PROBABILISTIC FLUTTER ANALYSIS AND RISK ASSESSMENT OF AN AIRFOIL

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Abstract

The explosive nature of the flutter phenomenon and these uncertainties in aeroelastic system mandate that flutter calculation and flutter flight testing be cautious and conservative. The traditional deterministic nonlinear flutter analysis method is no longer suitable to treat these uncertain parameters. In this paper, a probabilistic method, the Monte Carlo simulation together with non-parametric estimation, is proposed to quantify these uncertainties of airfoil. Through probabilistic flutter analysis we can get the PDF of flutter speed, basing on them, we assess the risk of flutter occurring, and then we carry out probabilistic sensitivity analysis to define the key value that acutely influences the flutter speed of airfoil.

1 Introduction

In classic flutter analysis, flutter has been studied by a very deterministic method with the intent to avoid catastrophic aircraft destruction. In this analytical method the attributes and tests for items such as mass, stiffness, inertias and dimension of the aircraft configurations are assumed to be the best estimation and deterministic values. In fact however, these values are variable from one aircraft to another due to the differences of manufacturing run and operational condition. Recently, measurements of component weight and hinge line variation were conducted on a small sample size of twenty-four samples [1]. It shows that the weight variation is about five percent and hinge line inertia is about twenty percent. These variations are in fact parametric uncertainties. As a consequence, in order to obtain the more reliable aeroelastic stability of system, we should take the effect of parametric uncertainties into the consideration and evaluate the risk of instability. Parametric uncertainties owe their origin to many sources, which include[2] (1) stochastic variations in material properties, (2) stochastic variations in structural dimensions, (3) stochastic variations in boundary conditions due to preload and relaxation variations in mechanical joints, (4) stochastic variations of external excitations.

Military aircraft require flight tests throughout their useful life as their operational demands changes and new external stores, but the conflict of increasingly constrained budgets and expanding requirements for performance induce designers to renovate our approach to design for and demonstration of aeroelastic stability. Recently the U.S. Air Force Office of Scientific Research and the U.S. Air Force Research Laboratory organized a workshop, which included the role of uncertainty quantification (UQ) in efforts to understand the physics of nonlinear aeroelasticity and to certify aeroealstic stability [2]. The participants of the workshop developed a strong consensus that UQ must play a prominent role in the future of aeroelasticity research.

Civil engineers have been involved in studying the influence of uncertainties of structural properties, in particular, damping, velocity, on the reliability analysis of flutter of a bridge and other buildings [3,4,5]. Civil engineers are interested in determining a probability of the bridge or a structure failure due to flutter for a given period.

To figure out the effect of uncertain parameters many researchers have done highly effective jobs. The impact of structural and material uncertainties on the flutter characteristics of plate and shell has been studied by Liaw and Yang [6,7] and Lindsley [8,9]. Castravete and Ibrahim [2] researched effect of Stiffness Uncertainties on the Flutter of a Cantilever Wing using a stochastic finite element approach. Krudi and Lindsley have studied the uncertainty effects on the Goland wing [10]. Heeg [11] applied stochastic treatment to wind tunnel data to predict flutter margins. Pettit [12] gave a good overview and review about uncertainties in aeroelasticity.

Recently, the influence of parameter uncertainties on the response of a typical airfoil section was considered by a few researchers [13, 14]. In 2003, Pettit and Philip researched the impact of parametric uncertainties in cubic nonlinear twist stiffness on the response of airfoil with pitch and plunge degree of freedom [15].

In this paper, in order to research the effect of uncertainties on flutter characteristics of aircraft, we employ a two-degree-of-freedom airfoil and use linear flutter analysis method together with non-intrusive uncertainty quantification (Monte Carlo simulation) to give the distribution of flutter speed. Basing on the probabilistic information of uncertain parameters and flutter speed, critical parameter was given by probabilistic sensitivity analysis and risk of flutter was evaluated by quantitative risk analysis. As work above is built on probabilistic concept, here we call it probabilistic flutter analysis.

2 Model Description

2.1 Pitch and Plunge Airfoil

The two degrees of freedom pitch (α) and plunge (h) airfoil is depicted in Fig. 1. The pitch and plunge DOF have linear stiffness (K_h and K_α). The equations of motion for a linear pitch and plunge airfoil are shown below.

