ADAPTIVE CONTROL USING SUPPORT VECTOR REGRESSION FOR HYPERSONIC AIRCRAFT CONTROL SYSTEM

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Abstract

This paper presents support vector regression (SVR)-based adaptive controller for the longitudinal dynamics of a generic hypersonic aircraft. SVR has been proven to generate global solutions contrary to neural networks, because SVR basically solves quadratic programming (QP) problems. With this advantage, the nominal dynamics of the input-output feedback-linearized hypersonic airplane is trained off-line. In order to compensate the offline-training error and unknown uncertainties in the control process, and to avoid the controller singularity problem, an adaptation algorithm of the offline-trained SVR is proposed using the concept of virtual control input. Stability of the overall system is analyzed by the Lyapunov stability theory. Numerical simulations validate the performance of the proposed approach.

1 Introduction

Dynamics of hypersonic aircrafts has highly nonlinear characteristics because flight conditions are set at high altitudes and Mach numbers. This means that there exist modeling inaccuracies and uncertainties in the generic hypersonic air vehicle model, and they can significantly deteriorate the control performance of the aircraft. As a result, adaptive or robust control, which compensates the effects of the uncertainties and modeling inaccuracies, respectively, has been studied with various ideas such as adaptive sliding model control [1], neural network (NN) based adaptive control [2] or nonlinear dynamic inversion (NDI) with stochastic robustness analysis [3].

In this study, support vector regression (SVR)-based adaptive controller is proposed. SVR transforms the original problem into a quadratic programming (QP) problem whose global solution can be obtained by QP solvers, thus, it can be solved without the issues of the local minima [12, 13]. Furthermore, it is straightforward to design the parameters of the SVR since it has a fixed structure. With this advantage, the nominal dynamics of the input-output feedback-linearized hypersonic airplane can be trained off-line. In order to handle the offline-training error or uncertainties and modeling inaccuracies, an adaptation rule of the offline-trained SVR is proposed using the concept of the virtual control input [5, 6]. The proposed adaptive algorithm enables the controller to avoid the singularity problem by utilizing the affine property of the hypersonic aircraft dynamics [7, 8]. Stability of the overall system is also analyzed by the ultimately bounded property in the nonlinear system theory. Finally, numerical simulations validate the performance of the proposed approach.

2 ε-Support vector regression

Support vector machines have recently become popular learning tools for the classification and regression [12, 13]. Typically, SVM classification is used to build a function that predicts binary values. On the other hand, SVM regression, or SVR, generates functions whose outputs are scalars. Unlike least-square or empirical methods, SVM regression maintains the same motivation as SVM classification: minimizing a bound on the expected error for future test data. Thus, SVR inherits interesting generalization properties and sparsity from SVM. This section briefly reviews ε-SVR algorithm [4].

Consider the training dataset $D = \{X_k, Y_k\}_{k=1}^N$, where X_k is the k^{th} input data in the input space $\mathscr{X} \subset \Re^n$ and Y_k is the corresponding output value in the output space $\mathscr{Y} \subset \Re$. ε-SVR model is trained by the following relationship between the input and output data points:

$$F(X_k) = <\mathbf{w}, \Phi(X_k)> + c \tag{1}$$

where $\mathbf{w}$ is a vector in the feature space $\mathscr{F} \subset \mathbb{R}^n$, $\Phi(X_k)$ is a mapping from the input space to the feature space $\mathscr{F}$, c is the bias term, and $<\cdot,\cdot>$ stands for the inner product in $\mathscr{F}$.

The ε-SVR model based on Vapnik's ε-insensitive loss function can be formulated in the primal space, as the following[12]:

$$\min_{\mathbf{w},c,\xi,\xi^*} J_\varepsilon^P = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^N (\xi_i + \xi_i^*) \tag{2}$$

subject to the constraints

$$\left\{ \begin{array}{rcl} Y_i - <\mathbf{w}, \Phi(X_i)> - c & \leq & \varepsilon + \xi_i \\ <\mathbf{w}, \Phi(X_i)> + c - Y_i & \leq & \varepsilon + \xi_i^* \\ \xi_i,\ \xi_i^* \geq 0, & & i = 1, 2, \cdots, N \end{array} \right. \tag{3}$$

where ε is the maximum value of tolerable error, ξ_i's and ξ_i^*'s are slack variables, $\|\cdot\|$ is the Euclidean norm, and C is a regularization parameter that represents a trade-off between the model complexity and the tolerance to the error larger than ε.

The dual form of (2) becomes a quadratic programming (QP) problem as follows:

$$\begin{aligned} \min_{\eta,\eta^*} J_\varepsilon^D &= \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \kappa(X_i, X_j)(\eta_i - \eta_i^*)(\eta_j - \eta_j^*) \\ &\quad + \varepsilon\sum_{i=1}^N (\eta_i + \eta_i^*) - \sum_{i=1}^N Y_i(\eta_i - \eta_i^*) \end{aligned} \tag{4}$$

subject to $0 \leq \eta_i, \eta_i^* \leq C$, $\sum_{i=1}^N (\eta_i - \eta_i^*) = 0$, $i = 1, \cdots, N$ where $\kappa(X_i, X_j)$ is a kernel function given by $\kappa(X_i, X_j) = \Phi(X_i)^T\Phi(X_j) = \kappa_{ij}$. Motivated by Mercer's condition, the kernel function handles the inner product in the feature space and hence the explicit form of $\Phi(X_k)$ does not need to be known [12]. In this study, the Gaussian radial basis kernel function is used $\kappa(X_i, X_j) = \exp\left(-\frac{(X_i - X_j)^T(X_i - X_j)}{\sigma^2}\right)$.

The solution of the QP problem (4) is the optimum values of η_i's and η_i^*'s. The value of the bias c in the model can be determined by the condition that at the point of the solution the product between dual variables and constraints has to vanish [13]. When only the support vectors are considered, the model becomes

$$F(X_k) = \sum_{i=1,(i\in SV)}^{N_{SV}} \zeta_i \kappa(X_k, X_i) + c \tag{5}$$

where $\zeta_i = \eta_i - \eta_i^*$, and N_{SV} denotes the number of support vectors in the model. The obtained ε-SVR model is sparse in the sense that the whole training data are represented by the support vectors only and many of ζ_i's are zero.

The control design presented in this paper employs the ε-SVR model (5) to approximate any nonlinear function $G(X_k)$ over a compact set $\mathscr{X} \subset \mathbb{R}^n$. The nonlinear function $G(X_k)$ is expressed as $G(X_k) = \sum_{i=1,(i\in SV)}^{N_{SV}} \zeta_i^* \kappa(X_k, X_i) + c^* + \varepsilon = w^{*T}\phi(X_k) + \varepsilon$, where $w^* = \begin{bmatrix} \zeta_1^* & \zeta_2^* & \cdots & \zeta_{N_{SV}}^* & c^* \end{bmatrix}^T$ is ideal weights, ε is the approximation error in the sense of ε-insensitive model, and $\phi(X_k) = [\kappa(X_k, X_1)\ \ \kappa(X_k, X_2)\ \ \cdots\ \ \kappa(X_k, X_{N_{SV}})\ \ 1]^T$.

