Abstract

Coupled BE/FE Method and Modeling of structural-acoustic interaction has shown its promise and potential in the design and analysis of various structural-acoustic interaction applications, ranging from modern flexible aircraft subject to both aerodynamic and acoustic load to thin and flexible rotating computer hard disks subject to complex fluid-structure interactions.

Based on the understanding gained in the interaction between acoustic and aerodynamic effects on vibration of structures, the unsteady aerodynamic loading will be considered to consist of three components, i.e. the unsteady aerodynamic load component induced by the elastic structural motion in the absence of acoustic excitation, the unsteady aerodynamic load due to structural vibration which is induced by the ensuing acoustic pressure loading, and the unsteady aerodynamic flow-field induced by acoustic pressure disturbance.

Following development carried out in earlier work, unified combined acoustic and aerodynamic loading on the structure is synthesized using two approaches. First, by linear superposition of the acoustic pressure disturbance to the aeroelastic problem, the effect of acoustic pressure disturbance to the aeroelastic structure is considered to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, which is known as the scattering pressure, referred to here as the acoustic aerodynamic analogy.

Second, by synthesizing the acoustic and aerodynamic effects on elastic structure using an elegant, effective and unified approach, noting that both acoustic and aerodynamic effect on solid structural boundaries can be formulated as a boundary value problem governed by second order differential equations which lead to solutions expressible as surface integral equations. The unified formulation of the acousto-aeroelastic problem is amenable for simultaneous solution, although certain prevailing situations allow the solution of the equations independently.

1 Introduction

Acoustic-Aerodynamic-Elastic Structure interaction, in particular the vibration of structures due to sound waves, aerodynamics and their combined effects, is a significant issue that is found in a wide spectrum of engineering applications, among others aircraft and spacecraft on one end to computer hard disk on the other end, and has drawn the attention of many workers in this field. Acoustic excitation of the aircraft structure has been one of the main concerns during certain flight operations, such as the acoustic loads on B-52 wing during take off as reported by Edson [1], which reaches acoustic sound pressure levels as high as 164 dB. Modern new and relatively lighter aircrafts may be subject to higher acoustic sound pressure levels, such as those predicted for the NASP [2]. Typical structural acoustic and high frequency vibration problems that can severely and adversely affect spacecraft structures and their payloads have also been lucidly described by Eaton [3]. Various related acoustic, aerodynamic and elastic structure interactions have been addressed by various authors, such as Renshaw et al.[4], and recently by Kang and Raman[5], while structure-acoustic interaction by Tandon, Rao and Agrawal[6], and Hall, Kielb, and Thomas [7].
Applications that are based on the understanding gained in the interaction between acoustic and aerodynamic effects on vibration of structures is to introduce acoustic activated control on flow induced vibration (Ffowcs Williams, [8], Huang, [9], Lu & Huang, [10], and Nagai et al, [11]), which motivates the work reported here as part of a series of work carried out by the author and co-worker [12-17]. The following aspects will be reviewed. The first is how the combined effects of acoustic and fluid induced vibration can interact with each other to arrive at a favorable state of affairs judged from vibration alleviation considerations. The second aspect is the formulation of the acoustic and aerodynamic effects on elastic structure using an elegant, effective and unified approach, noting that both acoustic and aerodynamic effect on solid structural boundaries can be formulated as a boundary value problem, which is governed by second order differential equations which lead to solutions expressible as surface integral equations. Then the fluid-structure coupling constitutes the key issue in the fluid structure interaction problem. The formulation and solution of the acoustic elastic structure interaction has been approached using coupled BE-FE method (Chuh Mei and Pates [18], Holström [19]), and the aerodynamic elastic structure interaction, which is the domain of aeroelasticity, is well known (Bisplinghoff, Ashley and Halfman, [20], Förch, [21], Dowell et al [22], Zwaan, [23]). The deterministic approach of the combined effect of acoustic and aerodynamic interaction with elastic structure has been given attention by various authors [18-19], as well as the present author, in earlier work [12-17], with the focus on the formulation and solution of the BE-FE fluid structure interaction.

The interest in utilizing acoustic excitation for fluid induced vibration has evolved significantly, particularly since Ffowcs Williams demonstrated the viability of aircraft wing vibration active control by acoustic means during ICAS Congress in London in 1986. The work reported by Huang [9] is such manifestation. In his seminal paper, Ffowcs Williams [8] [24] has advocated this antisound concept and emphasized that, in principle, unsteady linear secondary fields can artificially be produced in antiphase with the primary fields to result in a suppression of the primary fields, which is amenable to active control and modification by the technique of anti-sound. Such idea has found many applications in various aerodynamic areas as an attractive and viable alternative for high frequency control compared to the less responsive conventional electro-mechanical actuators.

The first and second aspect of the Acoustic-Aerodynamic-Elastic Structure interaction problem has been combined in the BE-FE fluid-structure coupling, the aero-acousto elasticity problem, by the author and colleagues in earlier work as stability and dynamic response problem [14]. Gennaretti and Iemma carried out aeroacoustoelastic modelling for analysis of the acoustic field inside an aircraft cabin. The aim is the identification of a state-space format for aeracoustoelasticity equations applicable, for instance, for synthesis of an active control law devoted to cabin noise abatement [25-27]. Forth and Staroselsky [28] has utilized a hybrid FEM/BEM approach for aircraft engine structural health monitoring. Eller and Carlsson [29] recently developed an efficient aerodynamic boundary element method for aeroelastic simulations and its experimental validation. Active control of aerelastic flutter carried out by Huang [9] was carried out utilizing Nissim’s theory on Flutter Suppression Using Active Control [30], and followed by various authors in the effort to control aerelastic flutter, for example [10-11].

