Abstract
The present study provides a number of fully isotropic stacking sequences for relatively thin composite laminates made of (0°, +60°, -60°) layers with up to 27 plies. Five independent fully isotropic solutions with one anti-symmetric case are found for 18-ply laminates. For 27-ply laminates, 340 independent fully isotropic solutions with thirty six anti-symmetric cases are found.

1 Introduction
Most fiber-reinforced composites are usually constructed with multiple orthotropic layers in which the reinforcing fibers are aligned in predetermined directions. In most field applications with single material type, the thickness of each layer is usually an integer-multiple of the unit ply thickness. Even though the elastic constants of a laminate can be tailored by combining and adjusting the fiber orientation in each layer, it is challenging to find fully isotropic stacking sequences for relatively thin laminates.

A particular class of anti-symmetric angle-ply laminates with $B_{ij}=D_{16}=D_{26}=0$ was presented by Caprino and Visconti [1]. Their analysis utilizes the fact that $B_{ij}=0$ for any symmetric laminate, and $A_{16}=A_{26}=0$ for any balanced laminate, and $D_{16}=D_{26}=B_{11}=B_{22}=B_{12}=B_{66}=0$ for any anti-symmetric laminate. Their study shows that the simplest laminate is the anti-symmetric angle-ply laminate with 8 plies, $[(\pm \theta)_{s}/(\mp \theta)_{s}]$, in which all coupling terms, $A_{16}$, $A_{26}$, $B_{ij}$, $D_{16}$, and $D_{26}$ are rigorously zero.

Gunnink [2] presented a simple comment on the result of Caprino and Visconti [1], which states “if laminate B is anti-symmetric with respect to laminate A and if laminate A as well as B is balanced, then $A_{16}$, $A_{26}$, $B_{ij}$, $D_{16}$, and $D_{26}$ of the entire laminate AB become rigorously zero.”

Jong-Won Lee et al. [3] presented a simple analytical method to obtain fully orthotropic laminates (FOL’s) with anti-symmetric stacking sequences. In their study, Pascal’s number triangle and Pythagorean theorem are found to be very closely related to the analytical stacking sequences sufficient to guarantee FOL’s.

Vannucci & Verchery [4] proposed some new kinds of fully isotropic laminates (FIL’s) obtained by the so-called polar method. They found five independent solutions for the 18-ply laminates, one independent solution for the 24-ply laminates, 219 solutions for the 27-ply laminates, and 29 solutions for the 30-ply laminates.

Valot and Vannucci [5] presented some numerical solutions for FOL’s made of 6 to 18 plies stacked anti-symmetrically. One of their solution is identical to $[(\pm \theta)_{s}/(\mp \theta)_{s}]$ previously presented by Caprino and Visconti [1].

Jong-Won Lee et al. [6] proposed some sufficient conditions for analytically obtaining FIL’s. They studied a number of specially chosen anti-symmetric stacking sequences in which all coupling stiffness components including $B_{16}$ and $B_{26}$ become rigorously zero. They also presented a systematic way to insert fully orthotropic cores with arbitrary thicknesses and arbitrary elastic constants between plies to provide a stepping stone toward further generalization.

In the present study, a relatively simple analytical method is presented for obtaining FIL’s made of (0°, +60°, -60°) layers with up to 27 plies.
2 Mathematical Modeling

Let’s consider the laminates made of \((0°, +60°, -60°)\) layers with \(m\) plies for each fiber orientation, where \(m\) is a natural number no greater than 9. In order to obtain FOL’s initially, some plies must be shuffled properly so that \(D_{16}=D_{26}=0, \ B_{11}=B_{22}=B_{12}=0,\) and \(B_{16}=B_{26}=0.\) It is also self-evident that the contribution of each fiber orientation must be kept identical to make the entire laminate fully isotropic. In other words, the entire laminate stiffness components must be invariant with respect to the in-plane tensor transformation.

It has been well known that the stiffness matrix for FOL’s is given by

\[
\begin{pmatrix}
N_1 \\
N_2 \\
N_3 \\
M_1 \\
M_2 \\
M_3
\end{pmatrix} =
\begin{pmatrix}
A_1 & A_2 & 0 & 0 & 0 & 0 \\
A_2 & A_2 & 0 & 0 & 0 & 0 \\
A_3 & A_3 & 0 & 0 & 0 & 0 \\
D_1 & D_2 & 0 & \kappa_1 \\
D_2 & D_2 & 0 & \kappa_2 \\
D_3 & D_3 & 0 & \kappa_3
\end{pmatrix}
\begin{pmatrix}
N_1 \\
N_2 \\
N_3 \\
M_1 \\
M_2 \\
M_3
\end{pmatrix}
\]