Assumption 1 *There exists an ideal constant vector of weights w^* that minimizes $|\varepsilon|$ for all $X_k \in \mathscr{X}$:*

$$w^* \triangleq arg \min_{w\in\mathbb{R}^{N_w}} \left\{ \sup_{X_k\in\mathscr{X}} \left|G(X_k) - w^T\phi(X_k)\right| \right\}. \tag{6}$$

where $N_w = N_{SV} + 1$. The ideal weight vector, which consists of the ideal Lagrange multipliers corresponding to the support vectors and bias term, is required for the stability proof.

3 Hypersonic air vehicle model

Let us consider the longitudinal dynamics of the hypersonic aircraft [1, 2, 3]:

$$\dot{V} = \frac{T\cos\alpha - D}{m} - \frac{\mu\sin\gamma}{r^2}, \quad (7)$$

$$\dot{\gamma} = \frac{L + T\sin\alpha}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{Vr^2}, \quad (8)$$

$$\dot{h} = V\sin\gamma, \quad (9)$$

$$\dot{\alpha} = q - \dot{\gamma}, \quad (10)$$

$$\dot{q} = M_{yy}/I_{yy}, \quad (11)$$

where

$$L = \frac{1}{2}\rho V^2 S C_L, \quad (12)$$

$$D = \frac{1}{2}\rho V^2 S C_D, \quad (13)$$

$$T = \frac{1}{2}\rho V^2 S C_T, \quad (14)$$

$$M_{yy} = \frac{1}{2}\rho V^2 S\bar{c}\left[C_M(\alpha) + C_M(\delta) + C_M(q)\right], \quad (15)$$

$$r = h + R_E, \quad (16)$$

$$C_L = 0.6203\alpha, \quad (17)$$

$$C_D = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772, \quad (18)$$

$$C_T = \begin{cases} 0.02576\beta & \text{if } \beta < 1 \\ 0.0224 + 0.00336\beta & \text{if } \beta > 1 \end{cases}, \quad (19)$$

$$C_M(\alpha) = -0.035\alpha^2 + 0.036617\alpha + 5.3261^{-6}, \quad (20)$$

$$C_M(q) = \frac{\bar{c}}{2V}\left(-6.796\alpha^2 + 0.3015\alpha - 0.2289\right)q, \quad (21)$$

$$C_M(\delta_e) = c_e(\delta_e - \alpha). \quad (22)$$

The engine dynamics are modeled by a second order system:

$$\ddot{\beta} = -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta + \omega_n^2\beta_c. \quad (23)$$

In the dynamics of the hypersonic aircraft, states variables are composed of velocity V, angle of attack α, altitude h, pitch rate q and flight path angle γ, and control inputs are throttle setting β_c and elevator deflection δ_e. The interested outputs for the input-output feedback linearization are V and h.

As shown in [1], the above dynamics (7)-(11) are input-output feedback-linearized as in the following manner:

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2, \\
\dot{\xi}_2 &= \xi_3, \\
\dot{\xi}_3 &= f_1(\xi) + g_{11}(\xi)u_1 + g_{12}(\xi)u_2, \\
\dot{\xi}_4 &= \xi_5, \\
\dot{\xi}_5 &= \xi_6, \\
\dot{\xi}_6 &= \xi_7, \\
\dot{\xi}_7 &= f_2(\xi) + g_{21}(\xi)u_1 + g_{22}(\xi)u_2, \quad (24)
\end{aligned}$$

where $\xi_1 = V$, $\xi_4 = h$, $u_1 = \beta_c$, $u_2 = \delta_e$ and the detailed materials on the above input-output feedback-linearized system are given in [1]. For the sake of the simplicity, (24) is represented as:

$$\dot{\xi} = A\xi + B\big(f(\xi) + g(\xi)\mathbf{u}\big), \quad (25)$$

where $\xi = [\xi_1\ \xi_2\ \xi_3\ \xi_4\ \xi_5\ \xi_6\ \xi_7]^T$, $\mathbf{u} = [u_1\ u_2]^T$,

$$A = \begin{bmatrix} 0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0 \end{bmatrix}, B = \begin{bmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&1 \end{bmatrix},$$

$$f(\xi) = \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix},$$

and

$$g(\xi) = \begin{bmatrix} g_{11}(\xi) & g_{12}(\xi) \\ g_{21}(\xi) & g_{22}(\xi) \end{bmatrix}.$$

In this study, it is assumed that $f(\xi)$ and $g(\xi)$ are unknown smooth function and matrix function, respectively, and their nominal functions $\bar{f}(\xi)$ and $\bar{g}(\xi)$ are known. Then, (25) is represented as:

$$\dot{\xi} = A\xi + B\big(\bar{f}(\xi) + \Delta_f + (\bar{g}(\xi) + \Delta_g)\mathbf{u}\big) \quad (26)$$

where Δ_f and Δ_g are unknown uncertainties. Since $g(\xi)$ is a smooth matrix function, it is bounded within some compact set. Therefore, the following assumptions are satisfied as commonly made in the literature [7, 8].

Assumption 2 *The sign of the eigenvalues of $g(\xi)$ is known, and there exist positive constants g_{ub} and g_{lb} such that $g_{ub} \geq \|g(\xi)\| \geq g_{lb} > 0$, $\forall \xi \in \Omega \subset \mathbb{R}^7$. In fact, $g(\xi)$ in (25) has 2 positive eigenvalues for all ξ.*

Assumption 3 *There exists a constant $g^d_{ub} > 0$ such that $\|\dot{g}(\xi)\| \leq g^d_{ub}$, $\forall \xi \in \Omega \subset \mathbb{R}^7$.*

Assumption 4 *The nominal dynamics in (26), i.e., $\xi_2, \xi_3, \xi_5, \xi_6, \xi_7$, $\bar{f}(\xi)$ and $\bar{g}(\xi)$ are known.*

Assumption 5 *The uncertainties Δ_f, Δ_g are generated by the following modeling inaccuracies*

$$\begin{aligned} m &= m_0(1+\Delta m),\ I_{yy} = I_0(1+\Delta I),\\ S &= S_0(1+\Delta S),\ \bar{c} = \bar{c}_0(1+\Delta\bar{c}),\\ \rho &= \rho_0(1+\Delta\rho),\ c_e = c_{e_0}(1+\Delta c_e), \end{aligned}$$

where the maximum values of all the additive uncertainties are set to 0.1.

