Abstract
The weight of objects which are placed into low earth orbit should be closely managed historically a launch vehicles lift capability is fixed, so designers have paid close attention to the weight of satellite subsystems to optimize the payload mass. Also due to applications of these structures, their performance against the vibrations is important.

Simultaneous reduction of weight and band averaged vibration transmission in a satellite boom structure is the major task of the present study. This study approached the design problem by sizing periodic structure and geometric parameters to optimize the weight and system performance in a broadband-frequency region using genetic algorithm (GA).

1 Introduction
Because of the costly process to launch objects to the space and the direct dependence between cost and weight, in this study, weight of the structure is assumed to represent the cost.

Large space structures have received considerable attention in dynamics and control literature [1, 2, 3, 4]. The proposed applications are many, spanning diverse areas such as solar energy collectors, solar sails, large astronomical telescopes communication antennas, and space station structures [5, 3, 6].

A typical feature of these designs is the fact that the nominal structure has cyclic symmetry, or some form of periodicity. Periodic structures have some remarkable dynamical properties, which have been discovered and rediscovered in several diverse areas of engineering and solid state physics. It is well-known that the normal modes of vibration of such structures are extended; that is, the standing wave comprising the mode extends throughout the structure.

Large flexible space structures (LFSS) cause weight problem and occupy a large space. So they should be designed to satisfy weight and volume limitations in satellite or careers. Such structures are constructed from many members and joints. To keep the members together in a rigid lattice for structural mission, the joints should have strength enough, but this increases the weight. So, to minimize the weight, the structures are relatively flexible and require sophisticated control systems if accurate shape or directional control must be maintained [1, 7].

Periodic structures have been shown to be sensitive to certain types of periodicity-breaking disorder or imperfections, resulting in a phenomenon known as mode localization or confinement. Structures consisting of a large number of weakly coupled substructures with high modal densities belong to this class [1, 7].

In fact it is possible that the concept of robust controls needs to be redefined for structures where mode localization occurs. Designing disorder into a structure in order to reduce vibration transmission could be a passive vibration isolation mechanism [4, 8]. Vibration isolation is achieved here via energy reflection as opposed to energy dissipation [8].

Recently, much effort has been expended on so-called active vibration control measures for use in this area [4, 6]. However, they are inevitably expensive to install and maintain. Applying the active method requires the
electronic equipments, computer and suitable actuators. Therefore, this method causes a penalty on the weight and the cost. Passive solutions would be preferable if they could be found [8, 4, 6].

A typical solution to reduce vibration in structures is to use viscoelastic materials. Their damping characteristic is considerable but because of weight problems, their applications in space structures are strictly limited.

Several requirements in practical engineering problems lead to the multidisciplinary optimization as a useful solution. In multidisciplinary design optimization, different methods are proposed to consider the interference among objective functions [9, 10]. For example, taking an objective as main objective and put the others as constraints or constructing a unique objective function by multiple or sum based method. A major difference between multi-objective and single optimization is the number of optimal solutions, which in a multi-objective problem there might be several optimum solutions [9].

Genetic algorithm (GA) is known as a powerful tool for unconstrained optimization problems [10, 11, 12, 13, 14]. In this work, geometrics variables would be assumed as design variables. The approach is to change the configuration of structure till the optimizer achieves a structure with high performance. It is possible by a large number of geometric variables to control the shape of structure. These types of design variables, make a large nonconvex design space. Also optimization problems of the sort posed here are characterized by having many variables, highly non-linear relationships between the variables and the objective function and an objective function that has many peaks and troughs [15, 16]. In short they are difficult to deal with the search for methods that can cope with such problems has led to the subject of evolutionary computation [15]. Therefore the old optimization methods would not be recommended for these types of problems and evolutionary algorithms based on statistical principles are more useful.

2 Multidisciplinary Optimization Problem

The aim of this study is set as simultaneous reduction of weight and frequency averaged response of the end beam in the range 150-250 HZ as a multidisciplinary design optimization. The two objectives are composed using weighted sum method. Each objective is a function of geometric variables and is quite nonlinear. Furthermore, because many modes contribute to the response in a broadband-frequency region, it becomes impossible to arrive at general design rules for achieving reduction in the vibration levels [15, 7, 10]. This is the fundamental premise of the design approach developed by Keane, where a formal optimization technique was used to determine the magnitude and precise nature of disorder that enables a structure to filter vibrations across a 100HZ bandwidth [7].

The G.A. used here is fairly typical of those discussed in the well-known book by Goldberg [11] but encompasses a number of new ideas that are particularly suited to engineering design problems. The multidisciplinary design problem was a constrained one and some penalty functions were applied to solve problem using G.A: i.e., but finally only a penalty function to minimize the cost of calculations was assigned for next runs.

