermeable wall (porosity coefficient tends to zero) or results corresponding to the four deck disturbed flow structure (porosity coefficient tends to infinity).

It is important that new boundary condition on the wall describing relation between pressure change and vertical velocity is linear and doesn’t change uniformity of the problem. So it is possible to investigate as well linear stability problems incorporating early obtained results.

Presented are results of stability analysis describing longwave disturbances development.
These results may be useful to provide passive boundary layer flow control along with the buffet onset control.

2 Problem formulation

Porous wall structure supposes that due to pressure difference on the external and internal sides of porous surface may lead to the distributed suction (in the regions of relatively high pressure) or distributed injection. In many cases it may be supposed that mass transfer obeys the Darsy law (or linear dependence between vertical velocity on the wall and disturbed pressure distribution).

This boundary condition allow in fact to reconsider early obtained results [1-3] describing self-induced boundary layer separation for the case of passive flow control.

2.1 Equations

Using results obtained in [1] mathematical problem for flows near porous walls may be formulated as follows

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$y=0 \quad v=-\beta p,\ u=0 \qquad y\to\infty \ u=y+A(x)$$

$$x\to-\infty \quad u=y \qquad p=-\frac{\partial A}{\partial x}$$

This problem differs from problems describing disturbed flow near impermeable wall due to condition for the vertical velocity on the wall [1-3].In fact such conditions are well known in fluid mechanics.

For small values of self-induced pressure next form of solution may be considered

$$u=y+u_1,\ v=v_1,\quad p=p_1$$

This form of solution gives next form of equations for the first approximation

$$y\frac{\partial u_1}{\partial x}+v_1+\frac{\partial p_1}{\partial x}=\frac{\partial^2 u_1}{\partial y^2}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0$$

As usual solution may be presented in the normal mode approximation

$$(u_1,v_1,p_1,A)=e^{\alpha x}(U,V,P,B) \quad y\alpha U+V+\alpha P=U''$$

$$U(\infty)=B$$

$$y\alpha U'=U'''$$

After some transformations we will get Airy equation

$$y=\alpha^{-1/3}Y \qquad U'=F \qquad YF=F''$$

With the next solution

$$F=CAi(Y) \quad F=\frac{\partial U}{\partial y}=\alpha^{1/3}\frac{\partial U}{\partial Y}$$

$$F=\frac{\partial U}{\partial Y}=C\alpha^{-1/3}Ai(Y)$$

$$U(\infty)=\int_0^\infty FdY=C\alpha^{-1/3}\int_0^\infty Ai(Y)dY=\frac{C}{3\alpha^{1/3}}$$

$$P=-\alpha B=-\frac{C\alpha^{2/3}}{3}$$

$$(\alpha-\beta)P=\alpha^{2/3}U''_w=C\alpha^{1/3}Ai'(0)$$

$$-\frac{C\alpha^{2/3}}{3}(\alpha-\beta)=C\alpha^{1/3}Ai'(0)$$

Introducing new variables

$$\alpha^{1/3}(\alpha-\beta)=-3Ai'(0) \quad \alpha=\delta^3\beta$$

Eventually we will get next relation associating increment of growth and wall velocity parameter β

$$\delta(\delta^3-1)=-\frac{3Ai'(0)}{\beta^{4/3}}$$

Next two limits can be considered

The first one corresponding small porosity $\beta\to 0$

$$\delta\approx[-\frac{3Ai'(0)}{\beta^{4/3}}]^{1/4}\approx\frac{1}{\beta^{1/3}}[-3Ai'(0)]^{1/4}$$

Then since $\alpha=\delta^3\beta$

we will get

$$\alpha\approx[-3Ai'0)]^{3/4}$$

The same result as was obtained in classical case (impermeable plate) [1]
In the second limit-corresponding to large porosity $\beta \to \infty$

$$\delta \approx 1, \ \alpha \approx \beta$$

So, large α values correspond to small length of disturbed region.
It is important that new boundary condition on the wall is linear in the approximation considered so mathematical problem for linear regimes may be considered as a uniform one.
To investigate stability problems is needed to consider unsteady mass transfer regimes.