where \(\kappa_i (i=1, 2, 6)\) are the curvatures of the laminate mid-plane, respectively. The stiffness components, \(A_{ij}, B_{ij},\) and \(D_{ij}\) are calculated from

\[
A_{ij} = \int Q_{ij} dz \\
B_{ij} = \int Q_{ij} z dz \\
D_{ij} = \int Q_{ij} z^2 dz
\]

where

\[
Q_{11} = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta \\
Q_{22} = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta \\
Q_{33} = U_4 - U_3 \cos 4\theta \\
Q_{16} = (U_1 \sin 2\theta)/2 + U_3 \sin 4\theta, \\
Q_{26} = (U_2 \sin 2\theta)/2 - U_3 \sin 4\theta, \\
Q_{36} = U_4 - U_3 \cos 4\theta
\]

and

\[
U_1 = (3Q_{xx} + 3Q_{yy} + 2Q_{xy} + 4Q_{xx})/8, \\
U_2 = (Q_{xx} - Q_{yy})/2, \\
U_3 = (Q_{xx} + Q_{yy} - 2Q_{xy} - 4Q_{xx})/8, \\
U_4 = (Q_{xx} + Q_{yy} + 6Q_{xy} - 4Q_{xx})/8, \\
U_5 = (Q_{xx} + Q_{yy} - 2Q_{xy} + 4Q_{xx})/8
\]

The coordinates for the laminate and the ply are defined in Figure 1.

For FIL’s, the following relationships must be satisfied.

\[
A_{11} = A_{22} \quad (5-1) \\
A_{16} = A_{26} = 0 \quad (5-2) \\
A_{66} = (A_{11} - A_{12})/2 \quad (5-3) \\
B_{11} = B_{22} = B_{12} = 0 \quad (5-4) \\
B_{16} = B_{26} = 0 \quad (5-5) \\
D_{16} = D_{26} = 0 \quad (5-6) \\
D_{ij} = A_{ij} H^2/12 \quad (5-7)
\]

where \(H\) is the entire laminate thickness.

As an example, let’s consider the \((0°, +60°, -60°)_{m=3}\) stacking sequence shown in Table 1 in which the algebraic manipulation becomes much simpler with \([(k-9/2)^3-(k-9/2-1)^3]/3-1/12\) instead of \([(k-9/2)^3-(k-9/2-1)^3]/3\).

| Table 1. \((0°, +60°, -60°)_{m=3}\) Laminate |
|---|---|---|---|---|---|---|---|
| \(k\) | \(1\) | \(2\) | \(3\) | \(4\) | \(5\) | \(6\) | \(7\) | \(8\) | \(9\) |
| \([(k-9/2)^3-(k-9/2-1)^3]/2\) | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| \([(k-9/2)^3-(k-9/2-1)^3]/3-1/12\) | 16 | 9 | 4 | 1 | 0 | 1 | 4 | 9 | 16 |

In order to make \(D_{16}=D_{26}=0\) and \(D_{ij}\) invariant with respect to the in-plane tensor transformation, the 9 boxes on the third row in Table 1 must be divided into three groups so that each group occupies three boxes and the numbers in each group sum to 20. Since \((16+4+0)\) is the only combination satisfying this condition, it is obvious that no FIL solution exists for \((0°, +60°, -60°)_{m=3}\) laminates.

For \((0°, +60°, -60°)_{m=4}\) laminates shown in Table 2, it is more convenient to handle \([(k-6)^2-(k-6-1)^2]\) and \([(k-12/2)^3-(k-12/2-1)^3]/6\) instead of \([(k-6)^2-(k-6-1)^2]/2\) and \([(k-12/2)^3-(k-12/2-1)^3]/3\) respectively.

| Table 2. \((0°, +60°, -60°)_{m=4}\) Laminate |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \(k\) | \(1\) | \(2\) | \(3\) | \(4\) | \(5\) | \(6\) | \(7\) | \(8\) | \(9\) | \(10\) | \(11\) | \(12\) |
| \([(k-6)^2-(k-6-1)^2]\) | -11 | -9 | -7 | -5 | -3 | -1 | 1 | 3 | 5 | 7 | 9 | 11 |
| \([(k-6)^3-(k-6-1)^3]/6\) | 15 | 10 | 5 | 0 | 0 | 1 | 3 | 6 | 10 | 15 |

Since the twelve boxes on the third row in Table 2 sum to 70 which is not a multiple of 3, no FIL solution exists for \((0°, +60°, -60°)_{m=4}\) laminates.
laminates. Likewise, it can be shown that FIL solutions do not exist for \( m = 5, 7, \text{ and } 8 \).