In a certain case which only the vibration transmission is objective function, the optimum result would be compared with the results published in [15]. Then the weight of the structure is added to the first objective and the whole optimization process is repeated till the optimizer converges to the optimum solutions.

2.1 Structure

A simple model of the main structure is two dimensional and the members are connected in a regular pattern (Fig. 1). The defined mission for the structure is to support antenna, camera and sensitive sensors in a satellite or space station. Because of the considerable length and weak joints between members, it belongs to the large flexible space structures category (LFSSs) and the vibration response and deflection, both are the performance parameters in its mission.
Random vibration has a special property which excites the structure in all natural modes, not in a single frequency and several resonances would be occur simultaneously. Also, as mentioned due to their high modal densities, the vibrations might be catastrophic and cause destabilizing effects. The periodicity of the structure causes the mode to be extended through structure and the magnitude of deformation would be increased. The space truss in Fig. 1 is constrained at the end which the joints (0,0) and(1,0) are taken to be pinned to ground; all other joints are free to move in X-Y plane. The beams are all either 1m or 1.414m long. All members are made of Aluminum 2024. Other physical and mechanical properties are listed below:

\[ EA=69.67\text{MN} \]
\[ EI=2.86\text{MNm}^2 \]

Mass per unit length: \( m = 2.74\text{kg} / \text{m} \)

It is excited by a point transverse force halfway between (0,0) and (1,0) and. The damping of the structure is fixed so that the normal modes of the uncoupled beam elements all have a constant bandwidth of 20s\(^{-1}\).

A finite element model (FEM) for this structure is developed using Euler-Bernoulli beam theory and a consistent mass matrix formulation. A proportional damping model is in the form of Eq.1.

\[
[C] = \alpha[K] + \beta[M]
\]

(1)

is chosen, where \( \alpha = 0.0 \) and \( \beta = 20.0 \) to reflect a lightly mass proportional, damped structure.

2.2 Formulation

One of the objective of design problem considered here and other studies [15], is vibration response (Vibration Transmission) at the end beam of structure. This function is defined as Eq. 2.

\[
J = \int_{150\text{HZ}}^{250\text{HZ}} \sum_{j=10}^{11} \sqrt{v_{jx}^2 + v_{jy}^2} df
\]

(2)

The frequency response of initial structure is observed in Fig. 2. Many peaks and troughs are seen in the range of 150-250Hz, which remark the high modal density of the structure in this range.

Fig. 2. Frequency Response of Initial Structure in 0-350HZ.

The mission of structure implies that the vibrational response be reduced in the broadband-frequency region(100Hz bandwidth).
As mentioned earlier, simultaneous reduction of weight and vibration transmission is the main objective in this study. The two initial functions (i.e. weight and vibration transmission) were composed using weighted sum method and GA was applied, but GA always seeks the optimum design in a maximization problem, so our minimization problem should be adapted. It is shown in Fig. 3. Designer should determine the relative importance of two objectives with proper $C_1$ and $C_2$ coefficients. A concise form of formulations are in Eq. 3.

$$\begin{align*}
\text{Max} \rightarrow \varphi(x) &= 3 - (C_1 \overline{W} + C_2 \overline{J}) \\
\text{Subject to:} \overline{W} &\leq W_i \\
\overline{J} &\leq J_i \\
\overline{W} &= \frac{W}{W_{\text{max}}} = \frac{W}{231.2} \quad \overline{J} = \frac{J}{J_{\text{max}}} = 0.006 \\
C_1 + C_2 &= 1
\end{align*}$$

Where $\varphi(x)$ is the objective function in maximization form of optimization problem. $\overline{W}$ and $\overline{J}$ are non-dimensional forms of weight and vibration transmission, respectively. The optimum solution must be better than the initial structure in both weight and vibration characteristic. Using penalty functions, these constraints were considered. An optimality function represented by $F(x)$, is defined by Eq. 4.

$$\begin{align*}
F(x) &= \varphi(x) - \alpha P(x) \\
P(x) &= k_1 (\overline{W} - 1) + k_2 (\overline{J} - 1)
\end{align*}$$

Where $\alpha$: Penalty Factor
$P(x)$: Penalty Function

Also a relation to express vibration transmission in “decibel” unit is in Eq. (5).

$$VTR (dB) = 20 \log \frac{J_{\text{Initial Structure}}}{J_{\text{Optimized Structure}}}$$

### 2.3 GA parameters and design variables

In optimization process, design variables are $x$ and $y$ coordinates of middle joints. In Fig. 1, all of the joints within the structure would be kept within fixed distances from their original positions. This ensures that no beam is too long or short and also restricts the overall envelope of the structure.