Huang’s wind-tunnel experiment showed that an appropriately operated loudspeaker system can stabilize a fluttering airfoil and his preliminary theory elucidated the underlying mechanism. His experiments indicated that the loudspeaker-induced aerodynamic forces are large, and using hydraulic analogy, it was argued that the amplification effect was mainly attributed to the area of the aerodynamic surface exposed to the acoustic wave field. Lu & Huang [10] stipulated that the conversion and amplification of the incident acoustic wave into shedding vortices from the trailing edge, known as the trailing-edge receptivity (Daniels, [31] and Crighton, [32]) should hold the key to the acoustic flutter
suppression technique, which was later verified in a 2D analysis [10] and later on by Nagai et al for 3D analysis of cascades [11].

The physical understanding of the phenomenon could be utilized to synthesize a theoretical framework to analyze and predict the state of affairs, and as such it covers generic problems and solutions of acoustics, unsteady aerodynamics and structural dynamics relevant to the engineering problems at hand. To this end, some available techniques in the literature will be utilized, and a novel synthesis of these will be established into a computational routine which has been the focus in the development of the foundation for the computational scheme for the calculation of fluid-structure interaction with combined acoustic and aerodynamic excitation on the structure.

2 Problem Formulation

To address the problem associated with fluid structure interaction, in particular the vibration of structures due to sound waves, aerodynamics and their combined effects, a generic approach to the solution of the acousto-aeroelastic dynamic interaction has been followed. The development of the foundation for the computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of the structure [12-17], starts from a rather simple and instructive model to a more elaborate FE-BE fluid-structure one. The formulation of the problem is summarized in Fig. 1.

In general generic approach consists of three streams. The first is the acoustic streams. Taking into account physical considerations elaborated by Ffowcs Williams [8, 24], Huang [9], Dowling and Ffowcs Williams [33], Norton and Karscub [34], the acoustic wave propagation governed by the Helmholtz equation by using boundary element approach is formulated, which then allows the calculation of the acoustic pressure on the acoustic-structure boundaries. The influence of the acoustic excitation field has been given rigorous consideration by taking into account both the incident and scattering acoustic pressure, following the governing equations described by Dowling and Ffowcs Williams [33] and Norton [34]. Taking note on the effects of acoustic disturbance on the aerodynamic field as considered by Lu and Huang [10], Nagai et al [11] and Yang et al [35], this effect is incorporated in the acoustic stream as a separate logical sequence identified as Acoustically Induced Aerodynamics.

Another acoustic effect is included as a new contribution. Due to the action of the total acoustic pressure on the structure subject to the prevailing acoustic source, the structure will experience an acoustic excited vibration. Correspondingly, such structural vibration will induce unsteady aerodynamic load, which is here identified as Vibration Induced Aerodynamics.

The second stream addresses the structural dynamic problem using finite element approach and normal mode analysis described by Felippa & Clough [36], Weaver and Johnston [37] and Beer and Watson [38]. In the formulation of BE-FE coupling to treat the fluid-structure interaction, reference is made to the solution procedure for structural-acoustic interaction problems described by Chuh-Mei and Pates.

Fig. 1 Logical approach to treat the aeroacoustic effects on aeroelastic structure.
The third stream accounts for the unsteady aerodynamic loading on the structure using a conveniently chosen and well established unsteady aerodynamic computational method, to be utilized in the aeroelastic problem.

Referring to the earlier development already carried out in earlier work [12-17], the governing equations for the acousto-aeroelasticity problem for an elastic wing submerged in a fluid and subject to acoustic pressure disturbance can be represented by the structural dynamic equation of motion with a general form of

\[
\begin{bmatrix}
[K] - \omega^2 [M]
\end{bmatrix}
\{x\} = \{F_{\text{Aero}}\} + \{F_{\text{Acou}}\} + \{F_{\text{Ext}}\}
\]

which, with appropriate considerations, can be transformed to

\[
\begin{bmatrix}
[K] - \omega^2 ( [M] + [A_{\text{Aero}}] + [P_{\text{Acou}}] )
\end{bmatrix}
\{x\} = \{F_{\text{Ext}}\} \quad (1)
\]

where, with appropriate considerations on their actual form

- **K** - stiffness related terms
- **M** - inertial mass related term
- **F_{\text{Aero}} , A_{\text{Aero}}** - aerodynamic related term
- **F_{\text{Acou}} , P_{\text{Acou}}** - acoustic related term
- **F** - external forces other than those generated aerodynamically or acoustically.

The acoustic pressure is governed by Helmholtz equation, which in discretized form can be written as

\[
[H]
\{p_{\text{Acou}}\} = i \rho_0 \omega [G] \{v\} + \{p_{\text{inc}}\} \quad (2)
\]

where, with appropriate considerations on their actual form

- **H** - acoustic pressure influence coefficient
- **G** - acoustic velocity influence coefficient
- **v** - acoustic velocity
- **p_{\text{Acou}}** - acoustic pressure

The definitions of these variables are elaborated in subsequent sections. The acoustic related term in Eq.(1) is dependent on the acoustic pressure solution of Eq. (2). These equations are comprehensively elaborated in [13-16].

The unified treatment which is the focus of the present work capitalizes on the following:

a. The set of Eqs. (1) and (2) represent the unified state of affairs of acousto-aeroelastic problem being considered, which in principle can be solved simultaneously. However, prevailing situations may allow the solution of the two equations independently.

b. The acoustic term incorporated in Eq. (1), as can be seen in subsequent development, is considered to consist of three components to be elaborated below, and are also treated in unified fashion.

c. Analogous to the aerodynamic terms, the total acoustic pressure in the acoustic term incorporated in Eq. (1), will be separated into elastic structural motion dependent and independent parts, which is referred to here as acoustic-aerodynamic analogy. Such approach allows the incorporation of the structural motion dependent part to the aerodynamic term in Eq. (1).