4 Control system design for the hypersonic aircraft

4.1 Offline training of the input-output feedback-linearized hypersonic aircraft

In order to design the velocity and altitude tracking control system, we define e_1 and e_4 as

$$e_1 = \xi_1 - V_r, \tag{27}$$
$$e_4 = \xi_4 - h_r, \tag{28}$$

where V_r and h_r are reference signals for V and h. Then, if (26) are represented as the known $\bar{f}(\xi)$ and $\bar{g}(\xi)$ only, error dynamics is given by

$$\begin{aligned} \dot{e}_1 &= e_2,\\ \dot{e}_2 &= e_3,\\ \dot{e}_3 &= \bar{f}_1(\xi)+\bar{g}_{11}(\xi)u_1+\bar{g}_{12}(\xi)u_2-\dddot{V}_r,\\ \dot{e}_4 &= e_5,\\ \dot{e}_5 &= e_6,\\ \dot{e}_6 &= e_7,\\ \dot{e}_7 &= \bar{f}_2(\xi)+\bar{g}_{21}(\xi)u_1+\bar{g}_{22}(\xi)u_2-\ddddot{h}_r, \end{aligned} \tag{29}$$

where $e_i = \xi_i - V_r^{(i-1)}$, $i = 2,3$ and $e_j = \xi_j - h_r^{(k-1)}$, $j = 5,6,7$, $k = 2,3,4$.

If the control input $\mathbf{u}(= [u_1\ u_2]^T)$ is designed in the following manner

$$\mathbf{u} = \bar{g}^{-1}(\xi)(-\bar{f}(\xi)+\nu) \tag{30}$$

where $\nu = [\nu_1\ \ \nu_2]^T$,

$$\begin{aligned} \nu_1 &= -k_1e_1-k_2e_2-k_3e_3+\dddot{V}_r,\\ \nu_2 &= -k_4e_4-k_5e_5-k_6e_6+\ddddot{h}_r, \end{aligned}$$

and $k_i(>0,\ i=1,\cdots,7)$'s are properly chosen, then the final error dynamics is given by

$$\dot{e} = \bar{A}e \tag{31}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & 0 & \cdots & 0\\ -k_1 & -k_2 & -k_3 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & -k_4 & -k_5 & \cdots & -k_7 \end{bmatrix} \leq 0, \tag{32}$$

$e = [e_1\ e_2\ \cdots\ e_7]^T$, and (31) becomes an asymptotically stable system.

Since the control input (30) is based on the nominal model only, it is not possible to control the real plant (26) using (30). In order to compensate the unknown uncertainty included in (26), the control input (30) is trained offline using the SVR algorithm and then, adapted online.

The SVR-based offline-training for (30) is performed as:

$$\bar{g}^{-1}(\xi)\bar{f}(\xi) = \hat{w}^T_{fg}\phi_{fg}(\xi)+\varepsilon_{t,fg}, \tag{33}$$
$$\bar{g}^{-1}(\xi) = \hat{w}^T_g\phi_g(\xi)+\varepsilon_{t,g}, \tag{34}$$

where $\varepsilon_{t,fg}$, $\varepsilon_{t,g}$ are inherent offline-training errors. Then, (30) is rearranged as:

$$\begin{aligned} \mathbf{u} &= -\bar{g}^{-1}(\xi)\bar{f}(\xi)+\bar{g}^{-1}(\xi)\nu\\ &= -\hat{w}^T_{fg}\phi_{fg}(\xi)+\hat{w}^T_g\phi_g(\xi)\nu. \end{aligned} \tag{35}$$

The reason why the offline-training of $\bar{g}^{-1}(\xi)\bar{f}(\xi)$, $\bar{g}^{-1}(\xi)$ is performed instead of that of $\bar{f}(\xi)$, $\bar{g}(\xi)$, which has been studied in [5, 6], is that $\bar{g}(\xi)$ can be singular in the adaptation process if we do not add some manipulation on the adaptive rule. The above approach has been studied originally in [7, 8] under the backstepping approach. In this study, it is applied to the feedback linearization framework by introducing the virtual control for each ξ_i.

4.2 Adaptive feedback linearization for the hypersonic aircraft

Even though the error dynamics in (31) is commonly used in the feedback linearization-based control literature, it is not applicable in our study because the uniformly ultimately boundeness of the error dynamics and the controller singularity problem cannot be considered simultaneously using the error dynamics (31). In order to settle these two issues concurrently, the new error dynamics is reformulated by introducing the virtual control [6].

Let $z_1 = \xi_1 - V_r$ and $z_4 = \xi_4 - h_r$. Then, the derivatives of z_1 and z_4 are given by

$$\begin{aligned} \dot{z}_1 &= \dot{\xi}_1 - \dot{V}_r = \xi_2 - \dot{V}_r, && (36)\\ \dot{z}_4 &= \dot{\xi}_4 - \dot{h}_r = \xi_5 - \dot{h}_r. && (37) \end{aligned}$$

In order to control z_1 and z_4, a new variables z_2 and z_5 are defined as $z_2 = \xi_2 - \bar{\xi}_2$, $z_2 = \xi_5 - \bar{\xi}_5$, respectively. Then, (36) and (37) become

$$\begin{aligned} \dot{z}_1 &= \xi_2 - \dot{V}_r = z_2 + \bar{\xi}_2 - \dot{V}_r, && (38)\\ \dot{z}_4 &= \xi_5 - \dot{h}_r = z_5 + \bar{\xi}_5 - \dot{h}_r. && (39) \end{aligned}$$

If $\bar{\xi}_2$ and $\bar{\xi}_5$ are defined as

$$\begin{aligned} \bar{\xi}_2 &= -k_1 z_1 + \dot{V}_r, && (40)\\ \bar{\xi}_5 &= -k_4 z_4 + \dot{h}_r, && (41) \end{aligned}$$

and used as the virtual control for z_1 and z_4 dynamics, then $\dot{z}_1$ and $\dot{z}_4$ are obtained by

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2, && (42)\\ \dot{z}_4 &= -k_4 z_4 + z_5. && (43) \end{aligned}$$