Therefore, the FIL solutions with up to 27 plies are possible only for \((0^\circ, +60^\circ, -60^\circ)_m\) and \((0^\circ, +60^\circ, -60^\circ)_m\) stacking sequences.

### 3 FIL's with \((0^\circ, +60^\circ, -60^\circ)_m\) (18 plies)

When a laminate is constructed with \((0^\circ, +60^\circ, -60^\circ)_m\) layers with \( m \) plies for each fiber orientation, the total number of plies becomes \(3m\). Since this type of laminates is always balanced, i.e. the number of \(+60^\circ\) plies is equal to that of \(-60^\circ\) plies, \((5-1), (5-2)\) and \((5-3)\) are satisfied trivially.

In order to satisfy \((5-6)\) and \((5-7)\), the summation of all numbers occupying the third row in Table 3 must be a multiple of \(3\) so that the numbers on the third row can be split into three groups which sum to identical values.

**Table 3. \((0^\circ, +60^\circ, -60^\circ)_m\) with symmetric 0°'s**

<table>
<thead>
<tr>
<th>(k)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>((k, k, 0))</td>
<td>17 15 13 11 9 5 3 1 0 1 3 5 7 9 11 13 15 17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((k, 1, 0))</td>
<td>36 28 21 15 10 6 3 0 0 1 3 6 10 15 21 28 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The LHS summation of the numbers on the second row not occupied by \(0^\circ\) plies must cancel out the RHS counterpart to satisfy \((5-4)\). The LHS summation of the numbers on the second row occupied by \(+60^\circ\) plies must cancel out the RHS counterpart occupied by \(+60^\circ\) ones to satisfy \((5-5)\). The same must be true for \(-\theta\) plies.

Among numerous \((0^\circ, +60^\circ, -60^\circ)_m\) laminates, we can select a special case in which \(0^\circ\) plies are arranged symmetrically so that \(D_{16}=D_{26}=0\) and \(D_{ij}'s\) are invariant with respect to the in-plane tensor transformation of elastic constants. On the third row of Table 3, the only symmetric combination of six boxes which sum to \(80\) is \((36+3+1+1+3+36)\).

The stacking sequence on the fourth row in the LHS as well as four boxes the RHS of the anti-symmetric. A simple and straightforward The present study has found that (0°, +60°, -60°)

Table 7. (0°, +60°, -60°)_{m=6} FIL #4 (Asym.)

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k-9)/16</td>
<td>17</td>
<td>15</td>
<td>13</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k-9)/4-9-11/16</td>
<td>36</td>
<td>28</td>
<td>21</td>
<td>15</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>Stack. Seq.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 8. (0°, +60°, -60°)_{m=6} FIL #5 (Asym.)

| k  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| (k-9)/16  | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| (k-9)/4-9-11/16 | 36 | 28 | 21 | 15 | 10 | 6 | 3 | 1 | 0 | 0 | 1 | 3 | 6 | 10 | 15 | 21 | 28 | 36 |
| Stack. Seq. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The five independent FIL’s of (0°, +60°, -60°)_{m=6} shown in Tables 4-8 are identical to the result of Vannucci and Verchery [4].

4 (0°, +60°, -60°)_{m=9} with Symmetric 0°’s

The present study has found that (0°, +60°, -60°)_{m=9} stacking sequence has 340 independent FIL solutions among which thirty six cases are anti-symmetric. A simple and straightforward summation scheme for the remaining thirty six anti-symmetric solutions is presented herein. The method to obtain Table 1 for m=3 can be extended for m=9 as shown in Table 9.

Table 9. (0°, +60°, -60°)_{m=9} with sym. 0°’s

| k  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| α  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| β  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A 0° ply is assumed to occupy the laminate mid-plane to reduce the computational effort significantly. The remaining eight 0° plies must be arranged symmetrically with respect to the laminate mid-plane to obtain anti-symmetric FIL’s. Table 9 shows the four symmetric arrangements of 0° plies in which four boxes in the LHS as well as four boxes the RHS of the third row sum to 273 identically.

4.1 Type01 FIL’s with (0°, +60°, -60°)_{m=9}

The stacking sequence on the fourth row in Table 9 has ten FIL solutions among which nine solutions are anti-symmetric. The remaining 18 boxes on the second row must be divided into two groups so that each one sums to 31+31=62 for the nine anti-symmetric solutions, Type01-1A to 9A, as shown in Table 10.