In the work of Huang [9], by introducing acoustic pressure field using vibrating membrane of loud-speaker, his theoretical prediction of the direct influence of the acoustic pressure field to the aerodynamically induced structural vibration has been demonstrated to agree with his experiments. On the other hand, Lu & Huang [10] and Nagai et al [11] have introduced the influence of the acoustic pressure field to the aerodynamic flowfield, recognizing the influence of of trailing edge receptivity not accounted for in the earlier work.

With such perspective in mind, the unsteady aerodynamic loading will here be considered to consist of three components, i.e.:

1. The unsteady aerodynamic load component induced by the elastic structural motion in the absence of acoustic excitation;
2. The unsteady aerodynamic load due to structural vibration which is induced by the ensuing acoustic pressure loading (P_{inc});
3. The unsteady aerodynamic flow-field induced by acoustic pressure disturbance.

The combined acoustic and aerodynamic loading on the structure is then synthesized by linear superposition principle of small oscillation for the acoustic pressure disturbance to the aeroelastic problem, the latter due to aerodynamic-structure interaction. To facilitate the solution of the acousto-aeroelastic stability equation solved by V-g method, a novel approach is undertaken. Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic...
effects can be distinguished into motion independent and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastic problem) is considered to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, which is the scattering pressure. This is referred to in this work as the acoustic aerodynamic analogy. The governing equation for the acousto-aeroelastic problem is then formulated incorporating the total acoustic pressure (incident plus scattering pressure), and the acoustic aerodynamic analogy. A generic approach to solve the governing equation as a stability or dynamic response equation is formulated allowing a unified treatment of the problem.

3 Unsteady Aerodynamics Formulation

3.1. Basic Equations
Following the development of a unified BE-FE Acoustic-Aerodynamic-Elastic Structure Coupling by Djojodihardjo [12-17], which capitalized on the utilization of already existing technique of contributing elements in acoustics, aerodynamics and structure, the calculation of the unsteady aerodynamic loading on the structure can be carried out using panel or doublet lattice method, Rodden & Johnson, [40] or doublet point method Houbolt [41], Ueda & Dowell [42], and can be conveniently matched with the discretization of the structure as finite elements. Such routine is available in the commercial code such as MSC NASTRAN® [40] and ZAERO®[43].

The basic lifting surface theory assumes that the flow is inviscid, isentropic, subsonic, and has no flow separation. The thickness of the surface is neglected and the angle of attack is small such that the small-disturbance potential flow approach may be used to linearize the mixed boundary value problem. The compressibility effect is taken into account in the aerodynamic governing equation using the Prandtl-Glauert transformation.

The basic equation for the velocity potential in aerodynamics is governed by the following differential equation written in Cartesian coordinates as:

\[ \nabla^2 \phi = \frac{1}{a^2} \left\{ \frac{\partial}{\partial t} \left( \nabla \phi \cdot \nabla \phi \right) + \frac{\partial^2 \phi}{\partial t^2} + \nabla \phi \cdot \nabla \left( \frac{\nabla \phi \cdot \nabla \phi}{2} \right) \right\} \]

(3)

where the vector operator \( \nabla \) is defined by:

\[ \nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \]

(4)

If the lifting surface moves in the x direction and oscillates harmonically with frequency \( \omega \) (radian/second) and with a small perturbation about its equilibrium, then

\[ \phi(t) = \bar{\phi} e^{i\omega t} \]

(5)

The velocity potential \( \phi \) is related to the harmonically varying pressure difference \( \Delta p \) by the Bernoulli equation:

\[ \Delta p = -\rho \left( \frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial x} \right) \]

(6)

and

\[ \Delta p = \Delta \bar{p} e^{i\omega t} \]

(7)

and the downwash velocity on the aerodynamic surface is given by

\[ \bar{w} = \frac{\partial \bar{\phi}}{\partial z} \]

(8)

The velocity potential can be formulated into a boundary integral equation using Green’s theorem (Wrobel [44], Djojodihardjo& Widnall [45]). Using the Bernoulli equation relationship between the pressure and downwash at the aerodynamic surface can be obtained. Alternatively, following Kussner (Forsching, [21], Dowell et al.[22]; Zwaan, [23]) the general integral equation that relates the normal-wash \( \bar{w} \) at a point \((x, y, z)\) due to a pulsating pressure \( \Delta p \) on an infinitesimal area \( d\xi d\eta \) centered at point \((\xi, \eta, \zeta)\) as shown in Fig. 2., can be written as:

\[ \bar{w} = \frac{1}{8\pi} \iint_{S} \frac{\Delta \bar{p}}{q_{\phi}} K_{\phi}(x_0, y_0, z_0, k, M) d\xi d\eta \]

(9)

where the function \( K_{\phi}(x_0, y_0, z_0, k, M) \) represents the kernel function of the integral equation.

Basic method for unsteady aerodynamics calculation on the structure can be carried out using Doublet Point Method (DPM), as proposed by Houbolt [41] based on the pressure velocity potential concept, and independently by
Ueda and Dowell [42] based on the pressure normal wash concept.

Following the procedure, in the lifting surface method exemplified here by the Doublet Point Method, equation (10) can be recast into the discretized form:

\[
\{\vec{w}\} = [A/C]\begin{bmatrix} \vec{p} \\ q \end{bmatrix} \quad (10)
\]

or

\[
\begin{bmatrix} \vec{p} \\ q \end{bmatrix} = [A/C]^{-1}\{\vec{w}\} \quad (11)
\]

\[\begin{array}{l}
\{\vec{w}\} = [A/C]\begin{bmatrix} \vec{p} \\ q \end{bmatrix} \\
\begin{array}{c}
\{\vec{h}\} = [D]\{\vec{w}\} \\
\{\vec{h}\} = [D]\{\vec{w}\} \\
\{\vec{h}\} = [D]\{\vec{w}\} \\
\{\vec{h}\} = [D]\{\vec{w}\}
\end{array}
\end{array}\]

where \(\{\vec{w}\}\) is the downwash vector at the aerodynamic surface which is represented by a set of surface elements depicted in Fig. 2, where \(\vec{p}\) is the mean pressure on the surface and \(q = \frac{1}{2}\rho U^2\), and \(U\) the free-stream velocity.