2.2 Numerical results

In general case mathematical problem (1) should be solved numerically. Numerical method used was described in [4]. Obtained are induced pressure gradient (marked by red color) and skin friction (marked by green color) as functions of induced pressure.

Such form results presentation was adopted due to monotonic character of pressure distribution upstream from the zero skin friction. Results presented on the next figures corresponds to the next values of parameter $\beta = 0.25$, 0.5, 0.75.

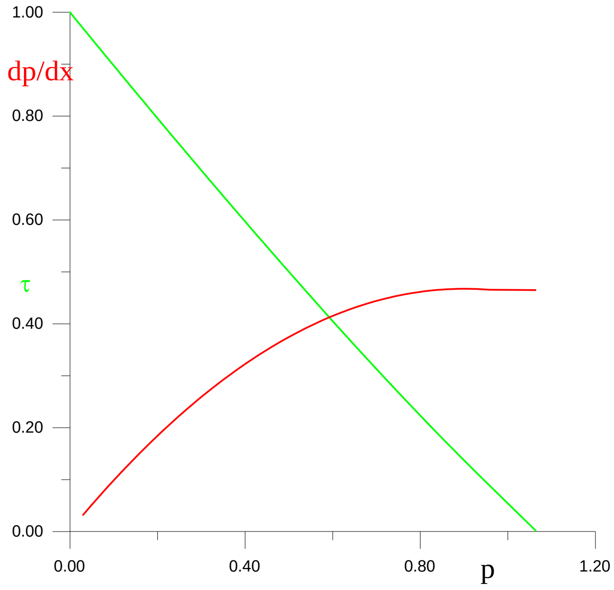

Fig. 1. Induced pressure gradient and skin friction distributions as functions of induced pressure for $\beta = 0.25$

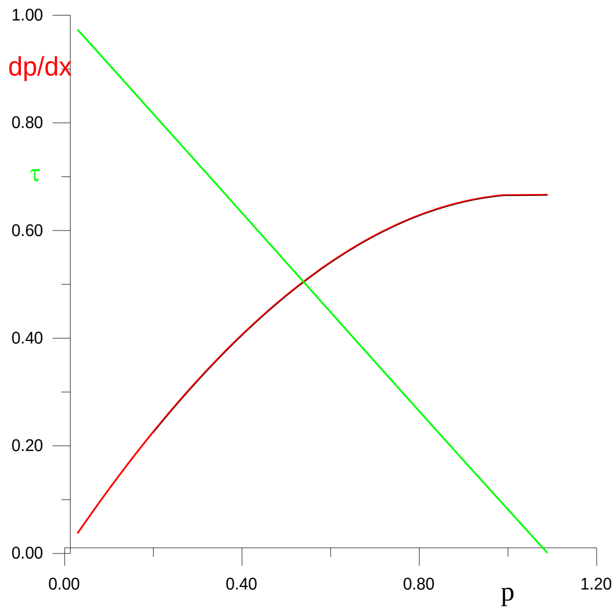

Fig. 2. Induced pressure gradient and skin friction distributions as functions of induced pressure for $\beta = 0.50$