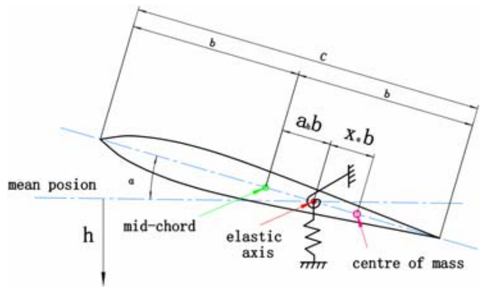

Fig. 1. Schematic of pitch and plunge airfoil

$$\begin{cases} m\ddot{h} + S_\alpha \ddot{\alpha} + K_h h = L \\ S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha = M_E \end{cases} \tag{1}$$

(h : plunge deflection, α : pitching angle, b : semi chord, m : mass, L : lift, M_E :moment about the elastic axis, S_α : static moment about the elastic axis, I_α : moment of inertia about elastic axis, K_α : linear stiffness in pitching, K_h :linear stiffness in plunge, ω_ξ :natural frequencies in plunge modes, ω_α :natural frequencies in pitching modes, U : velocity.)

2.2 Modal Parameters and Uncertainties

The parameters of airfoil are shown in Table 1. In fact some parameters can fluctuate due to differences in manufacturing process and environment. These are not deterministic but uncertain. Uncertainties can make flutter velocity fluctuate, in order to keep flight safe we should take the effect of uncertain parameters on stability of airfoil into consideration. In this paper, we assume that some uncertainties have normal distribution; the parameters of distribution (mean: μ , Standard Deviation: σ , Coefficient of Variation: C_v) were shown in Table 2.

Table 1 Parameters of two degree of freedom

Parameters	x_α	$x = x_\alpha \cdot b$ (m)	a_h	$a = a_h \cdot b$ (m)	$\varpi = \frac{\omega_h}{\omega_\alpha}$
Value	0.25	0.0625	-0.5	-0.125	0.2
Parameters	b (m)	m (Kg)	I_α (Kg·m^2)	μ	r_α
Value	0.25	24.04	0.3756	100	0.5

Table 2 Uncertain Values

Parameters	Distribution	μ	$C_v = \dfrac{\sigma}{\mu}$	σ
b	normal	0.25	0.2%	0.0005
x	normal	0.0625	2%	0.00125
m	normal	24.04	0.2%	0.04808
ρ	normal	0.7364 （H=5Km）	1.5%	0.011046
a	normal	-0.125	1%	0.00125

3 Probabilistic flutter analysis

3.1 Uncertainty quantification

In this paper we research the effect of parametric uncertainties of airfoil in Table 2 on flutter speed by Monte Carlo simulation (MCS). With the help of MCS we do not need modify the equations of motion for airfoil, it is non-intrusive uncertainty quantification. The samples of the responses of airfoil can be obtained by Monte Carlo simulation, in which the deterministic aeroelastic analysis was repeated for 1000 times. Based on those samples we can get the probability density function (PDF) of response by Kernel density estimation, a non-parametric estimation method [16], The probability density is estimated by the following function.

$$f(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \qquad (2)$$

Where $x_1, \cdots, x_n$ is sample independent and identically-distributed random variable of a random variable, and K is the kernel (here is standard Gaussian function) and h is the bandwidth (smoothing parameter). Fig. 2 shows the PDF of flutter speed of the airfoil.

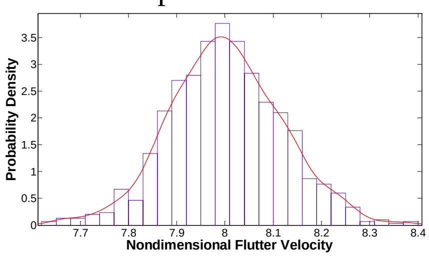

Fig. 2. PDF of nondimensional flutter velocity

Uncertain parameters make the flutter velocities fluctuate around deterministic flutter velocity (nondimensional flutter velocity: 8). When flutter velocity samples locate below deterministic flutter velocity, aircraft is intensively threatened by flutter occurring.

3.2 Probabilistic sensitivity analysis

In order to make clear which one of the uncertain parameters is the most important, probabilistic sensitivity analysis should be performed. In this paper we give sensitivities of these uncertainties through the scatter plot and standard regression coefficient (SRC). The scatter plot can give us qualitative information about sensitivity, but the standard regression coefficient (SRC) is quantitative index about sensitivity. Repeat deterministic flutter analysis with stochastic input values for 1000 times by Monte Carlo simulation, and then the following scatter plot can be obtained.