The kinematic boundary condition on the surface of the wing implies that:

\[
\{\vec{w}\} = [D]\{h\} \quad (12)
\]

where \(\{h\}\) is the vertical displacement vector of the aerodynamic surface due to the prevailing load, and

\[
D \equiv \left( i\omega + U \frac{\partial}{\partial x} \right) \quad (13)
\]

for harmonic oscillation case with \(x\) the streamwise direction. Using these relationships, Eq. (10) can be further recast into a computationally convenient discretized form:

\[
\mathbf{F}_{\text{Aero}} = q_{\infty}\text{[Area]}[A/C]^{-1}[D]\{h\} \quad (14)
\]

where \([A/C]\) is the Aerodynamic Influence Coefficient.

3.2. Calculation of \(\mathbf{F}_{\text{Aero}}\) using Doublet Point Method

Following the DPM procedure, if the doublet point and the control point are located at elements \(i\) and \(j\) respectively, then the aerodynamic influence coefficient \([A/C]\) may be written as

\[
\mathcal{F}_{ij} = \frac{\Delta S}{8\pi} K\left(x_i - \xi_j, y_i - \eta_j, z_i - \zeta_j, k, M\right) \quad (15)
\]

where

\[
\Delta S = 2\varepsilon \Delta x = \text{the area of sending element}
\]

The detail of the computational procedure is elaborated by Djojodihardjo and Safari [14-15].

3.3. Aero-Structure Coupling

For the coupling of the aerodynamic forces defined in a specific way on the aerodynamic surface with the structural inertial forces in the structure defined using finite element discretization procedure to be incorporated in the structural dynamic equation of motion (1) for the structural responses, consideration should be given to the grid of the discretized aerodynamic model, which is placed on the external surface, and that of the structural model, which is placed on the internal load-carrying component. Hence care should be taken for translating the corresponding displacement vectors commensurate with each system, which gives appropriate treatment the data-transferral problem between the two computational grid systems. A procedure for this data transferral problem that relates the displacements at the structural finite element grid points to the control points of aerodynamic boxes follows that of Harder and Desmarais [46], who defined a spline matrix generated by infinite plate spline (IPS) method, and assembles the total spline matrix \(\mathbf{G}_{\text{ips}}\) expressed in:

\[
\{\vec{h}\} = \mathbf{G}_{\text{ips}}x \quad (16)
\]

where \(\{\vec{h}\}\) is the “interpolated” displacement vector at aerodynamic boxes, including the translational displacements and their slopes with
respect to \( x \). Correspondingly, Eq. (14) can be recast into the following form:

\[
\{ F_{\text{Aero}} \} = q_0 \left[ A(ik) \right] \{ h \} = q_0 \left[ A(ik) \right] \left[ G_{\text{ips}} \right] \{ x \}_{\text{panel}}
\]

(17)

where \( \{ x \}_{\text{panel}} \) deflection matrix at aerodynamic surface panel nodes. The procedure to generate the aerodynamic-structural coupling matrix \( G_{\text{ips}} \) is well known [46-47].

**4. Discretization of Helmholtz Integral Equation for the Treatment of the Acoustic Field**

The development for the discretization of the Helmholtz equation which governs the acoustic domain depicted in Fig. 3 has been elaborated by Djojodihardjo and Safari [14-15] and Djojodihardjo [16]. Only some aspects relevant to the discussion in this paper will be reproduced for clarity.

**Exterior Problem**

![Fig. 3 Schematic of Acoustic Domain](image)

For an exterior acoustic problem, the problem domain \( V \) is the free space \( V_{\text{ext}} \) outside the closed surface \( S \). \( V \) is considered enclosed between the surface \( S \) and an imaginary surface \( \Lambda \) at a sufficiently large distance from the acoustic sources and the surface \( S \) such that the boundary condition on \( \Lambda \) satisfies Sommerfeld’s acoustic radiation condition as the distance approaches infinity.

For time-harmonic acoustic problems in fluid domains, the corresponding boundary integral equation is the Helmholtz integral equation [31-32, 44]

\[
cp(R) = \int_S \left[ p(R) \frac{\partial g}{\partial n_0} - g(R-R_0) \frac{\partial p}{\partial n_0} \right] dS
\]

(18)

where \( n_0 \) is the surface unit normal vector, and the value of \( c \) depends on the location of \( R \) in the fluid domain, \( R_0 \) denotes a point located on the boundary \( S \), and \( g \) the free-space Green’s function, as given by

\[
g(R-R_0) = \frac{e^{-ik|R-R_0|}}{4\pi|R-R_0|}
\]

(19)

To solve Eq. (1) with \( g \) given by Eq. (2), one of the two physical properties, acoustic pressure and normal velocity, must be known at every point on the boundary surface. At the infinite boundary \( \Lambda \), the Sommerfeld radiation condition in three dimensions can be written as [31-32]:

\[
\lim_{|R-R_0| \to \infty} r \left( \frac{\partial g}{\partial r} + ikg \right) = 0 \quad \text{as} \quad r \to \infty, r = |R-R_0|
\]

(20)

which is satisfied by the fundamental solution.

