Abstract
The aim of this paper is to discuss and define methodologies for reliability analysis of bolted joint elements in composite structures. The performance of a composite bolted joint is affected by uncertainties associated with loads, shimming material and its thickness, bolt hole clearance as well as laminate material and thickness. These uncertainties have traditionally been accounted by the use of safety factors. The safety factors in most designs have a probabilistic basis to ensure that various uncertainties are properly accounted for. However, in cases where they are not covered by the design principle, these safety factors may not be enough and a probabilistic design methodology is required. The development of numerical method based on probabilistic approach to tackle this kind of problem is a very difficult task.

The objectives of this work are to demonstrate methods and codes that simulate the progressive damage in composite bolted joints with shimming material and probabilistically evaluate the effect of design variable uncertainties on structural failure. The methodologies and corresponding computer codes are used to determine the uncertainty in the damage load on bolt joints with shimming material.

Finite element analysis (FEA) is used for the determination of contact stresses in the contact region around the bolt hole of a single lap joint. In particular, degradation of the structural behaviour will be quantified by using a FE method in which the damaged plies are identified. The present method is based on a point and average stress criteria for predicting failure stress and failure modes. Both of these criteria predict net-section, shear-out and bearing failures when the stress components at specific locations reach their corresponding unnotched strength level. The FEA is carried out using ABAQUS, a commercial available FE program.

Furthermore, a probabilistic failure method is used to simulate progressive fracture and the statistical scatter response of the bolted composite joint. In addition probabilistic sensitivity factors are computed to identify parameters that have significant influence on a structural reliability. The design can therefore be improved by controlling distribution parameters associated with the scatter of various variables. This can be implemented during the manufacturing process to obtain maximum benefit with minimum modification.

1 Introduction
Bolted composite joints are widely used in the aerospace industry. In many cases, particularly those that are weight sensitive, composite materials are superior from a performance perspective, but increasingly they must also be cost-effective, compared to their metallic competitors. This can be achieved by manufacturing large and integrated complex shaped components in one step, and thus compensating high raw material costs for low manufacturing costs. A drawback of using large complex shaped geometries is the occurrence of a higher manufacturing tolerance, therefore shim materials are often used between bolted joints.
The phenomena of fracture in bolted composite structures are compounded by the inherent uncertainties (scatter range) in the material properties, geometry and loading of the structure. The objective of the present investigation is to demonstrate methods and commercial code that simulate the initiation and progressive damage in bolted composite joints and probabilistically assess the effect of various variables on the structural failure. To meet this objective, stress analysis of bolted composite joints with shimming material is carried out by both the FE method using ABAQUS [4] and a semi-analytical method [5], which takes progressive fracture into account. Furthermore, a probabilistic approach is used to take into account the sensitivity and the scatter of the most significant parameter on progressive fracture of composite bolted joints. The accuracy and applicability of this approach has been verified with tests data.

2 Computational Simulation of progressive fracture

2.1 Progressive Fracture

The simulation of progressive fracture of bolted composite joints with shimming material is performed to predict behavior of test specimens. The concept governing the simulation of structural fracture is shown in Fig. 1a and Fig. 1b.

2.2 Geometry

The test specimen is rigidly clamped into an all-purpose testing machine. They are fixed into the clamping device so that no initial bending moment is introduced. The load is introduced linearly until structural failure occurs. The test set up is shown in Fig.2.

2.3 FE Model

Implementation of the failure criteria in the simulation of the progressive damage are conducted with the USDFLD user subroutine of ABAQUS. This routine allows access to quantities, i.e. stresses or strains, in the material points (Gauss points) and is executed within the definition of material properties that are regarded as a function of field variables (FIELD).

The FE model is created using ABAQUS. The laminates are modelled with one solid element per ply. The boundary conditions are
shown in Fig. 3. The nodes are held fixed in all three directions in the fixed-end. The free-end is declared as a rigid body and a reference node is assigned to this surface. Thus the motion of the nodes and elements of the selected surface are governed by the motion of the reference node, which is held fixed in 2- and 3-direction, while in 1-direction a force is applied. The titanium bolt is modelled using isotropic material properties.

To determine the strength of a structure the continuum damage mechanics approach is used. The failure process is described by progressive degradation of the material stiffness. Damage is applied to the material if a failure criterion is violated. The well established failure criterion of HASHIN [1] distinguishes for different failure modes. Once a failure mode is detected the stiffness properties are degraded to 10% of their original value.

Calculation of the deformation and the progressive damage is accomplished by running this deterministic FE model. To enable the reliability analysis, this deterministic model must be able to provide the output of interest.

3 Reliability analysis

3.1 Non deterministic approach

In the most general sense, the reliability of a structure is its ability to fulfill its design purpose for a specified duration.

The performance of the structural analysis values the probabilities of certain events defined in terms of random variables and limit-state functions [2]. The bolted composite joint is normally affected and influenced by several uncertainties associated with material properties (i.e. stacking sequence, thickness, laminate material), mechanical geometric properties (i.e. shimming material, bolt-hole clearance) and others i.e. like loads or boundary conditions. In the probabilistic approach, these variables are defined using distributions function [3].