According to the definition of z_2, $\dot{z}_2$ is obtained by

$$\begin{aligned} \dot{z}_2 &= \dot{\xi}_2 - \dot{\bar{\xi}}_2 = \xi_3 - (-k_1 \dot{z}_1 + \ddot{V}_r)\\ &= \xi_3 - (-k_1(-k_1 z_1 + z_2) + \ddot{V}_r)\\ &= -k_1^2 z_1 + k_1 z_2 + \xi_3 - \ddot{V}_r\\ &= -k_1^2 z_1 + k_1 z_2 + z_3 + \bar{\xi}_3 - \ddot{V}_r, && (44) \end{aligned}$$

where $z_3 = \xi_3 - \bar{\xi}_3$. $\bar{\xi}_3$ is defined as

$$\bar{\xi}_3 = k_1^2 z_1 - (k_1 + k_2) z_2 + \ddot{V}_r. \tag{45}$$

Using the virtual control $\bar{\xi}_3$, $\dot{z}_2$ is derived as

$$\dot{z}_2 = -k_2 z_2 + z_3. \tag{46}$$

Similar to the above, $\dot{z}_5$ and $\dot{z}_6$ are derived as:

$$\begin{aligned} \dot{z}_5 &= -k_5 z_5 + z_6, && (47)\\ \dot{z}_6 &= -k_6 z_6 + z_7, && (48) \end{aligned}$$

where $z_6 = \xi_6 - \bar{\xi}_6$, $z_7 = \xi_7 - \bar{\xi}_7$ and

$$\begin{aligned} \bar{\xi}_6 &= k_4^2 z_4 - (k_4 + k_5) z_5 + \ddot{h}_r, && (49)\\ \bar{\xi}_7 &= -k_4^3 z_4 + (k_4^2 + k_4 k_5 + k_5^2) z_5\\ &\quad + (k_4 + k_5 + k_6) z_6 + \dddot{h}_r. && (50) \end{aligned}$$

Finally, the time derivatives of z_3 and z_7 are described as

$$\begin{aligned} \dot{z}_3 &= \dot{\xi}_3 - \dot{\bar{\xi}}_3 = f_1 + g_{11} u_1 + g_{12} u_2 + k_1^3 z_1\\ &\quad -(k_1^2 + k_1 k_2 + k_2^2) z_2 + (k_1 + k_2) z_3 - \dddot{V}_r, && (51)\\ \dot{z}_7 &= \dot{\xi}_7 - \dot{\bar{\xi}}_7 = f_2 + g_{21} u_1 + g_{22} u_2 - k_4^4 z_4\\ &\quad +(k_4^3 + k_4^2 k_5 + k_4 k_5^2 + k_5^2) z_5\\ &\quad -(k_4^2 + k_4 k_5 + k_5^2 + k_4 k_6 + k_5 k_6 + k_6^2) z_6\\ &\quad +(k_4 + k_5 + k_6) z_7 - \ddddot{h}_r. && (52) \end{aligned}$$

Then, the transformed error dynamics is given by

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2,\\ \dot{z}_2 &= -k_2 z_2 + z_3,\\ \dot{z}_3 &= f_1 + g_{11} u_1 + g_{12} u_2 - c_1 z_1 - c_2 z_2\\ &\quad - c_3 z_3 - \dddot{V}_r,\\ \dot{z}_4 &= -k_4 z_4 + z_5,\\ \dot{z}_5 &= -k_5 z_5 + z_6,\\ \dot{z}_6 &= -k_6 z_6 + z_7,\\ \dot{z}_7 &= f_2 + g_{21} u_1 + g_{22} u_2 - c_4 z_4 - c_5 z_5\\ &\quad - c_6 z_6 - c_7 z_7 - \ddddot{h}_r, && (53) \end{aligned}$$

where

$$\begin{aligned} c_1 &= -k_1^3, \\ c_2 &= k_1^2+k_1k_2+k_2^2, \\ c_3 &= -(k_1+k_2), \\ c_4 &= k_4^4, \\ c_5 &= -(k_4^3+k_4^2k_5+k_4k_5^2+k_5^2), \\ c_6 &= k_4^2+k_4k_5+k_5^2+k_4k_6+k_5k_6+k_6^2, \\ c_7 &= -(k_4+k_5+k_6). \end{aligned}$$

For the sake of the simplicity, (53) is rearranged as:

$$\begin{aligned} \dot{\mathbf{z}}_1 &= A_z\mathbf{z}_1 \\ \dot{\mathbf{z}}_2 &= f(\xi)+g(\xi)\mathbf{u}-\Pi \\ &= g(\xi)\left[g^{-1}(\xi)f(\xi)+\mathbf{u}-g^{-1}(\xi)\Pi\right] \end{aligned} \tag{54}$$

where $\mathbf{z}_1=[z_1\ \ z_2\ \ z_4\ \ z_5\ \ z_6]^T$, $\mathbf{z}_2=[z_3\ \ z_7]^T$,

$$A_z=\begin{bmatrix} -k_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_6 & 1 \end{bmatrix},$$

and

$$\Pi=\begin{bmatrix}\Pi_1\\ \Pi_2\end{bmatrix}=\begin{bmatrix} c_1z_1+c_2z_2+c_3z_3+\dddot{V}_r \\ c_4z_4+c_5z_5+c_6z_6+c_7z_7+\ddddot{h}_r \end{bmatrix}.$$

Then, the control input $\mathbf{u}$ is defined as using (33), (34)

$$\mathbf{u}=-\hat{w}_{fg}^T\phi_{fg}(\xi)+\hat{w}_g^T\phi_g(\xi)\Pi-K_2\mathbf{z}_2 \tag{55}$$

where

$$K_2=\begin{bmatrix} k_3 & 0\\ 0 & k_7\end{bmatrix}.$$

By applying the above control input to (54), we obtain

$$\begin{aligned} \dot{\mathbf{z}}_1 &= A_z\mathbf{z}_1 \\ \dot{\mathbf{z}}_2 &= g(\xi)\Big[g^{-1}(\xi)f(\xi)-\hat{w}_{fg}^T\phi_{fg}(\xi) \\ &\quad +\hat{w}_g^T\phi_g(\xi)\Pi-K_2\mathbf{z}_2-g^{-1}(\xi)\Pi\Big] \\ &= g(\xi)\left[\tilde{w}_{fg}^T\phi_{fg}(\xi)-\tilde{w}_g^T\phi_g(\xi)\Pi-K_2\mathbf{z}_2+\varepsilon\right] \end{aligned} \tag{56}$$

where $\tilde{w}_{fg}\triangleq w_{fg}^*-\hat{w}_{fg}$, $\tilde{w}_g\triangleq w_g^*-\hat{w}_g$,

$$\begin{aligned} g^{-1}(\xi)f(\xi) &= {w_{fg}^*}^T\phi_{fg}(\xi)+\varepsilon_{a,fg} \\ g^{-1}(\xi) &= w_g^{*T}\phi_g(\xi)+\varepsilon_{a,g} \\ \varepsilon &= \varepsilon_{a,fg}-\varepsilon_{t,fg}+(\varepsilon_{t,g}-\varepsilon_{a,g})\Pi \end{aligned}$$

w_{fg}^* and w_g^* are the ideal weights for $g^{-1}(\xi)f(\xi)$, $g^{-1}(\xi)$, respectively, and $\varepsilon_{a,fg}$, $\varepsilon_{a,g}$ are the inherent online-approximation errors.