The total pressure, which consists of incident and scattering pressure, serves as an acoustic excitation on the structure. The integral equation for the total wave is given by

\[
cp(R) - p_{\text{inc}}(R) = \int_S \left[ p(R) \frac{\partial g(R-R_0)}{\partial n_0} - \frac{\partial p}{\partial n_0} g(R-R_0) \right] dS
\]

(21)

where \( p = p_{\text{inc}} + p_{\text{sc}} \), and where

\[
c = \begin{cases} 
1 & , R \in V_{\text{ext}} \\
1/2 & , R \in S \\
\Omega/4\pi & , R \in S \quad \text{(non smooth surface)} \\
0 & , R \in V_{\text{int}}
\end{cases}
\]

(22)

The discretization of the Helmholtz equation by appropriate division of the boundary surface \( S \) into \( N \) elements yields the following convenient form for computation:

\[
[H] \{ p \} = i \rho_0 \omega [G] \{ V \} + \{ p_{\text{inc}} \}
\]

(23)

where the elements of \( [H] \) and \( [G] \) are given by:

\[
H = \int_S g dS 
\]

(24)

\[
G = \int_S g dS 
\]

(25)

where

\[
g = \frac{\partial g}{\partial n}
\]

(26)

and where, for the monopole Green’s free-space fundamental solution, it follows that:
This matrix equation can be solved if the boundary condition
\[ \nu = \frac{\partial \rho}{\partial n} \]
and the incident acoustic pressure field \( p_{\text{inc}} \) are known. The value of \( \nu \) due to acoustic pressure disturbance on the solid boundary is governed by the following kinematic condition
\[ \nu = i \omega (T \cdot w) \] (28)
where \( T \) is the kinematic coupling between the acoustic surface interfacing with the solid surface and for each BE element is given by
\[ T_{ij} = \{ n \}^T \] (29) and \( w \) is the displacement vector on the associated solid finite element nodes and \( n \) is the normal to the interfacing BE-FE element.

At this point, a few remarks are necessary. Proper interpretation should be given to the diagonal terms of \( [H] \) in Eq. (23) as implied by the original boundary integral (18) and accordingly, \( [H] \) could be written as
\[ [H] = [H]^D + [H]^{OD} \] (30)
i.e. the diagonal and the off-diagonal part. The matrix \([H]^D\) as implied in Eq. (30) can be written as \([H]^D = [\bar{H}]^D + [C]\) where \( C \) is space angle constant as implied in Eq.(22) which is the quotient of \( \Omega/4\pi \) and \( \bar{H} \) is related to \( H \) following the relationship
\[ H_{ij} = \frac{1}{2} \delta_{ij} - \bar{H}_{ij}, \]
and is \( \delta_{ij} \) Kronecker delta.

5 BE-FE Acousto-Aeroelastic Coupling

The acousto-aeroelastic equation governing the structural dynamic motion of the structure, as further elaboration of Eqs. (1) and (2), can then be given by:
\[
\begin{align*}
\begin{bmatrix}
\mathbf{K}^* - \omega^2 [\mathbf{M}] + \rho_0 [\mathbf{F}_{ac}(ik_w)] \\
\end{bmatrix} \{ q \} &= \{ P^*_{\text{inc}} \} \\
+ \frac{\rho}{2} \left( \frac{b_y}{k} \right)^2 \left[ \mathbf{A}(ik) \right] \{ q \} &= \{ P^*_{\text{inc}} \}
\end{align*}
\] (32)
and
\[
\begin{align*}
\rho_0 \omega^2 [\mathbf{G}_{i11}] [\mathbf{T}] [\mathbf{q}] &= [\mathbf{H}_{i11}] \{ p_{\text{in}} \} + [\mathbf{H}_{i12}] \{ p_{\text{inc}} \} = \\
i \rho_0 \omega^2 [\mathbf{G}_{i11}] [\mathbf{T}] [\mathbf{q}] &= [\mathbf{H}_{i12}] \{ p_{\text{inc}} \} + [\mathbf{H}_{i22}] \{ p_{\text{inc}} \} = \\
+ i \rho_0 \omega^2 [\mathbf{G}_{i12}] [\mathbf{T}] [\mathbf{q}] &= [\mathbf{H}_{i22}] \{ p_{\text{inc}} \} + [\mathbf{H}_{i23}] \{ p_{\text{inc}} \}
\end{align*}
\] (33)

where
\[
\begin{align*}
\mathbf{K}^* &= \Phi^T [\mathbf{K}] \Phi \\
\mathbf{M}^* &= \Phi^T [\mathbf{M}] \Phi \\
\mathbf{F}_{ac}(ik_w) &= \Phi^T [\mathbf{L}] [\mathbf{P}_{ac}(ik_w)] \Phi \\
\mathbf{A}(ik)^T &= \Phi^T \mathbf{G}_{ips} [\mathbf{A}(ik)] \mathbf{G}_{ips} \Phi
\end{align*}
\] (35a-d) and where \( \Phi \) is the eigenmodes obtained from modal analysis of the purely elastic structure governed by
\[
\begin{align*}
\begin{bmatrix}
\mathbf{K} - \omega^2 \mathbf{M} \end{bmatrix} \{ x \} &= \{ 0 \}
\end{align*}
\] (36) and where \( \mathbf{L} \) is a coupling matrix of size \( \mathbf{M} \times \mathbf{N} \), where \( \mathbf{M} \) is the number of FE degrees of freedom and \( \mathbf{N} \) is the number of BE nodes on the coupled boundary \( a \), depicted in Fig.4. For each coinciding Boundary and Finite Elements, the acoustic pressure elemental coupling term is given by (Holström [19], Marquez et al [39])
\[ L_e = \int_{S_e} N_F^T n N_B dS \] (37)
where \( N_F \) and \( N_B \) are the shape function of the coinciding Finite Element and Boundary Element, respectively, and \( n \) is the normal vector to the corresponding coinciding surface. Here \( \{ x \} \) is the structural deformation and transformed to the generalized coordinates \( \{ q \} \) using the following relationship:
\[ x = \Phi q \] (38) where \( \Phi \) is the modal matrix as solution to the eigenvalue problem corresponding to the free vibration of the structure. By appropriate mathematical derivation, it was readily shown by Djojodihardjo and Safari [14] that
\[
\{ p_{\text{a}} \} = -\rho_0 \omega^2 [\mathbf{P}_{ac}(ik_w)] \{ x \} \] (39)
where a new “acoustic equivalent pressure $P_{ac}(ik_w)$” in (22) is defined as

$$[P_{ac}(ik_w)] = [E_{11}]^{-1} [D_{11}] [T] \quad (40)$$

and $[D_{11}]$ and $[E_{11}]$ are matrices derived from Eq.23 for specific regions denoted in Fig.4, as elaborated in [14-16]. Fig. 4 schematically depicts various acoustic domain associated with the motion of the structure a in an acoustic (and aerodynamic) domain, where the boundary conditions for the governing equations (19)-(21) are defined.