The implementation of the reliability analysis is based on a description of the limit state of interest by a performance or limit-state function $G(x)$ where $x=(x_1,x_2,...x_n)$ is a n-dimensional vector of random variables. Some of these may affect the demand or response of the structure, denoted by $D$, while the others may influence the structure capacity or tolerance $C$ to withstand the demand. The performance function $G$ may be written in terms of $C$ and $D$ as

$$G(x)=C/D.$$ (1)

For reliability analysis purpose the structure can be defined as the entire process, the demand as the structure response and the capacity as the allowable.

The reliability has not been performed if the combination of the random variables (e.g. material properties, shimming, process conditions) results in $G(x) > 0$.

The corresponding probability of such an event, $P(G < 0)$ is called the probability of failure $P_f$.

Conversely, the combination of the random variables resulting in ($G(x) > 0$) will make the structure perform as required and the
corresponding probability, \( P(G(x) > 0) \) is termed the failure and the reliability, \( Pr \).

The limit-state failure surface, \( P(G(x) = 0) \), defines the boundary between failure and survival, as shown in Fig.4 in two-dimensional space.

![Fig.4 Limit state function in two-dimensional space](image)

### 3.2 Uncertainties characterisation

The tests and the simulation have shown that the shim thickness is one of the most critical values for the progressive fracture of a bolted composite joint. In fact the scatter of this variable reflects in the performance and functionality of the component. In general, this unpredictability also makes it difficult for the manufacturer to adjust the structure response and the allowable.

In this paper the structure analysis on a probabilistic approach is modeled to provide a description for the structural response, considering only the variability of the shim thickness and the important variables of the material properties.

The probabilistic description is in general achieved by selecting shim thickness randomly with given parameters and probability distributions. The Gauss probability density function (PDF) and cumulative distribution function (CDF) for the shim thickness are shown in Fig. 5.

The desired statistics of the response quantities, such as the mean, variance and probabilities are evaluated on the basis of the test data.

![Fig. 5 PDF and CDF of the shimming thickness](image)

\[ \mu = 1.5, \sigma = 0.3 \]

### 3.3 Probabilistic analysis of progressive fracture

Considering the failure modes and the reliability analysis defined above the limit-states function assuming the form

\[ G(x) = \frac{C(x_{\text{material}})}{D(x_{\text{shim}})} \] (2)

where \( C \) is a specific allowable for the bolted composite joint and \( D \) is the structural response, which is a function of the parameter “shim thickness”.

The scatter of the shimming on the bolted composite joints is quantified and shown in Fig. 6, where \( F_x \) is the fracture load, \( F_S \) the maximum shear load of the bolt, \( t_i \) the plate thicknesses and \( D_{\text{bolt}} \) the bolt diameter. The results of two test configuration (test A and B) are shown with different shim thicknesses (0.5 mm, 1.5 mm, 2.5 mm and 3.0 mm).

The deterministic analysis is carried out for a series of sample points (a set of random
variables) generated in accordance with the probability distribution.

Consequently, PDF and CDF for the structure response parameters are obtained and compared to the test data.

The structural fracture is simulated by the ABAQUS FE-model and the semi-analytical model using the statistical shim thicknesses as shown in Fig. 5.

The results obtained by the FE method and Hashin’s fiber failure criterion is conservative. Therefore it is assumed that another failure criterion or modification of Hashin’s criterion leads to a better agreement of results from experimental and numerical analysis [6].

The probability distribution of progressive damage as a function of shimming thickness is compared to corresponding distribution functions from test results (s. Fig. 7).

The failure probability \( (P_f < 10^{-9}) \) is calculated as sum of the probability for each Hashin’s failure mode minus the probability associated with the intersections.

4 Summary and Conclusions

The probabilistic method and computer codes for composite bolted joint with shimming have been discussed.

The progressive fracture in composite structural is simulated via ABAQUS as well as an semi-analytical approach. A probabilistic analysis is followed to calculate the scatter of the stochastic shim thickness and the probability failure \( P_f \).

The methodologies and corresponding computer codes have been used to demonstrate the effect of the shim thickness uncertainties on the failure load of a bolted joint.

The scatter of the structural response is probabilistically quantified. The probabilistic failure using FORM approach for the failure modes is calculated on the basis of the scatter of slaming variable and allowable.

The influence of the shim thickness on reliability and Hashin’s failure criterion demonstrated, that the safety factor is conservative. The scatter of shim thickness from tests must be considered in the semi-analytical method.

The proposed method showed that the choice of the dominant variable (shim thickness) and the limit states failure has an impact on the simulation of the tests.

In particular, the degradation of the structural behaviour will be quantified by using FE method and semi-analytical approach. The semi-analytical approach uses a point and average stress criteria for predicting failure stress and failure modes. Both of these criteria predict net-section, shear-out and bearing failures when the stress components at specific locations reach their corresponding unnotched strength level. The performance of the material can be improved using the probabilistic approach to simulate the test data.
As a conclusion the design can therefore be improved by controlling or selecting distribution parameters associated with the scatter of the uncertainly variables. This can be implemented during the manufacturing process to obtain maximum benefit with minimum alterations.

5 References

[4] ABAQUS. Commercial codes

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