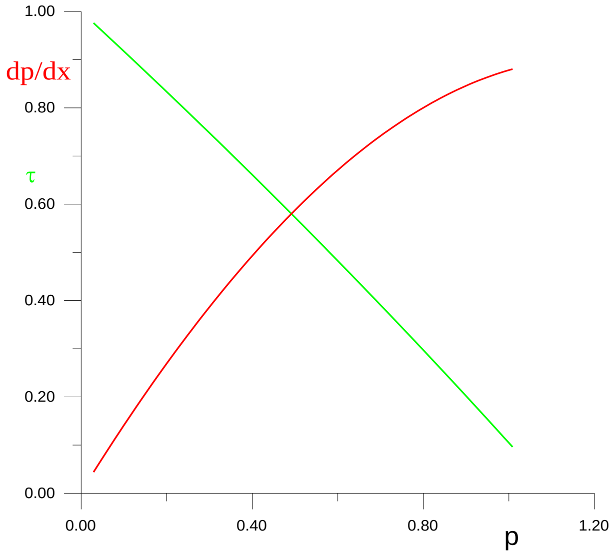

Fig. 3. Induced pressure gradient and skin friction distributions as functions of induced pressure for $\beta = 0.75$

All results presented show that growth of porosity coefficient leads to the growth of the induced pressure gradient. At the same time induced pressure at the point of zero skin friction is approximately the same (near unity). To analyze it in the next part of the paper limiting case is considered corresponding to the large porosity coefficient values.

2.3 Limiting regime $\beta >> 1$

For large porosity coefficients $\beta >> 1$ it may be supposed that interaction region will have more complex –four deck structure. The main change of the total displacement thickness will form in the region with the nonlinear velocity changes an where viscosity influence will be negligible in the first approximation. Near the wall then relatively thin region will be located where viscosity forces will be important. Such disturbed flow structure change is associated with the diminishing of the longitudinal length of disturbed flow. Original problem analysis both for linear and nonlinear regimes will lead to the next estimate for longitudinal length of the interaction region

$$\Delta x \sim \frac{1}{\beta}$$

Solution in the region of nonlinear inviscid disturbances may be written as follows

$$u = y + A(x)$$

Then longitudinal momentum equation will take the next

$$A\frac{\partial A}{\partial x} + v_w + \frac{\partial p}{\partial x} = 0\,, \quad p = -\frac{\partial A}{\partial x}$$

$$v_w = -\beta p$$

$$A\frac{\partial A}{\partial x} + \beta\frac{\partial A}{\partial x} - \frac{\partial^2 A}{\partial x^2} = 0$$

Comparison of the second and third terms in the last equation lead to the estimate for the disturbed flow length

$$x \sim \frac{1}{\beta}$$

Interaction condition gives the estimate for the displacement thickness change

$$A \sim \frac{p}{\beta}$$

Supposing that the induced pressure value is limited the next equation may be deduced

$$\beta\frac{\partial A}{\partial x} - \frac{\partial^2 A}{\partial x^2} = 0$$

This equation solution has the next form

$$p = c\,exp(\,\beta x\,)$$

$$A = -\frac{c}{\beta}exp(\,\beta x\,)$$

In the nearwall region solution may be written in the next form

$$u = y + u_1(\,x, y\,)$$
$$v = v_w - y\frac{\partial u_1}{\partial x}$$

Then equation for the longitudinal impulse has the form

$$(y+u_1)\frac{\partial u_1}{\partial x}+(v_w-y\frac{\partial u_1}{\partial x})(1+\frac{\partial u_1}{\partial y})+\frac{\partial p}{\partial x}=\frac{\partial^2 u_1}{\partial y^2}$$

Taking into account the induced pressure distribution we can arrive at

$$u_1\frac{\partial u_1}{\partial x}+v_w\frac{\partial u_1}{\partial y}-y\frac{\partial u_1}{\partial x}\frac{\partial u_1}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}$$

This equation analysis shows that thickness of viscous region can be estimated as follows

$$y\sim\frac{1}{p\beta}$$

And longitudinal momentum equation has the next form

$$v_w\frac{\partial u_1}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}$$

Substitution of the vertical velocity on the wall dependence gives eventually the next equation

$$-\beta p\frac{\partial u_1}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}$$

Solution corresponding to the boundary conditions on the wall and on large distances upstream has the next form

$$u_1=A(1-exp(-\beta py))$$

Then skin friction distribution may be written as follows

$$\frac{\partial u}{\partial y}=1+A\beta p$$

or

$$\frac{\partial u}{\partial y}=1-c^2\,exp(\beta x)$$

If the the beginning of the Cartesian coordinate system corresponds to the zero skin friction then $c=1$ and correspondingly disturbed pressure pressure value at this point

$$p=c\,exp(\beta x)=1$$

This analytical result confirms numerical data obtained for finite porosity coefficient β values.