Then Eq.(31) can be written as

$$\begin{bmatrix} K' - \omega^2 \left( M' + \frac{b_i}{2k} [A'(ik)] + \rho_s [F_{ac}(ik_w)] \right) \end{bmatrix} q = \{0\} \quad (41)$$

where, as elaborated in [14-16]:

$$[F_{ac}(ik_w)] = \Phi^T [L] [P_{ac}(ik_w)] \Phi \quad (42)$$

$$[F_{ac}(k_w)] = [F_{ac}(k_w)] + [F_{ac}(k_w)] \quad (43)$$

and which can be simplified as:

$$\begin{bmatrix} M^{**} - \lambda K \end{bmatrix} q = 0 \quad (44)$$

where

$$M^{**} = K' - \omega^2 \left( M' + \frac{b_i}{2k} [A'(ik)] + \rho_s [F_{ac}(k_w)] + [F_{ac}(k_w)] \right) \quad (45)$$

The variables with asterisk or double asterisks are appropriately transformed variables and defined for convenience, which then allows Eq. (41) be reorganized in a standard eigenvalue problem as

$$\begin{bmatrix} M^{**} - \lambda K^* \end{bmatrix} \{q\} = \{0\} \quad (46)$$

and which can be treated as acousto-aeroelastic flutter stability problem. Frequency, damping, and velocity can be obtained following the procedure for K-Method, and using

$$Q^* = \Phi^T Q \Phi \quad (47)$$

where $Q$ represents $M'$, $K'$, $A'$ as appropriate, and

$$F_{ac}(k_w) = \Phi^T [F_{ac}(k_w)] \Phi \quad (48)$$

$$\left( F_{ac}'' \right)_{\omega_i} = \frac{(\omega_i^2 q_i)}{\omega_i^2 q_i} \quad (49)$$

and by assuming

$$\lambda = \frac{1 + ig}{\omega^2} \quad (50)$$

Eq. (43) is solved as an eigenvalue problem for a series of values for parameters $k$ and $\rho$. Since $M^{**}$ is in general a complex matrix, the eigenvalues $\lambda$ are also complex numbers. For $n$ structural modes, there are $n$ eigenvalues corresponding to $n$ modes at each $k$. The air speed, frequency and structural damping are related to the eigenvalue $\lambda$ as follows:

$$\omega = \frac{1}{\sqrt{\text{Re}(\lambda)}} \quad (51a)$$

$$U = \frac{\omega b}{k} \quad (51b)$$

$$g = \frac{\text{Im}(\lambda)}{\text{Re}(\lambda)} \quad (51c)$$

To evaluate the flutter speed, $V-g$ and $V-f$ diagrams are constructed following well known procedure [20-23]. The flutter velocity $U_f = \frac{\omega_f b_k}{k}$ is obtained when $\lambda$ crosses the zero-axis in the $V-g$ diagram.

The set of Eqs.(32)-(34) can be solved simultaneously, or alternatively by solving each equation independently, depending on the assumptions for the boundary conditions. Correspondingly, the following situations may prevail:

1. The acoustic pressure on the boundary b should not influence the solution of the acousto-aeroelastic equation (32).
2. Interpreting $\{p_b\}$ considered as $\{\Delta p_b\}$, i.e. pressure differential between the upper and lower surface of the wing, and $\{p_s\}$ considered...
as \{ \Delta p_a \} on the horizontal plane outside the wing, then certainly the notion mentioned in 1. is valid and the total acoustic pressure \{ p_a \} on the upper surface of the wing structure due to the ensuing acoustic pressure disturbance from an acoustic source can readily be evaluated. Alternatively, Eq. (41) can be treated as dynamic response problem, as reported by Djojodihardjo and Safari [14].

5. Incorporation of the acoustic contribution to the acousto-aeroelastic problem

The three components of the unsteady aerodynamic load mentioned in section 2: Problem Formulation, can readily be incorporated in Eq.(41) in the aerodynamic term \( [\mathbf{A}'(i\omega)] \). Correspondingly, these will be identified as \( [\mathbf{A}'(i\omega)]^a \), \( [\mathbf{A}'(i\omega)]^r \) and \( [\mathbf{A}'(i\omega)]^s \).

The first unsteady aerodynamic loading, characterized by \( [\mathbf{A}'(i\omega)]^a \) has already been treated in great detail in section 3: Unsteady Aerodynamics Formulation. The acousto-aeroelastic problem incorporating the total acoustic pressure (consisting of incident and scattered acoustic pressure components) loading on the upper surface of the wing structure, characterized by \( [\mathbf{A}'(i\omega)]^s \) has already been addressed by Huang [9] as induced by active acoustic excitation with structure fixed and aligned in the free-stream direction (zero angle of attack) is here treated as the flowfield induced pressure distribution due to the ensuing acoustic pressure disturbance. Work is in progress for a unified approach to this third unsteady aerodynamic contribution due to disturbed flow field induced acoustically.

Then equation (31) can be rewritten as:

\[
\left[ \mathbf{K} + \omega^2 \mathbf{M} \right] \mathbf{x} + \rho_0 \mathbf{G}_1 \mathbf{y}_b + \rho_0 \mathbf{G}_2 \mathbf{y}_b + \Delta p \mathbf{g} = \{ p \} \nonumber
\]

The other equations (33) and (34) are solved simultaneously or independently.