The main parameters used to control the method of optimization may be summarized as follows:

- $N_{\text{gen}}$: the number of generations allowed to stop the process;
- $N_{\text{pop}}$: the initial population size or number of trials used to start the process;
- $N_{\text{mpop}}$: the number of middle population after applying the Genetic operators on initial population.

$P[$mutation$]$: The proportion of the new generations’ genetic material that is randomly changed.

$P[$crossover$]$: The proportion of the surviving population that are allowed to breed.

$P[$elitist$]$: The proportion of the current population that have high order optimality and would be entered to next generation.

The values assigned to these parameters are as below and during several optimization processes, would not be changed.

<table>
<thead>
<tr>
<th>GA operators</th>
<th>GA parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P[$mutation1$]$: 100%</td>
<td>$N_{\text{gen}}$</td>
</tr>
<tr>
<td>$P[$mutation2$]$: 100%</td>
<td>$N_{\text{pop}}$</td>
</tr>
<tr>
<td>$P[$crossover1$]$: 60%</td>
<td>$N_{\text{mpop}}$</td>
</tr>
<tr>
<td>$P[$crossover2$]$: 60%</td>
<td></td>
</tr>
<tr>
<td>$P[$elitist$]$: 2%</td>
<td></td>
</tr>
</tbody>
</table>

Because of the mission of structure, it should work in an environment which induces vibrations, the portion of vibration transmission in Eq. 3 is assigned by designer equals to $70\% (C_2 = 0.7)$ and for weight this portion is $30\% (C_1 = 0.3)$. Also, in one stage, to compare this study with other works, $C_1$ value would set as zero number. ($C_1 = 0.0, C_2 = 1.0$).

Two different penalty factors are used in optimization process, one is death penalty factor (DPF) and the other is linear dynamic penalty factor (LDPF). DPF removes infeasible chromosomes and they wouldn’t be alive in next generation, but LDPF decreases the values of
optimality of infeasible solutions and never eliminates them. During calculations, penalty factors are defined as Eq. 6.

\[
DPF : \alpha = 1000 \\
LDPF: \alpha = \frac{\text{number of generations}}{20}
\]

The length of structure is kept constant as a constraint and the end beam length in right hand side would be unchanged.

3 Results of Optimization

3.1 Simultaneous Reduction of Weight and Vibration Transmission (Non-Zero Weight Factors) in a Broad band Frequency Region (150-250HZ), C1=0.3 and C2=0.7.

3.1.1 Limits of ±30% on All Joint Positions.

The limits of ±30% are applied on joints coordinates and the coefficients C1 and C2 are assigned as: C1 = 0.3, C2 = 0.7.

In this part, multidisciplinary optimization was performed using the two mentioned penalty factors.

1. LDPF: The process was continued till 90 generations. The limits of ±30% was applied for 36 x and y variables within the structure. The optimized configuration is shown in Fig. 4 and the vibrational behavior of optimized structure in the range 150-250 HZ in comparison with the initial structure is shown in Fig. 5. The band averaged frequency response for 100 HZ bandwidth is 

\[J = 0.44 \times 10^{-3} \text{ m/s}^2\] (for unit forcing), which by using Eq. 5 the filtering capability was improved as 35.6dB. The weight reduction was 8% of initial structure. The convergence diagram is illustrated in Fig. 7.

2. DPF: The death penalty function was set as \(\alpha = 1000\) in 90 generations. In Fig. 5 vibrational behavior of initial structure and optimized structure are compared, which the improvement in whole 100HZ frequency bandwidth is observed. Averaged frequency response for 100HZ bandwidth is 

\[J = 1.65 \times 10^{-4} \text{ m/s}^2\] and its filtering capability is improved as 24dB and weight reduction is 3.23%.

Comparison between the results of two penalty factors for limits of ±30% states that convergence rate is more rapidly for LDPF, therefore its optimum solution is obtained in a few generation numbers (Fig. 7). There are many researches discussed about this subject. Ghiasi and Abedian [17] have investigated the effects of composite structures and they verified the effect of DPF in decelerating the convergence rate, so the cost of calculations and run time would be increased.

DPF make disturbance in results distribution through generations and it’s observed that the final optimum results are not similar in several runs of GA program.

In other words, results are not converged to unique solution, this fault originates from eliminating the infeasible solutions in each generation. Therefore by applying DPF in optimization problem, GA has to find the chromosome which has appeared as feasible patterns in first generation. So, it concludes that, LDPF would reduce the cost of calculations in our optimization problem and in continue it would be used in optimization processes.
3.1.2. Limits of ±10% and ±25% on All Joint Positions.