Theorem 1 *The error states $\mathbf{z_1},\mathbf{z_2}$ and the weight estimation error $\tilde{w}_{fg}$ and $\tilde{w}_g$ are uniformly ultimately bounded in the following compact set $\mathfrak{D}$:*

$$\mathfrak{D}=\Big\{\mathbf{z}_1\in\Re^5,\mathbf{z}_2\in\Re^2,\ \tilde{w}_{fg}\in\Re^{N_{SV,fg}+1},\ \tilde{w}_g\in\Re^{N_{g,2}+1}\Big|\ \mathbf{z}_1^T\mathbf{z}_1+\mathbf{z}_2^Tg^{-1}(\xi)\mathbf{z}_2 + \frac{1}{max\{\gamma_{fg},\gamma_g\}}\Big(tr\big[\tilde{w}_{fg}^T\tilde{w}_{fg}\big]+tr\big[\tilde{w}_g^T\tilde{w}_g\big]\Big)<\frac{C}{\tau}\Big\}$$

where

$$C=\frac{k_{fg}}{2}\|w_{fg}^*\|_F^2+\frac{k_g}{2}\|w_g^*\|_F^2+\frac{\|\varepsilon\|^2}{4k_{22}},$$

and subscript F is the Frobenius norm, if the following adaptation rule is chosen

$$\dot{\hat{w}}_{fg} = \gamma_{fg}\phi_{fg}(\xi)\mathbf{z}_2-\kappa_{fg}\gamma_{fg}\hat{w}_{fg} \tag{57}$$

$$\dot{\hat{w}}_g = -\gamma_g\phi_g(\xi)\Pi\mathbf{z}_2-\kappa_g\gamma_g\hat{w}_g \tag{58}$$

with positive constants γ_i, k_i $(i=fg,g)$.

Proof. Consider the following Lyapunov function:

$$V = \frac{1}{2}\mathbf{z}_1^T\mathbf{z}_1+\frac{1}{2}\mathbf{z}_2^Tg^{-1}(\xi)\mathbf{z}_2+\frac{1}{2\gamma_{fg}}\mathrm{tr}\big[\tilde{w}_{fg}^T\tilde{w}_{fg}\big] +\frac{1}{2\gamma_g}\mathrm{tr}\big[\tilde{w}_g^T\tilde{w}_g\big]. \tag{59}$$

Let us first differencitate the Lyapunov function (59). Then

$$
\begin{aligned}
\dot{V} &= -\sum_{i=1}^{2}\left(k_i z_i^2 - z_i z_{i+1}\right) - \sum_{j=4}^{6}\left(k_j z_j^2 - z_j z_{j+1}\right) \\
&\quad -\mathbf{z_2}^T g^{-1}(\xi)\dot{g}(\xi)g^{-1}(\xi)\mathbf{z_2} + \mathbf{z_2}^T g(\xi)^{-1}\dot{\mathbf{z}}_2 \\
&\quad +\frac{1}{\gamma_{fg}}\mathrm{tr}\left[\tilde{w}_{fg}^T\dot{\tilde{w}}_{fg}\right] + \frac{1}{\gamma_g}\mathrm{tr}\left[\tilde{w}_g^T\dot{\tilde{w}}_g\right] \\
&\le -\mathbf{z}_1^T K_1'\mathbf{z}_1 - \mathbf{z_2}^T g^{-1}(\xi)\dot{g}(\xi)g^{-1}(\xi)\mathbf{z_2} \\
&\quad -\mathbf{z}_2^T(K_{21}+K_{22})\mathbf{z}_2 + \frac{1}{\gamma_{fg}}\mathrm{tr}\left[\tilde{w}_{fg}^T\left(\dot{\tilde{w}}_{fg}\right.\right. \\
&\quad \left.\left.+\gamma_{fg}\phi_{fg}(\xi)\mathbf{z}_2\right)\right] + \frac{1}{\gamma_g}\mathrm{tr}\left[\tilde{w}_g^T\left(\dot{\tilde{w}}_g - \gamma_g\phi_g(\xi)\Pi\mathbf{z}_2\right)\right] \\
&\quad +\mathbf{z}_2^T\varepsilon \\
&= -\mathbf{z}_1^T K_1'\mathbf{z}_1 - \mathbf{z_2}^T\left(K_{21} + g^{-1}(\xi)\dot{g}(\xi)g^{-1}(\xi)\right)\mathbf{z_2} \\
&\quad +\frac{1}{\gamma_{fg}}\mathrm{tr}\left[\tilde{w}_{fg}^T\left(\dot{\tilde{w}}_{fg} + \gamma_{fg}\phi_{fg}(\xi)\mathbf{z}_2\right)\right] \\
&\quad +\frac{1}{\gamma_g}\mathrm{tr}\left[\tilde{w}_g^T\left(\dot{\tilde{w}}_g - \gamma_g\phi_g(\xi)\Pi\mathbf{z}_2\right)\right] \\
&\quad -\mathbf{z}_2^T K_{22}\mathbf{z}_2 + \mathbf{z}_2^T\varepsilon
\end{aligned}
\qquad (60)
$$

where

$$
K_1' = \begin{bmatrix} k_1' & 0 & 0 & 0 & 0 \\ 0 & k_2' & 0 & 0 & 0 \\ 0 & 0 & k_4' & 0 & 0 \\ 0 & 0 & 0 & k_5' & 0 \\ 0 & 0 & 0 & 0 & k_6' \end{bmatrix},
$$

$$
\begin{aligned}
k_i &= k_i' + 1/2,\ k_i' > 0,\ \ i = 1,4, \\
k_j &= k_j' + 1,\ k_j' > 0,\ \ j = 2,5,6,
\end{aligned}
$$

$$
K_2 = K_{21} + K22,
$$

$$
K_{21} = \begin{bmatrix} k_{31} & 0 \\ 0 & k_{71} \end{bmatrix},\ K_{22} = \begin{bmatrix} k_{32} & 0 \\ 0 & k_{72} \end{bmatrix},
$$

and

$$
k_i = k_{i1} + k_{i2} + 1/2,\ \ k_{i1}, k_{i2} > 0,\ \ i = 3,7.
$$

In the above, 1 and 1/2 are used to cancel the cross terms $z_i z_{i+1}$, $i = 1,2,4,5,6$.