6. Acoustic-Aerodynamic Analogy

H. Djojodihardjo [17,48]. The governing equation for the acoustic-elastic coupled FE/BE acoustic-structure model (without structural damping) is then given by:

\[
\mathbf{K} + \omega^2 \mathbf{M} \mathbf{x} + \rho_0 \mathbf{G}_1 \mathbf{y}_b + \rho_0 \mathbf{G}_2 \mathbf{y}_b + \Delta p \mathbf{g} = \{ p \}
\]
Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic effects can be distinguished into motion independent (self-excited) and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastic problem) can be viewed to consist of structural motion independent incident acoustic pressure (excitation acoustic pressure) and structural motion dependent acoustic pressure, which is known as the scattering pressure. However the scattering acoustic pressure is also dependent on the incident acoustic pressure.

Solution of Eq. (41) will be facilitated by the use of modal approach, i.e. transforming \( \{x\} \) into the generalized coordinate \( \{q\} \) following the relationship \( x = \Phi q \), where \( \Phi \) is the modal matrix. In the examples worked out, a selected lower order natural modes will be employed in \( \Phi \). Pre-multiplying Eq. (41) by \( \Phi^T \) and converting dynamic pressure \( q_{\infty} \) into reduced frequency \( (k) \) as elaborated in [12-14], Eq. (41) can then be written as:

\[
\Phi^T \left[ K - \omega^2 \left( M + \frac{\rho^2}{2} \frac{l}{k} \right) \right] \Phi [q] + \rho \Phi \left[ F_{\text{ac}} (k_w) \right] = \Phi^T [F]
\]

(56)

Since all of the acoustic terms are functions of wave number \( (k_w) \), Eq. (43) will be solved by utilizing iterative procedure. Incorporation of the scattering acoustic component along with the aerodynamic component in the second term of Eq.(56) can be regarded as one manifestation of what is referred to here as the acousto-aerodynamic analogy followed in this approach.

7. Numerical Results

7.1. Acoustic Boundary Element Validation

In order to verify the validity of the computational method developed, the method is applied to some test cases, where exact results or worked out examples are available.

For a pulsating sphere an exact solution for acoustic pressure \( p(r) \) at a distance \( r \) from the center of a sphere with radius \( a \) pulsating with uniform radial velocity \( U_a \) is given by

\[
p(r) = \frac{a}{r} U_a e^{-ika} e^{ikr}  
\]

(56)

where \( z_0 \) is the acoustic characteristic impedance of the medium and \( k \) is the wave number. Fig. 5 shows the discretization of the surface elements of an acoustics pulsating sphere representing a monopole source. Excellent agreement of the BEM calculation for scattering pressure from acoustic monopole source with exact results is shown in Fig. 5. The calculation was based on the assumption of \( f=10 \text{ Hz}, \rho=1.225 \text{ Kg/m}^3, \) and \( c=340 \text{ m/s} \). The excellent agreement of these results with exact calculation serves to validate the developed MATLAB® program for further utilization.

Fig. 5. Comparison of monopole source exact and BE scattering pressure results

Fig. 5 shows the convergence trend of the computational scheme to the grid size on the calculation of the sound pressure level on the pulsating sphere for various frequencies of the pulsating sphere.
Fig. 6 a. Normal displacement and b. Surface pressure distribution, on a completely flexible sphere modeled using BEM-FEM Coupling, and subjected to external normal force at a point on the surface.

The BEM-FEM simulation results on a spherical shell subject to acoustic pressure is shown in Figs. 6 and 7. Analytical values are given by Junger & Feit [49].

Fig. 7 (a) Typical Mode shape, showing the triangular FEM Elements on the spherical shell.

Fig. 7 (b) Triangular FEM Shell elements adopted by Chen et al [50] which is also used here for benchmarking.

Fig. 7 (c) Normal displacement along a meridian arc for $k=1.6$.

Fig. 7 (d) Normal displacement along a meridian arc for $k=1.6$ obtained by Chen et al [41].

7.2. Coupled BEM-FEM Numerical Simulation

Fig. 8b compares the computational results obtained using present method to the example worked out by Holström [19] for vibrating membrane on top of a box, depicted in Fig. 8a. The excellent agreement lends support to the present method. Fig. 8c shows the resulting total pressure distribution in a color-coded diagram. Top surface is modeled as BE-FE, others as BE.
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Fig 8. b. Convergence trend of the sound pressure level frequency response of a vibrating top membrane of an otherwise rigid box due to monopole source at the center of the box (Fig.8a) studied in [19] as a function of frequency calculated using present computational scheme for various grid sizes.

Next the method is applied for the acoustic effects on the aeroelastic stability of a wing structure, here using the BAH wing [20]. The BAH wing and the surrounding boundary representing a quarter space of the problem are discretized as shown in Fig.9.

The problem domain is divided into two parts. First the near field region is a quarter space with radius of two times the BAH wing span and is relatively more densely discretized; second, the intermediate to far field region is a quarter space with radius ten times the BAH wing span and is less densely discretized compared to the near field region, and is modeled with BE only. The BAH wing which is modeled as FE and BE is subjected to an excitation due to an acoustic monopole source; the acoustic medium is air with density $\rho = 1.225 \text{ kg/m}^3$ and the sound velocity is $c = 340 \text{ m/s}$. The monopole acoustic source is placed at the intersecting line of the half span and half chord planes of the BAH wing structure, and at about 0.1 m above the wing surface.

The monopole source has the frequency of 10 Hz, radius of $a = 0.1 \text{ m}$, air density $\rho = 1.225 \text{ kg/m}^3$ and sound velocity $c = 340 \text{ m/s}$. The result of applying Eq. (32) for $F = 0$ is presented as color-coded diagrams in Fig. 10, which exhibits the real, imaginary and absolute values of the incident, total and scattering pressures and which qualitatively exhibits the expected behavior. Typical deformation and total acoustic pressure response is exhibited in Fig. 11.