Multidisciplinary design optimization is performed for limits of ±10%, ±25% and using LDPF. In the case of ±10%, the configuration of optimal solution is shown in Fig. 10. Averaged frequency response for 100HZ bandwidth is J=2.89×10⁻⁴ m/s² which in comparison with initial structure, the improvement of vibrational behavior is 19.3dB and weight reduction is 2.55%. Fig. 8 illustrates the frequency response of optimal solution and Fig. 7 shows the convergence process versus generation number.

For limits of ±25%, the optimization process converged to an optimal solution, illustrated in Fig. 7. Vibration transmission of optimal structure is compared with initial structure in the range 150-250 HZ (Fig. 8). This run gave rise to the configuration shown in Fig.

11. Filtering capability is improved as 31.2dB and weight reduction is 4.48%.

3.2 Minimization of Vibration Transmission (C₁=0 C₂=1) in 150-250HZ Frequency Bandwidth

Assigning the weight factors as C₁=0 and C₂=1 in Eq. 3, gives a constrained single objective function. Limits of ±25% was applied on design variables. The run was attempted with 400 generations, including around 33600 evaluations. The convergence process is illustrated in Fig. 9.

Fig. 12 shows that Vibrational behavior is improved in comparison with initial structure in 150-250 HZ bandwidth. Its configuration is shown in Fig. 13. Band averaged frequency response for 100HZ bandwidth is J=3.966×10⁻⁵ m/s² The filtering capability is improved as 36.75dB and the weight reduction is 2.6%.

Keane [15] optimized the initial structure to achieve a structure with enhanced filtering capability for whole 100HZ bandwidth (150-250HZ). GA as a stochastic method was used and coordinates of x and y for all middle joints were selected as design variables. He set the parameters as shown in Table 2, and achieved an optimum solution with improved vibrational behavior. Optimization has been done in 15 generations with 4500 evaluations.

Analysis the structure during the optimization was done by receptance method. Function of objective which was used, is different from the function of objective used here, but fundamentally both of them are the same. Here using FEM, the magnitude of band averaged frequency response for the range of 150-250 HZ bandwidth is J=4.09×10⁻⁴ m/s². In other words, the filtering capability is increased as 16.2 dB and the weight reduction is 1.2%.
MULTIDISCIPLINARY DESIGN OPTIMIZATION OF A SATELLITE BOOM STRUCTURE TO REDUCE WEIGHT AND VIBRATION TRANSMISSION USING GA.

Figure 7. Convergence Diagram of Multidisciplinary Objective Function ($\phi(f)$) Versus Generation Number for $C_1=0.3$, $C_2=0.7$.

4 Discussion

As explained in section 3.1, linear dynamic penalty factor was recognized as a optimal penalty factor to reduce the calculations during optimization to achieve optimum solutions.

In a same evolutionary generations number (here was set as 90), it was demonstrated that, by increasing the limits of variations for x and y coordinates, the designs with more reductions in weight and vibration transmission in multidisciplinary optimization will be obtained (Figs. 10, 11, 4 7).

As observed in Figs. 4, 10, 11 and 13 for every set of $C_1$ and $C_2$, the weight is reduced in optimum design and vibration filtering capability (vibration transmission) is improved for all 150-250 HZ frequency bandwidth. It must be stressed that the improved vibration isolation characteristics arise from the constructive reflections of travelling waves caused by the discontinuities introduced.

In current work for $C_1=0$ and $C_2=1$, the optimum design was achieved in a process which GA parameters have been set as Table 1. Keane has set these parameters as Table 2. In his work, it seems that convergence to the exact optimum solution has been stopped in 15 generations, so his solution was a local optimum solution.

Table 2. GA Parameters Used by[15]

<table>
<thead>
<tr>
<th>GA operators</th>
<th>GA parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{mutation}}$</td>
<td>5% $N_{g_{\text{max}}}$ 15</td>
</tr>
<tr>
<td>$P_{\text{crossover}}$</td>
<td>80% $N_{c_{\text{max}}}$ 700</td>
</tr>
<tr>
<td>$P_{\text{Elitist}}$</td>
<td>80% $N_{m_{\text{max}}}$ 200</td>
</tr>
<tr>
<td>$P_{\text{invert}}$</td>
<td>50%</td>
</tr>
<tr>
<td>Number of evaluations</td>
<td>4500</td>
</tr>
</tbody>
</table>

Fig. 8. Frequency Response of Optimized Designs for $C_1=0.3$, $C_2=0.7$ with Limits of ±10% and ±25%, Using LDPF.