When *Assumptions* 2 and 3 hold, the following inequality is satisfied:

$$
\begin{aligned}
&-\mathbf{z_2}^T(K_{21} + g^{-1}(\xi)\dot{g}(\xi)g^{-1}(\xi)\mathbf{z_2} \\
&\qquad \le -\left(k_{21} - \frac{g_{ud}^d}{2{g_{lb}}^2}\right)\|\mathbf{z}_2\|^2,
\end{aligned}
\qquad (61)
$$

where $k_{21} = \lambda_{\min}(K_{21})$ and $\lambda(\cdot)$ is the eigenvalues of $(\cdot)$. Then, k_{21} is chosen as

$$
k_{21}^* \triangleq \left(k_{21} - \frac{g_{ud}^d}{2{g_{lb}}^2}\right) > 0. \qquad (62)
$$

The term $-\mathbf{z}_2^T K_{22}\mathbf{z}_2 + \mathbf{z}_2^T\varepsilon$ is bounded in the following manner:

$$
\begin{aligned}
-\mathbf{z}_2^T K_{22}\mathbf{z}_2 + \mathbf{z}_2^T\varepsilon &\le -k_{22}\left(\|\mathbf{z}_2\|^2 - \frac{\varepsilon^T}{k_{22}}\mathbf{z}_2\right. \\
&\quad \left.+\frac{\|\varepsilon\|^2}{4k_{22}^2}\right) + \frac{\|\varepsilon\|^2}{4k_{22}} \\
&\le \frac{\|\varepsilon\|^2}{4k_{22}},
\end{aligned}
\qquad (63)
$$

where $k_{22} = \lambda_{\min}(K_{22})$.

Then, with the adaptation rule in (57) and (57), $\dot{V}$ satisfies the following inequality:

$$
\begin{aligned}
\dot{V} &\le -k_1'\|\mathbf{z}_1\|^2 - k_{21}^*\|\mathbf{z}_2\|^2 - \frac{k_{fg}}{2}\|\tilde{w}_{fg}\|_F^2 \\
&\quad -\frac{k_g}{2}\|\tilde{w}_g\|_F^2 + \frac{k_{fg}}{2}\|w_{fg}^*\|_F^2 + \frac{k_g}{2}\|w_g^*\|_F^2 + \frac{\|\varepsilon\|^2}{4k_{22}}
\end{aligned}
\qquad (64)
$$

where

$$
\begin{aligned}
\mathrm{tr}\left[k_{fg}\tilde{w}_{fg}^T\hat{w}_{fg}\right] &= \mathrm{tr}\left[k_{fg}\tilde{w}_{fg}^T\left(w_{fg}^* - \tilde{w}_{fg}\right)\right] \\
&\le k_{fg}\|\tilde{w}_{fg}\|_F\|w_{fg}^*\|_F - k_{fg}\|\tilde{w}_{fg}\|_F^2 \\
&\le \tfrac{k_{fg}}{2}\|w_{fg}^*\|_F^2 - \tfrac{k_{fg}}{2}\|\tilde{w}_{fg}\|_F^2
\end{aligned}
,\qquad (65)
$$

$$
\begin{aligned}
\mathrm{tr}\left[k_g\tilde{w}_g^T\hat{w}_g\right] &= \mathrm{tr}\left[k_g\tilde{w}_g^T\left(w_g^* - \tilde{w}_g\right)\right] \\
&\le k_g\|\tilde{w}_g\|_F\|w_g^*\|_F - k_g\|\tilde{w}_g\|_F^2 \\
&\le \tfrac{k_g}{2}\|w_g^*\|_F^2 - \tfrac{k_g}{2}\|\tilde{w}_g\|_F^2
\end{aligned}
,\qquad (66)
$$

and $k_1' = \lambda_{\min}(K_1')$. Note that $\dot{\tilde{w}}_i = \dot{w}_i^* - \dot{\hat{w}}_i = -\dot{\hat{w}}_i$ $(i = fg, g)$ is used to derive (64).

In order to derive uniformly ultimate boundedeness, we choose

$$
k_{21}^* \ge \frac{\mu}{g_{lb}} \qquad (67)
$$

for some $\mu > 0$, by designing k_{21} in the following manner:

$$
k_{21} > \frac{\mu}{g_{lb}} + \frac{g_{ub}^d}{2{g_{lb}}^2}. \qquad (68)
$$

Then, the time derivative $\dot{V}$ of the Lyapunov function (64) is given by

$$\begin{aligned}\dot{V} \;\le\;& -k_1'\|\mathbf{z}_1\|^2 - \frac{\mu}{g_{lb}}\|\mathbf{z}_2\|^2 - \frac{k_{fg}}{2}\|\tilde{w}_{fg}\|_F^2 \\ & -\frac{k_g}{2}\|\tilde{w}_g\|_F^2 + C \\ \le\;& -k_1'\|\mathbf{z}_1\|^2 - \mu\mathbf{z}_2^T g^{-1}(\xi)\mathbf{z}_2 - \frac{k_{fg}}{2}\|\tilde{w}_{fg}\|_F^2 \\ & -\frac{k_g}{2}\|\tilde{w}_g\|_F^2 + C \qquad (69)\end{aligned}$$

where

$$C = \frac{k_{fg}}{2}\|w_{fg}^*\|^2 + \frac{k_g}{2}\|w_g^*\|^2 + \frac{\|\varepsilon\|^2}{4k_{22}},$$

and

$$0 < \tau < \min\left\{k_1',\ \mu,\ \frac{k_{fg}\gamma_1}{2},\ \frac{k_g\gamma_2}{2}\right\}. \qquad (70)$$

Therefore, the error states $\mathbf{z}_1, \mathbf{z}_2$ and the weight estimation error $\tilde{w}_{fg}$ and $\tilde{w}_g$ are uniformly ultimately bounded in the compact set $\mathfrak{D}$.

5 Numerical simulations

This section presents the numerical simulation results for the proposed SVR-based adaptive control scheme. As explained before, the proposed control scheme consists of two steps: first, the offline training of the nominal plant and second, the online adaptation with the offline-trained model.

In order to train the nominal $\bar{g}^{-1}(\xi)\bar{f}(\xi)$ and $\bar{g}^{-1}(\xi)$ offline, input-output data pairs are extracted from the response of the nominal plant about the sinusoidal reference signal with the controller (30). The simulation for the offline training data is performed for 100 seconds with 0.01 sampling time, thus, the length of data is 10,000. The offline-training has been performed using **LIBSVM** [14].

Figs.1-3 show the results of the offline-trained $\hat{w}_{fg}^T\phi_{fg}(\xi)$ and $\hat{w}_g^T\phi_g(\xi)$, and the number of the support vectors for each SVR is given in Table 1. As shown in Figs. 1-3 and Table 1, the offline-trained $\hat{w}_{fg}^T\phi_{fg}(\xi)$ and $\hat{w}_g^T\phi_g(\xi)$ show the sufficient approximation performance, and do not need the total input data in order to learn the nominal system, thanks to the sparse property explained in Sec. 2.