7.3. Flutter Calculation for Coupled Unsteady Aerodynamic and Acoustic Excitations

Fig. 12 shows the unsteady aerodynamics pressure ($C_p$) and mode shape of the structure when flutter occurs, using present method.

Fig 8. b. Convergence trend of the sound pressure level frequency response of a vibrating top membrane of an otherwise rigid box due to monopole source at the center of the box (Fig.8a) studied in [19] as a function of frequency calculated using present computational scheme for various grid sizes.

![Convergence trend of the sound pressure level frequency response of a vibrating top membrane of an otherwise rigid box due to monopole source at the center of the box.](image)

![Total pressure distribution on the surface of the wing.](image)

![3-D domain representing a wing structure and its surrounding boundary.](image)

![Typical deformation and total acoustic pressure response.](image)

![Incident, Total and Scattering Pressure [Real, Imaginary, and Magnitude] on a wing due an acoustic monopole source above the wing.](image)
Fig. 11. Pressure distribution on symmetric equivalent BAH wing: a) Incident pressure [dB] from monopole acoustic source as an acoustic excitation and b) deformation and total acoustic pressure response [dB]

Fig. 12. (a) Unsteady aerodynamics pressure distribution \( (C_p) \), and (b) Mode shape of the wing structure when flutter occurs

Rigorous consideration has been given to the acoustic scattering problem. by solving Eq. (32) as a stability equation in a "unified treatment". Here the disturbance acoustic pressure already incorporates the total pressure, which has been “tuned” to behave like the aerodynamic terms in the modal Eq.(46).

Fig 13. (a) Damping and (b) frequency diagram for BAH wing calculated using V-g method written in MATLAB® for the acousto-aeroelastic problem (the total acoustic pressure already incorporates the scattering pressure). (c) enhanced V-g curve at flutter.
The solution is illustrated in Fig. 13. The V-g diagrams are drawn for pure aeroelastic as well as for acousto-aeroelastic cases. The figure illustrates the influence of the acoustic disturbance. To look into various characteristics of this disturbance, a parametric study has been carried out. The influence of the acoustic monopole source position across the wing is shown in Figs. 14 (a)-(c) obtained by considering only the direct effect or “hydrodynamic part” of the acoustic pressure.

Fig. 15. The Influence of Acoustic Monopole Source on flutter velocity as a function of Monopole vertical position above the wing at wing-root section along the chord

The acoustic monopole source strength was varied. The figures indicate that the acoustic source frequency influences the flutter inception, and the influence is more pronounced as the acoustic monopole source is located near the wing tip trailing edge. This result confirms that obtained by Huang [9]. The influence of as expected, the cont intensity on flutter velocity as a function of monopole position over the wing is shown in Fig. 14. The influence of the
vertical location of the acoustic monopole source above the wing is shown in Fig. 15, while of the frequency of the acoustic source is shown in Fig. 16. Fig. 17 illustrates the comparison of the acoustic disturbance power to that of the fluid in the vicinity of the flutter inception.

8. Concluding Remarks

The viability of the method in dealing with acousto-aero-elastomechanic problem is demonstrated by application of the method for the combined acoustic excitation and unsteady aerodynamic load in the flutter stability calculation. The results shown have indicated that the set of equations (32-34), which are formulated in a unified fashion is efficient and accurate. Existing enabling approaches, i.e. the DPM for the unsteady aerodynamic load computation, the V-g method for solving the flutter stability problem, and mathematical formulation to incorporate the acoustic effects in the same group of terms as the unsteady aerodynamic effects, have been incorporated. Here the acoustic term, analogous to the aerodynamic term, is treated as the “motion induced” forces. For the acoustic term this is commensurate to the acoustic scattering part of the acoustic pressure disturbances.

The computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of a structure has been developed using a unified treatment and acoustic-aerodynamic analogy. By considering the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastodynamic problem) to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, the scattering acoustic pressure can be grouped together in the aerodynamic term of the aeroelastic equation. By tuning the incident acoustic pressure, it can also be incorporated along with the scattering acoustic term, forming the acousto-aeroelastic stability equation. For this purpose the problem domain has been defined to consist of those subject to acoustic pressure only and that subject to acoustic structural coupling, which is treated as acousto-aeroelastic equation. Using BE and FE as appropriate, an integrated formulation is then obtained as given by the governing Eq. (32), which relates all the combined forces acting on the structure to the displacement vector of the structure. The solution of Eq. (32) – and after using modal approach in structural dynamics, Eq.(41) - can be obtained by solving it as a stability equation in a “unified treatment”. The disturbance acoustic pressure already incorporates the total pressure (incident plus scattering pressure), which has been “tuned” to behave like the aerodynamic terms in the modal Eq.(41). Such approach allows the application of the solution of the acousto-aeroelastic stability equation in the frequency domain using V-g method. Such technique forms the first generic approach to solve Eq.(32). Alternatively, the acousto-aeroelastic equation part can also be treated as a dynamic response problem, which forms the second generic approach and which has been dealt with in [14].

The method developed has been demonstrated to be capable of solving the acoustic-aero-elastomechanic coupling problem. Specifically, the results of the example worked out show the influence of various parameters related to the disturbance acoustic source to the inception of flutter. At the present stage, only the direct, or hydrodynamic effects, of the acoustic disturbance has been computed. However, these results indicate that the most effective location of the disturbance for delaying the inception of flutter is at the wing
tip section trailing edge, confirming the results obtained by previous investigators.

The results thus far obtained which only take into account the direct contribution of the acoustic pressure disturbance to the acousto-aerelastic stability of the structure have served as a preliminary indication that further work which take into account the other two indirect contributions of the acoustic pressure field on the flow field could produce more significant results.

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