Table 10. Type01 FIL with (0°, +60°, -60°)_{m=9}

| k  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| α  | 31 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 |
| β  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

An asymmetric solution, Type01-10, is obtained by dividing the same 18 boxes on the second row into two groups so that each one sums to 32+30=62

4.2 Type02 FIL’s with (0°, +60°, -60°)_{m=9}

The stacking sequence on the fifth row in Table 9 has nine anti-symmetric and three asymmetric FIL solutions. The remaining 18 boxes on the second row must be divided into two groups so that each one sums to 31+31=62 for the nine anti-symmetric solutions, Type02-1A to 9A, and an asymmetric solution, Type02-10, as shown in Table 11.

Table 11. Type02 FIL with (0°, +60°, -60°)_{m=9}

| k  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| α  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| β  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Two additional asymmetric solutions, Type02-11 and Type02-12, are obtained by dividing the same 18 boxes on the second row into two groups so that each one sums to 32+30=62 and 33+29=62, respectively, as shown in Table 11.
4.3 Type03 FIL’s with $(0^\circ, +60^\circ, -60^\circ)_{m=9}$

The stacking sequence on the sixth row in Table 9 has nine anti-symmetric and four asymmetric FIL solutions. The remaining 18 boxes on the second row must be divided into two groups so that each group sums to 30+30=60 for the nine anti-symmetric solutions, Type03-1A to 9A as shown in Table 12.

An asymmetric solution, Type03-10, is obtained by dividing the same 18 boxes on the second row into two groups so that each group sums to 31+29=60. Three asymmetric solutions, Type03-11 to Type03-13, are additionally obtained by dividing the same 18 boxes on the second row into two groups so that each group sums to 33+28=60 as shown in Table 12.

4.4 Type04 FIL’s with $(0^\circ, +60^\circ, -60^\circ)_{m=9}$

The stacking sequence on the seventh row in Table 9 has nine anti-symmetric and two asymmetric FIL solutions as shown in Table 13.

The remaining boxes on the second row must be divided into two groups so that each group sums to 30+30=60 for the nine anti-symmetric solutions, Type04-1A to 9A as shown in Table 13.

In addition, two asymmetric solutions, Type04-10 and 11 are obtained by dividing the same 18 boxes on the second row into two groups so that each group sums to 31+29=60 and 32+28, respectively.

5 $(0^\circ, +60^\circ, -60^\circ)_{m=9}$ with Asymmetric $0^\circ$’s

As before, a $0^\circ$ ply is placed on the laminate mid-plane and four boxes in the LHS and four boxes in the RHS on the third row of Table 9 are assumed to be occupied by $0^\circ$ plies. However, the symmetric arrangement of the remaining eight $0^\circ$ plies need not be kept for obtaining asymmetric FIL’s classified as Type05 to Type51. And yet the numbers on the third row in Table 3 occupied by $0^\circ$ plies must sum to 546.

5.1 Type05~06 FIL’s with $(0^\circ, +60^\circ, -60^\circ)_{m=9}$

For these types, four boxes in the LHS and four boxes in the RHS are occupied asymmetrically by $0^\circ$ plies, and each group sum to 273 as shown in Table 14 and 15.

5.2 Type07~10 FIL’s with $(0^\circ, +60^\circ, -60^\circ)_{m=9}$

For these types, four boxes in the LHS and four boxes in the RHS are occupied by $0^\circ$ plies on the third row of Table 9 and sum to 263+283 or 243+303. The entire boxes occupied by $0^\circ$ plies
sum to 546 as do the previous types. Four types exist for the combination of numbers as listed in Table 16.

5.3 Type11–14 FIL’s with (0°, +60°, -60°)_{m=9}
For these types, four boxes in the LHS and four boxes in the RHS are occupied by 0° plies on the third row of Table 9 and sum to 264+282 or 284+262. And yet the entire boxes occupied by 0° plies sum to 546. Four types exist for the combination of numbers as listed in Table 17.

5.4 Type15–26 FIL’s with (0°, +60°, -60°)_{m=9}
For these types, four boxes in the LHS and four boxes in the RHS are occupied by 0° plies on the third row of Table 9 and sum to 255+291, 265+281, 275+271, 285+261, or 295+251. The entire boxes occupied by 0° plies sum to 546 as do previous cases. Twelve types of FIL solutions are listed in Table 18.

5.5 Type27–41 FIL’s with (0°, +60°, -60°)_{m=9}
For these types, four boxes in the LHS and four boxes in the RHS are occupied by 0° plies on the third row of Table 9 and sum to 246+300, 276+270, 286+260, or 316+230. The entire boxes occupied by 0° plies sum to 546 as do previous cases. Fifteen types exist for the combination of numbers as listed in Table 19.