Table 1: The number of support vectors for $\bar{g}^{-1}(\xi)\bar{f}(\xi)$ and $\bar{g}^{-1}(\xi)$

	$\hat{w}_{fg,1}$	$\hat{w}_{fg,2}$	$\hat{w}_{g,11}$	$\hat{w}_{g,12}$	$\hat{w}_{g,12}$	$\hat{w}_{g,12}$
# of SVs	46	96	63	44	52	44

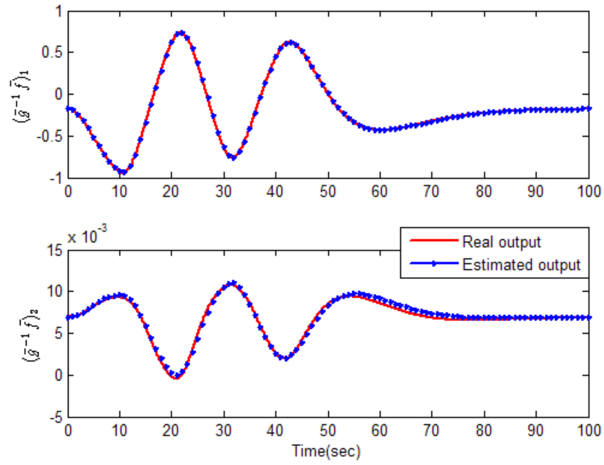

Fig. 1 : Results of the offline-trained $\bar{g}^{-1}\bar{f}$

In the online process, the offline-trained $\bar{g}^{-1}(\xi)\bar{f}(\xi)$ and $\bar{g}^{-1}(\xi)$ are adjusted to compensate the difference Δ_f, Δ_g between the nominal model $\bar{f}(\xi), \bar{g}(\xi)$and real system $f(\xi), g(\xi)$. In order to validate the performance of the proposed adaptive rule, the simulations are performed on (25) without any uncertainties. The reference commands V_r, h_r are given in Fig. 4. As shown in Fig. 5, the performance of the proposed adaptive control is better than that of the offline-trained control only even when there is no uncertainty, because the exact or global estimation of the nonlinear function cannot be achieved by the SVRs trained over some extracted domain. In Fig. 6, 7, the weights of the $\hat{w}_{fg}^T\phi_{fg}(\xi)$ and $\hat{w}_g^T\phi_g(\xi)$ are adjusted to learn the nonlinear term which is not trained offline exactly.