3 Local Surface Heating

Among different methods of boundary layer flow control one of the mostly investigated now is a method associated with energy release due electrical discharge, surface heating or cooling.

The aim of this paper is the analysis of possible application of the local surface heating to determine response of the boundary layer flow and to find optimal heating elements parameters.

It is supposed that on the surface of the body are located heated parts, having temperatures different from the gas temperature in ambient boundary layer flow. It is supposed that temperature difference may change with time. Practically this method is easy to fulfill using electrically conducting strips. Example of such method application is described for example in [5].

The most important factor due to energy release (surface heating) is density change in the region influenced by the heating. This region structure is controlled by the convection and thermal conductivity processes. At the same time density change (diminishing due to temperature rise) will change boundary layer thickness. Situation is similar to the local flow nearby local surface distortion, but in our case effective surface distortion is created due to temperature (density) change. The difference is that the distortion shape is not known beforehand but is formed due to energy release in the boundary layer and due to the region with smaller density formation. Previous analysis of the disturbed flow nearby local surface distortions allowed to develop corresponding mathematical problems and to find distortions parameters influencing boundary layer flow [2].

In the paper corresponding asymptotical analysis was applied to analyze energy release influence on the separation of the boundary layer flow along with the influence on the boundary layer flow instability.

Investigated are local flows in the laminar boundary layer nearby heating elements located on the surface and flows near porous surface. On the basis of asymptotic analysis mathematical models are derived and similarity parameters are found. Described are unsteady local heating regimes providing boundary layer separation and laminar-turbulent transition control. Presented are numerical and analytical analysis results.

At present time intensive investigations are conducted associated with new methods of boundary layer control due to energy release. The energy source may be due to the electrical discharge, surface heating or cooling. Along these methods mechanical devices are analyzed, so called MEMS, or methods associated with suction or injection. Application of such methods allows suppressing boundary layer separation, to change boundary layer transition position along with the influence on the turbulent boundary layer parameters.

This paper is aimed at analysis of possible regimes of local surface heating to determine arising disturbances in the boundary layers. It is supposed as well to formulate corresponding mathematical problems and to determine similarity parameters. Analogous analysis is done for disturbed flows near porous surface.

It is supposed that on the streamlined surface are located parts having temperature which differs from the temperature of an ambient gas. It is supposed as well that this temperature depends on time. Practically is easy to fulfill such method of control having conducted wires or strips and using electrical current for local heating [5].

Let us consider physical aspects of such flows. The most important factor associated with the surface heating is gas density change in the region where energy release influence is significant. Dimensions of this region are determined by convection and heat conduction processes. Temperature increase will lead to the density decrease which will change boundary layer thickness. This change may induce in the external inviscid flow corresponding pressure disturbances. Situation is very similar to the flow in the boundary layer with the surface distortions. The main difference is that in the considered case geometry of surface distortions are not known beforehand but is determined by energy release and by corresponding decreased density region formation. Flow analysis in the boundary layer disturbed by an abrupt change of surface temperature and catalytic properties distribution is presented in [7-9].

For the subsequent analysis we will use the following papers [8- 10] results where local surface distortions located on the bottom of the surface were analyzed. It is supposed that the Reynolds number is large but doesn't exceed the critical value corresponding to the laminar-turbulent transition. Subsequent analysis is based on the derivation of estimates of possible physical mechanisms and similarity parameters determination.