Fig. 9. Convergence Diagram in Minimization of Vibration Transmission ($C_1=0$) Versus Generation Number for Limits of ±25%.
However, in this work because of the magnitude of improvement in filtering capability and number of calculations in a larger generations number, the conclusion is that our optimum solution has higher filtering capability and is lighter than Keane’s structure.

It might be interesting to know the behavior of each objective function (weight and vibration transmission) during optimization process. In other words, it would be considered that what happens for each object when the composed objective function is minimized through evolutionary generations.

For limits of ±25% and $C_1=0$, the weight and vibration functions for optimum solution

versus generation number are illustrated in Figs. 14 and reveals that as a result of minimization of the vibration transmission as main objective function, the weight function would be minimized consequently.

Fig. 16 demonstrates that both objective functions (weight and vibration transmission) would be minimized during generations, independent of multidisciplinary ($C_1, C_2\neq0$) or single ($C_1=0$) optimization to reduce vibration for whole 100HZ frequency bandwidth.

Considering the weight of structure ($\overline{W}$) in conjunction with vibration transmission($\overline{J}$) (Assigning a nonzero value for $C_1$) accelerates the convergence process to an optimum solution with more reduction in weight function.

The configurations of optimum solutions in Figs. 4, 6, 10, 11 and 13 are so disordered and are not favorable for practical applications. But during the optimization, after 50 generations, for limits of ±25% by weight factors of $C_1=0.3$ and $C_2=0.7$, a local optimum design is observed which its configuration is shown in Fig. 15.

Its pattern is similar to an asymmetric geometry and illustrates a strong overall pattern where the design clearly workable in comparison with other disordered optimum designs. The vibration transmission as shown in Fig. 12, is reduced for all 150-250HZ frequency bandwidth in comparison with the initial structure and the band averaged frequency response is $J=1.867 \times 10^{-4}$ m/s² which means its filtering
capability is increased as 23.04dB and the weight reduction is 4.3%.

![Image of satellite boom structure]

**Fig. 13. Optimized Design for C1=0, C2=1 with Limits of ±25%, Using LDPF, 33600 Evaluations Over 400 Generations (Weight Reduction is 2.6% and filtering capability is increased as 36.75 dB).**

![Graphs showing objective function behavior]

**Fig. 14. Behavior of Each Objective Function During Evolutionary Optimization with the Limits of ±25% for C1=0, C2=1 and 400 Generations.**

**Fig. 15. Local Optimum Design for C1=0.3, C2=0.7 with Limits of ±25%, Using LDPF, 4200 Evaluations Over 50 Generations (Weight Reduction is 4.3% and filtering capability is increased as 23.04 dB).**

**Fig. 16. Behavior of Each Objective Function During Evolutionary Optimization with the Limits of ±25%, for C1=0.3, C2=0.7 and 90 Generations.**

5. Conclusion

In this work, optimization of a space truss to reduce the weight and vibration transmission in a 100HZ bandwidth was performed, using GA. The role of weight function is important for designer to discover its effect on optimization process and to minimize the payload mass.

Some remarkable results are:

The convergence of problem, governed by Eq. 3, to achieve optimum solutions for some values of weight factors (C1 and C2)

1- was demonstrated, In other words, multidisciplinary optimization problem has solution.

2- The results revealed that linear dynamic penalty factor has an accelerating effect on convergence to optimal solutions and reduces the cost of calculations. In contrast the death penalty factor, decreases the convergence rate.

3- Increasing C1 from 0 to 0.3 demonstrates that, convergence to a lighter stricture would
be accelerated and optimum solutions will be obtained in a few generations.

4- It was shown that, when a purely periodic structure deviates from ideal periodicity, the vibration transmission would be decreased dramatically. This phenomenon arises from the changes of natural frequencies distribution in 100HZ bandwidth.

5- Vibration transmission was decreased for all frequencies in a range of 150-250HZ. The magnitudes of reduction of vibration transmission strongly depend on weight factors (C1,C2), limits of variations for coordinates of x and y, penalty factor and number of generations.

6- The optimization process for C1=0 was performed in large number of generations (Ngen=400), which mathematically (Fig. 9) it seems to be the exact solution.

7- From the point of performance for configuration, a workable design was obtained during optimization which has 4.3% weight reduction and 23.04dB filtering capability.

8- The optimum designs (with reduction in weight and vibration transmission) which obtained here, revealed that, GA as a stochastic and modern optimization technique is so powerful to deal with problems with no general design rules for achieving reduction in weight and vibration levels simultaneously.

References


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