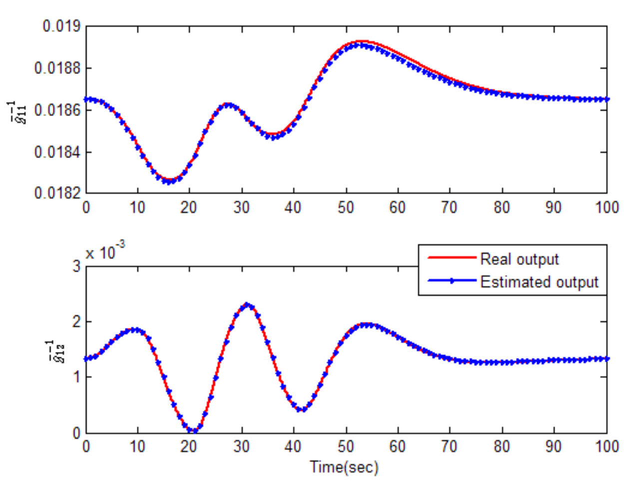

Fig. 2 : Results of the offline-trained $\bar{g}_{11}^{-1}, \bar{g}_{12}^{-1}$

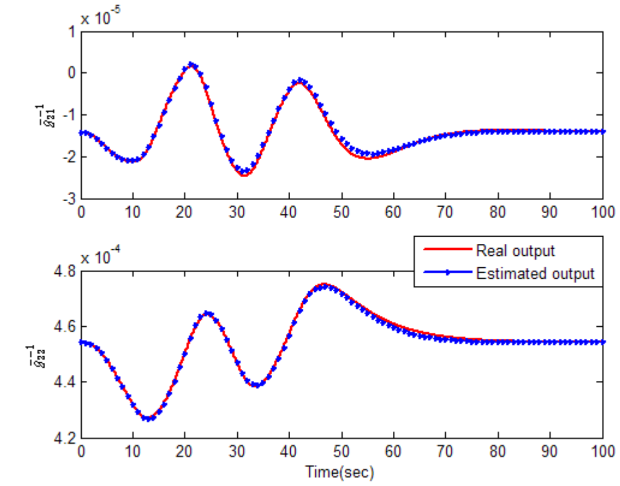

Fig. 3 : Results of the offline-trained $\bar{g}_{21}^{-1}, \bar{g}_{22}^{-1}$

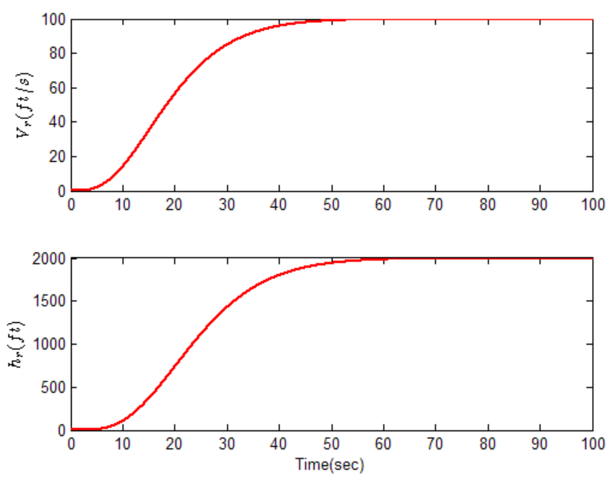

Fig. 4 : Reference commands V_r, h_r

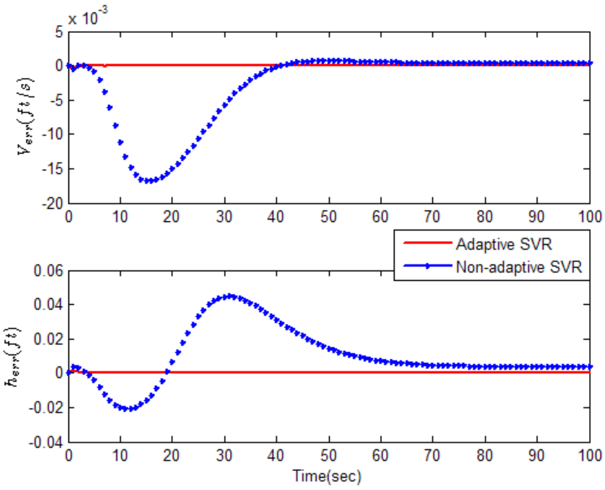

Fig. 5 : Tracking errors with no uncertainty

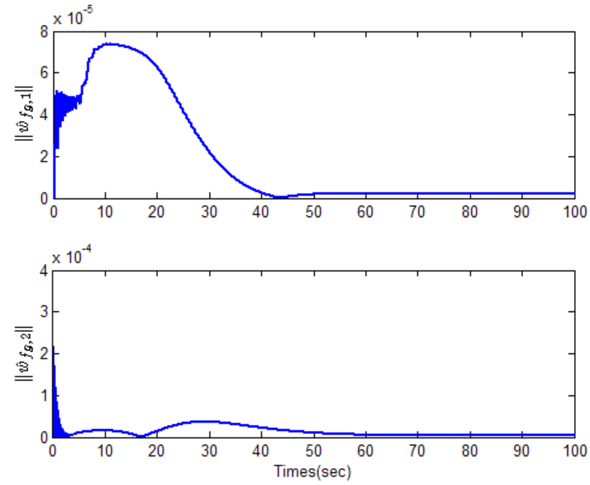

Fig. 6 : Norm histories of $\hat{w}_{fg}$

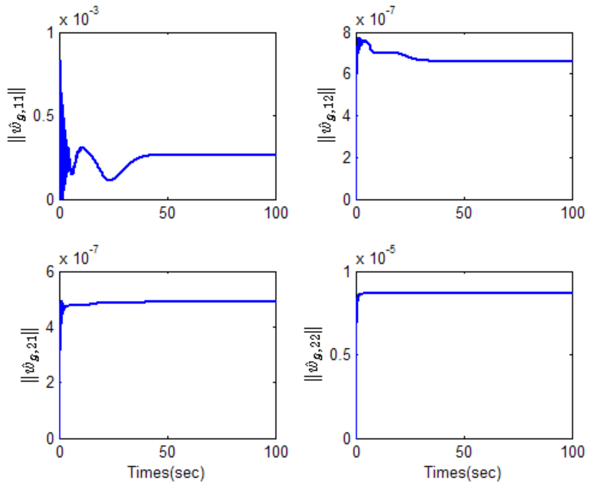

Fig. 7 : Norm histories of $\hat{w}_g$

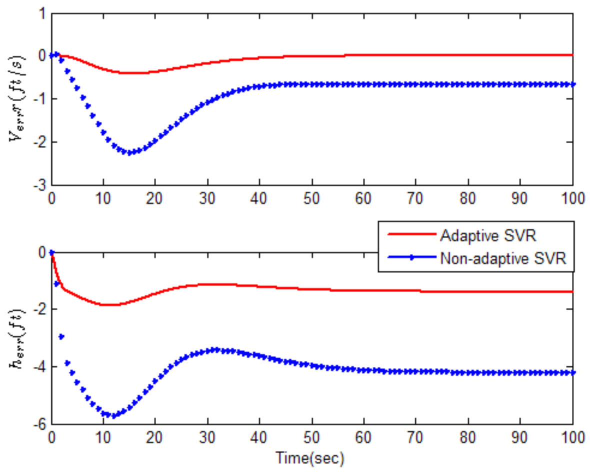

Fig. 8 : Tracking errors with uncertainty

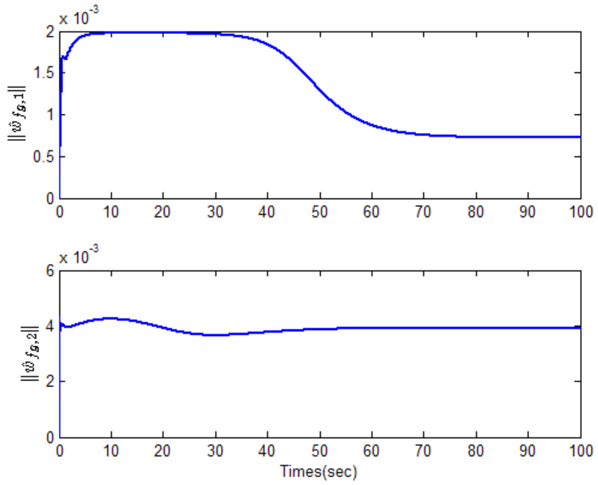

Fig. 9 : Norm histories of $\hat{w}_{fg}$

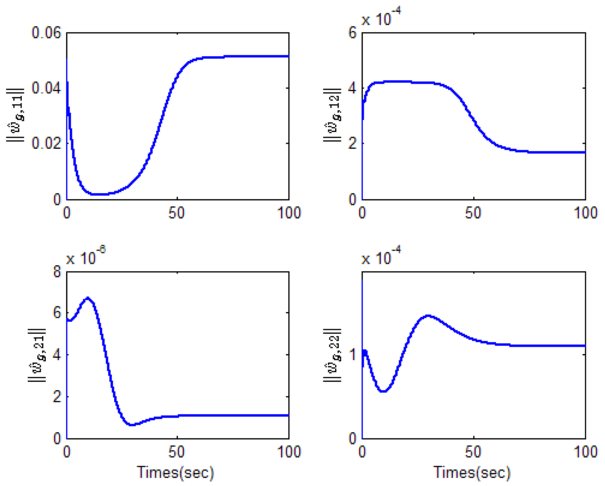

Fig. 10 : Norm histories of $\hat{w}_g$

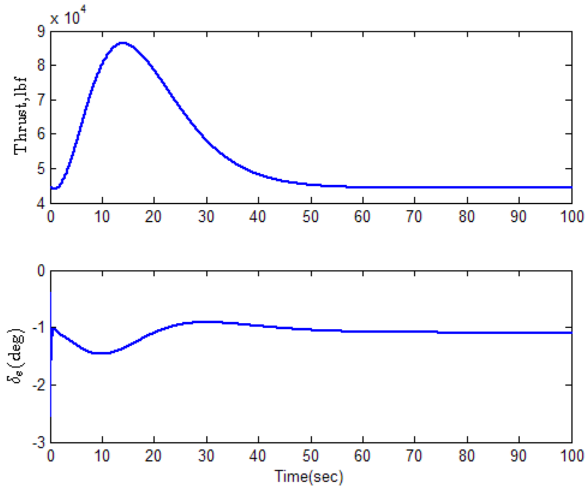

Fig. 11 : Control input histories

Finally, the additive uncertainties in *Assumption* 5 are added to the nominal values. As shown in Fig. 8, the performance of the SVR-based control is improved with the adaptation, while the non-adaptive control (i.e., using the offline-trained SVRs only) degrades the overall performance. The boundedness of the weight norm and control input are shown in Figs. 9, 10 and 11.

6 Conclusions

In this study, an adaptive feedback linearization-based control algorithm is presented using support vector regression for hypersonic aircraft control. The main idea of the proposed algorithm is that the offline estimator is designed by the SVR algorithm and the adaptation rule for the weight value of the offline-trained SVR is derived by defining a virtual control. By introducing the virtual control, online adaptation rule of the offline-trained SVR is derived using all the error states, and by utilizing the affine property of the system dynamics, the controller singularity problem can be avoided. Uniformly ultimate boundedness of the overall error dynamics is analyzed by the Lyapunov stability. Finally, numerical simulation was performed in order to validate the effectiveness of the proposed algorithm using the longitudinal dynamics of the hypersonic aircraft.

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