3.1 Problem formulation

Considered is the supersonic or subsonic viscous gas flow near a flat semi-infinite plate. It is supposed that the Reynolds number is large but is subcritical corresponding to the laminar flow $Re = \rho_\infty u_\infty l / \mu_\infty = \varepsilon^{-2} \to \infty$, where $\rho_\infty,\ u_\infty,\ \mu_\infty$ - are density, longitudinal velocity and dynamical viscosity coefficient in undisturbed flow over the region where heated part of plate is located, l - is a distance from a leading edge to a zone of energy release. The next nondimensional values are choosed for the Cartesian coordinates, velocity vector components, density, pressure, dynamical viscosity coefficient $xl,\ yl,\ zl,\ lu_\infty^{-1}t,\ u_\infty u,\ u_\infty v,\ u_\infty w,\ \rho_\infty \rho,\ u_\infty^2 R^{-1} T,\ \mu_\infty \mu$ -.

In general case it is supposed that temperature change in the local region is finite $\Delta T \sim T \sim O(1)$, and the region of increased temperature is characterized by the next longitudinal size $a \le O(1)$, next transversal size $b \le O(1)$ and characteristic time of temperature change $O(\tau)$.

Undisturbed flow is 2-D and steady but disturbed flow is supposed to be unsteady and 3-D. Preliminary analysis is conducted for 2-D flow and then results are generalized for 3-D flows.

Following to the method of matched asymptotic expansions [11], in the beginning let's consider the region having identical sizes comparable with the body length $x \sim y \sim z \sim O(1)$. For large Reynolds number the flow in this region is described by the Euler equations. For the flat plat having zero angle of attack and zero thickness these equations solution is a solution describing undisturbed flow. To fulfill no - slip conditions it is needed to introduce boundary layer - region located nearby the surface and having the next sizes $x \sim z \sim O(1), y \sim O(\varepsilon)$.

Local surface heating may lead to the effective distortion formation, thickness of which may be evaluated using longitudinal impulse equation.

Surface temperature change will cause corresponding gas density change in the layer located nearby the surface $\Delta\rho \sim \rho \sim O(1)$. If the Prandtl number is finite $Pr \sim O(1)$ in general case it may be deduced that thickness of local viscous layer and thickness of temperature conducting layer are comparable. In the near wall layer longitudinal velocity is proportional to the distance from the wall y/ε, $u \sim O(y/\varepsilon)$. If values of convective and diffusion terms in the longitudinal momentum equation have the same order then the next estimate can be obtained for the thickness of local layer as a function of its longitudinal size a

$$y \sim O(\varepsilon a^{1/3}) \le O(\varepsilon)$$

Subsequent analysis depends on the longitudinal size of heated part of the surface. At least three different regimes described by different mathematical models may be formulated.

The first one corresponds to the longitudinal size smaller than the boundary layer thickness. If sizes of the disturbed region have the same orders

$$a \sim O(\varepsilon a^{1/3}),\ \ a \sim \varepsilon^{3/2}$$

we will get the disturbed region where the flow is described by Navier-Stokes equations with boundary conditions taking into account rarefied gas effects (slip conditions). Characteristic time in this region has the next order $\tau \sim \varepsilon^{-1}$.

For relatively larger sizes of the heated part $a^2 Re^{3/2} = Re_1 \to \infty$ the disturbed flow will be described by so called equations for compensation regime [9]. This regime will exist for length comparable with the boundary layer thickness as well as for larger sizes but lesser than the length scale comparable with the so called free interaction scale. Corresponding to this regime

$$a \sim O(\varepsilon^{3/4}),\ \ \Delta y \sim O(\varepsilon^{5/4}\Delta T),\ \ \Delta p \sim O(\varepsilon^{1/2}\Delta T)$$