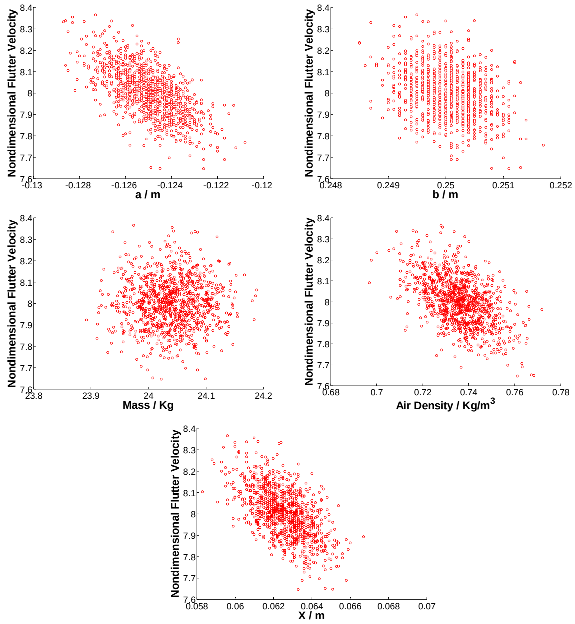

Fig. 3. Scatter plot of these uncertainties and non-dimensional flutter velocity

Fig. 3 shows that x , a and air density have the sharper influence on flutter velocity than b and mass, but in the scatter plot the distributions we can't tell the difference about these sensitivities.

However regression analysis provides an algebraic representation of the relationships between y and uncertain parameters (x_j, $j = 1,2,...,nX$).

$$\hat{y} = b_0 + \sum_{j=1}^{nX} b_j x_j \tag{3}$$

In fact the regression coefficients b_j , are not very useful in sensitivity analysis because each b_j is influenced by the units in which x_j is expressed and also does not incorporate any information on the distribution assigned to x_j . Because of this, the regression models in Eq. (3) are usually reformulated as [17]

$$(\hat{y} - \bar{y})/\hat{s} = \sum_{j=1}^{nX} (b_j \hat{s}_j / \hat{s})(x_j - \bar{x}_j)/\hat{s}_j \tag{4}$$

where

$$\bar{x}_j = \sum_{i=1}^{nS} x_{ij} / nS \ , \quad \bar{y} = \sum_{i=1}^{nS} y_i / nS$$

$$\hat{s} = \left[\sum_{i=1}^{nS} (y_i - \bar{y})^2 / (nS - 1) \right]^{1/2} ,$$

$$\hat{s}_j = \left[\sum_{i=1}^{nS} (x_{ij} - \bar{x}_j)^2 / (nS - 1) \right]^{1/2}$$

The coefficient $b_j \hat{s}_j / \hat{s}$ in Eq. (4) is referred to as SRC. As long as the x_j are independent, the SRCs provide a useful measure of variable importance, with (i) the absolute values of the coefficients $b_j \hat{s}_j / \hat{s}$ providing a comparative measure of variable importance (i.e., variable x_u is more important than variable x_v if $|b_u \hat{s}_u / \hat{s}| > |b_v \hat{s}_v / \hat{s}|$) and (ii) the sign of $b_j \hat{s}_j / \hat{s}$ indicating whether x_j and y tend to move in the same direction or in opposite directions[17].

We give quantitative sensitivities (SRC) of these uncertain parameters in Table 2 by above probabilistic sensitivity analysis (PSA).

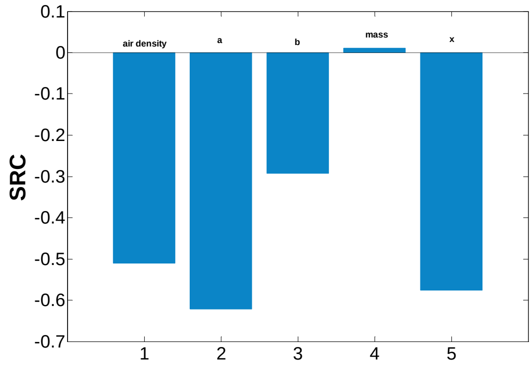

Fig. 4. standard regression coefficients (SRC) of uncertain parameters

Fig. 4 shows that the most important parameter is elastic axis (a), the second is centre of mass (x), the third is air density, the forth is span ($2b$), the last is mass.