5.6 Type42–51 FIL’s with (0°, +60°, -60°)_{m=9}
For these types, four boxes in the LHS and four boxes in the RHS are occupied by 0° plies on the third row of Table 9 and sum to 267+279, 277+269, 287+259, 297+249, or 307+239. The entire boxes occupied by 0° plies sum to 546 as do previous cases. Ten types exist for the combination of numbers as listed in Table 20.
6 Type 52~71 FIL’s with \((0^\circ, +60^\circ, -60^\circ)_{m=9}\)

For these types, three boxes in the LHS and five boxes in the RHS are occupied by 0° plies on the third row of Table 9 and sum to 302+244, 282+264, 293+253, 314+232, 315+231, 326+220, 317+229, 308+238, 299+247, 339+207, or 329+217. The entire boxes occupied by 0° plies sum to 546 as do previous cases. Twenty types for the combination of numbers are listed in Table 21.

![Table 21. Type 52~71 FIL with \((0^\circ, +60^\circ, -60^\circ)_{m=9}\)](image)

<table>
<thead>
<tr>
<th># of Type</th>
<th>LHS + Center + RHS</th>
<th>Summation</th>
<th>No. of FIL Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>11°+10°+9°+0°+4°+1°+3°+7°+8°+11°</td>
<td>302°+0+244=546</td>
<td>3</td>
</tr>
<tr>
<td>53</td>
<td>11°+10°+9°+0°+4°+1°+3°+7°+8°+11°</td>
<td>302°+0+244=546</td>
<td>6</td>
</tr>
<tr>
<td>54</td>
<td>13°+8°+7°+0°+1°+2°+3°+9°+13°</td>
<td>282°+9+264=546</td>
<td>4</td>
</tr>
<tr>
<td>55</td>
<td>12°+10°+7°+0°+4°+1°+3°+5°+7°+13°</td>
<td>293°+0+233=546</td>
<td>4</td>
</tr>
<tr>
<td>56</td>
<td>13°+12°+1°+4°+0°+1°+2°+3°+7°+13°</td>
<td>314°+0+232=546</td>
<td>5</td>
</tr>
<tr>
<td>57</td>
<td>13°+9°+8°+0°+1°+2°+3°+6°+7°+11°</td>
<td>314°+0+232=546</td>
<td>3</td>
</tr>
<tr>
<td>58</td>
<td>12°+11°+7°+0°+1°+2°+3°+6°+7°+11°</td>
<td>314°+0+232=546</td>
<td>4</td>
</tr>
<tr>
<td>59</td>
<td>13°+11°+5°+0°+2°+3°+4°+7°+12°</td>
<td>315°+0+231=546</td>
<td>5</td>
</tr>
<tr>
<td>60</td>
<td>13°+11°+5°+0°+1°+3°+6°+8°+11°</td>
<td>315°+0+231=546</td>
<td>4</td>
</tr>
<tr>
<td>61</td>
<td>13°+11°+5°+0°+2°+3°+4°+9°+11°</td>
<td>315°+0+231=546</td>
<td>6</td>
</tr>
<tr>
<td>62</td>
<td>13°+11°+6°+0°+2°+4°+6°+8°+10°</td>
<td>326°+0+220=546</td>
<td>8</td>
</tr>
<tr>
<td>63</td>
<td>13°+11°+6°+0°+4°+1°+5°+7°+9°</td>
<td>326°+0+220=546</td>
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<tr>
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<td>317°+0+229=546</td>
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<td>308°+0+238=546</td>
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<tr>
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<td>299°+0+247=546</td>
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</table>

7 Concluding Remarks

The five independent FIL’s of \((0^\circ, +60^\circ, -60^\circ)_{m=6}\) obtained by Vannucci and Verchery [4] are confirmed by the present study.

In contrast to the 219 independent FIL’s of \((0^\circ, +60^\circ, -60^\circ)_{m=9}\) stacking sequences obtained by Vannucci and Verchery [4], 340 FIL solutions are obtained by the present study. The authors are still checking whether the present result is the complete set of the fully isotropic solutions of \((0^\circ, +60^\circ, -60^\circ)_{m=9}\) stacking sequences.

The present method will be extended for obtaining FIL’s made of \((0^\circ, +45^\circ, -45^\circ, 90^\circ)\) layers up to 24 plies and \((0^\circ, +72^\circ, -72^\circ, +144^\circ, -144^\circ)\) layers up to 30 plies.

FULLY ISOTROPIC LAMINATES WITH UP TO TWENTY SEVEN PLIES

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References


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