Corresponding mathematical problem may be written as follows

$$Sh_2\frac{\partial u_b}{\partial t_b} + u_b\frac{\partial u_b}{\partial x_b} + v_b\frac{\partial u_b}{dy_b} + \Pi_1 w_b\frac{\partial u_b}{\partial z_b} + T_b\frac{\partial p_b}{\partial x_b} = \frac{\partial^2 u_b}{\partial y_b^2}$$

$$Sh_2\frac{\partial w_b}{\partial t_b} + u_b\frac{\partial w_b}{\partial x_b} + v_b\frac{\partial w_b}{dy_b} + \Pi_1 w_b\frac{\partial w_b}{\partial z_b} + T_b\frac{\partial p_b}{\partial z_b} = \frac{\partial^2 w_b}{\partial y_b^2}$$

$$Sh_2\frac{\partial T_b}{\partial t_b} + u_b\frac{\partial T_b}{\partial x_b} + v_b\frac{\partial T_b}{dy_b} + \Pi_1 w_b\frac{\partial T_b}{\partial z_b} = \frac{\partial^2 T_b}{\partial y_b^2}$$

$$\frac{\partial u_b}{\partial x_b} + \frac{\partial v_b}{\partial y_b} + \frac{\partial w_b}{\partial z_b} = 0$$

$$u_b(x_b,0,z_b,t_b) = w_b(x_b,0,z_b,t_b) = v_b(x_b,0,z_b,t_b) = 0$$

$$T_b(x_b,0,z_b,t_b) = T_w(x_b,z_b,t_b)$$

$$u_b \to y_b + d,\ d_1 = \int_0^\infty (1 - T_1) d\eta + d,\ T_1(x,\infty) \to 1, y_b \to \infty$$

$$d(-\infty) \to 0,$$

$$p_b(x_b,t) = -B_2 \frac{\partial d_1}{\partial x_b}$$

3.2 Numerical results

This problem was numerically solved. Numerical results were obtained for the next surface temperature distribution

$$T_w(|x_b| \le 0.5,t) = 1. + (1 - exp(-t))(0.25 - x_b^2),$$
$$T_w(|x_b| > 0.5,t) = 1., B_2 = 1$$

On the fig.4 induced pressure distribution is presented $p_b(x_b,t \to \infty)$ for large time values.

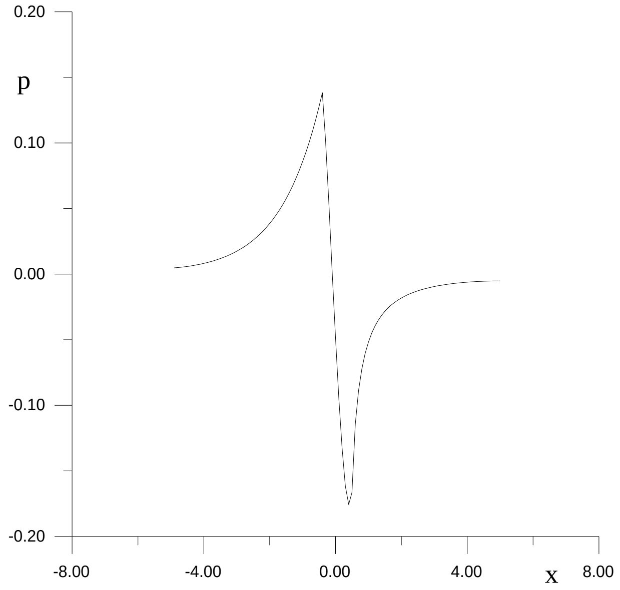

Fig.4. Induced pressure distribution

4. Conclusions

Presented are numerical and analytical results describing viscous-inviscid interaction processes in laminar flows near porous walls. This method of passive boundary layer flow control looks promising in some cases like separation prevention or laminat-turmulent transition delay.

This method may be useful as well to prevent buffet onset.

New interesting features were discovered. So it was shown that porous wall (passive control tool) doesn't change seriously pressure value near the point of separation, but changes pressure gradient in longitudinal direction. It means that length of disturbed flow region diminishes if porosity coefficient grows. For subsequent analysis of the flow stability unsteady Darsy law should be taken into account.

5 Aknowledgements

The work was partially supported by the Russian Foundation for Basic Research (grant N. 08-01-00594 a).

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