4 Flutter risk assessment

We quantify the risk of flutter by the probability of the occurrence of flutter (call it flutter probability for short : P_f) and the equal probability flutter boundary. First of all, we define the structure performance function of flutter:

$$Z = g(V_f, V) = V_f - V \tag{5}$$

so the probability of the occurrence of flutter:

$$P_f = P(Z \le 0) = P(V_f - V \le 0) \tag{6}$$

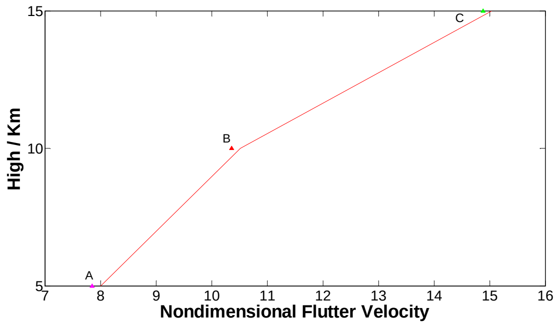

Fig. 5. flutter boundary and Calculate point

Assume coefficient of variation of the nondimensional velocities of calculate points is 2%. With the help of Eq. (5) and Eq. (6) we can calculate the flutter probability of these points which are close to flutter boundary. The flutter probabilities are shown in Table 3.

Table 3 Flutter probability of calculate points

Calculate point (Altitude,nondimensionalvelocity)	Flutter probability P_f
A（5km，7.85）	22.4%
B（10km，10.36）	28.5%
C（15km，14.88）	32.6%

Table 3 shows the quantitative risk information (flutter probability) when flying condition of airfoil (velocity and altitude) is close to flutter boundary.

In order to gain the equal probability flutter boundary, basing on these samples and PDF of flutter speed distribution at the some altitude we pick out the velocity point, at which the probability that flutter speed samples is larger than this velocity (V_0) is P. At the different altitudes we repeat the above calculation, and then we obtain some points ($(V_0,H_0);(V_1,H_1);(V_2,H_2);\cdots$) that have the same probability (P), connect these points to get the equal probability flutter boundary that the probability of the occurrence of flutter is P. This method was shown in Fig. 6.

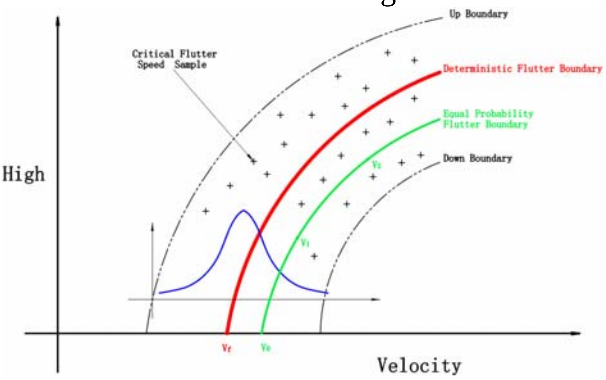

Fig. 6.Method of equal probability flutter boundary

For two degrees of freedom of airfoil with uncertainties in the Table 2, we can obtain these equal probability flutter boundaries shown in Fig. 7 by the method described above.

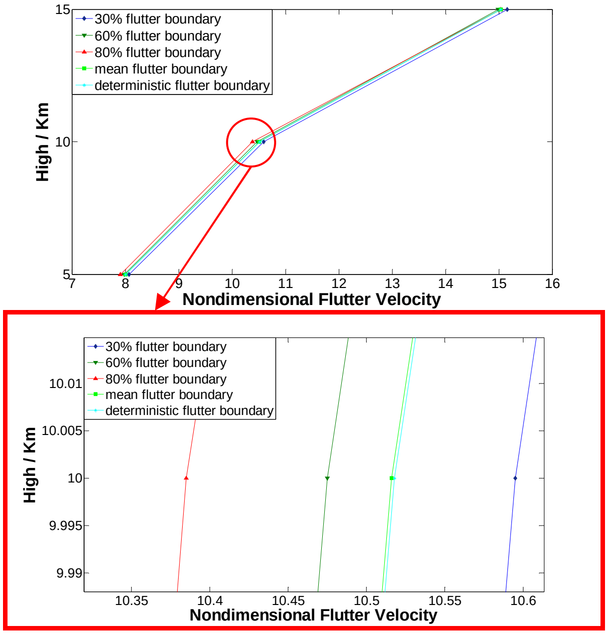

Fig. 7. Equal probability flutter boundary of airfoil

Fig. 7 shows that the flutter risk is increasing when flutter boundary moves towards left sides of figure. The conventional deterministic flutter boundary is close to mean flutter boundary (mean value of flutter speed distribution) and the deterministic flutter boundary, of which the probability of the occurrence of flutter is 60%.

5 Conclusions

In this paper we studied the effect of uncertainties in the two degrees of freedom airfoil on the flutter speed by MCS, and calculate the PDF of flutter speed distribution, basing on which we give the quantitative sensitivities of these uncertainties by SRCs, the results are shown in Fig. 4, and then with the help of the concept of structure performance function, we calculate the probability of the occurrence of flutter that are shown in Table 3 and obtain the equal probability flutter boundaries in Fig. 7.

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