

# FITTING FITNESS IN AIRCRAFT DESIGN

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## Abstract

*The design and analysis methods applied in multidisciplinary design and optimization of aircraft are continuously being improved in accuracy and reliability. The related computational complexity easily leads to high costs in terms of time, effort and money, needed for these analyses. In order to limit these costs, meta-models on the basis of fitting methods can be used. This paper presents a study in which various advanced interpolation and approximation techniques and optimization algorithms are applied in a response surface optimization approach for aircraft design problems. The results demonstrate the flexibility and the potential of this approach by tackling various complex design optimization problems at relatively low computational cost.*

## 1 Introduction

The continuous development of the methodologies for aircraft design and analysis is aimed at achieving higher levels of detail in shorter analysis turn around cycles. Moreover, driven by ever increasing technical and commercial requirements due to global competition, more detailed design analyses are being required and applied in earlier phases of the aircraft design where there are still very many degrees of freedom and few restrictions on the design space [1]. The analyses traditionally used in the early phases of the aircraft design are mostly based on semi-

empirical rules [2]. Although computationally efficient, these analyses often have a limited range of validity, accuracy and flexibility. Therefore these methods are gradually being replaced by the more generic ‘geometry and physics based’ detailed design analysis methods that are generally applicable and potentially highly accurate [3]. However, these methods are mostly computationally expensive. Also, the required design analyses in aircraft multidisciplinary design and optimization (MDO) are various and may be difficult to combine into an integrated aircraft design system. Therefore such integrated aircraft design system is usually developed for a specific range of design problems, like optimization of blended-wing-body [4], [5] or transonic transport aircraft planform [6]. Moreover, such integrated aircraft design system often requires specific software (e.g. particular analysis tools) and hardware (e.g. dedicated compute servers), and is therefore prone to operational issues such as temporal unavailability of servers or licenses. The computational cost of (some of) the analyses in the integrated aircraft design system is another issue to be handled, especially when used within automated search or optimization loops that typically may require many design analysis evaluations (e.g. thousands). Approximation and interpolation methods (also known as meta-modeling or data fitting) have been proposed and have shown to effectively deal with such issues [7], [8], [9] by providing compact,

accurate and computationally efficient representations of the considered properties of the underlying aircraft design (in optimization context also termed as design objectives or fitnesses). The key of this approach lies in the de-coupling of, on the one hand the computationally expensive integrated aircraft design analyses, and on the other hand the search process by automated optimization algorithms. The search process now makes use of the compact and computationally efficient meta-model and allows for high flexibility for further investigations. For the creation of the meta-model many different fitting methods are available (e.g. [7]), each with different advantages for different types of problems.

In the present study a number of different fitting methods are applied to aircraft design problems. For effective sampling of the multi-dimensional design domain, use is made of Design Of Experiments (DOE) methods. In the selected sample points the design objectives and constraints are evaluated by parallel computations with integrated aircraft design analysis systems. NLR's multi-dimensional and multi-method data fitting tool MultiFit [10] is used to statistically analyze the data sets that result from the design evaluations and to generate meta-models using different fitting methods. The representativeness of the meta-models is investigated and the most suitable meta-models are applied in the aircraft design process where several optimization algorithms are used to find the most promising aircraft designs.

In this paper the methodology for obtaining and assessing the meta-models is described. For illustration of the approach and its benefits, two complementary aircraft design cases will be described, in which the meta-modeling approach is applied in the multi-disciplinary design and optimization of aircraft wings. The first one concerns a wing planform design study for single objective optimization of mission range, which is part of an MDO case study of a generalized transonic wing design that is currently ongoing in the European project Vivace [11]. The second one concerns a generalized transport aircraft winglet design

study by multi-objective optimization of wing drag and bending moment.

## 2 The MultiFit response surface approach

In aircraft design problems the aim is to improve or optimize the characteristics (design objectives such as performance, behavior, etc.) of the product by variation of its properties (design parameters such as shape, material, etc.). In general the product's properties ( $\mathbf{x}$ ) and characteristics ( $\mathbf{y}$ ) are expressed as real-valued (continuous) quantities and their inter-dependency ( $f$ ) is non-linear ( $\mathbf{y}=\mathbf{f}(\mathbf{x})$ ). Evaluation of the function  $f$  is often costly (in terms of time and computer resources) and may involve (iterative) computational analyses (e.g. finite element or computational fluid dynamics). Because of these computational complications it is desirable to retrieve efficiently, i.e. using as few as possible function evaluations, the desired product's characteristics ( $\mathbf{y}$ ) in the considered design domain (i.e. for the set of allowable values of the product's properties  $\mathbf{x}$ ). This may be achieved, for example, by direct optimization of  $\mathbf{y}$  for  $\mathbf{x}$  using efficient gradient based optimization algorithms (e.g., [12]). However, lack of accurate gradient information ( $d\mathbf{y}/d\mathbf{x}$ ), limited robustness and reliability of the computational analyses, or convergence into local sub-optima, may hamper the effectiveness of this approach. It is therefore beneficial to also apply 'gradient-free' global search methods, such as genetic algorithms and pattern search (e.g., [13]), besides the gradient based optimization algorithms. The large number of evaluations of the objective function ( $\mathbf{y}=\mathbf{f}(\mathbf{x})$ ) that are typically needed by these search methods do not allow for a high computational cost per evaluation. An efficient approximate representation ( $\mathbf{y}^*=\mathbf{f}^*(\mathbf{x})$ ), or meta-model, of the design problem is therefore required. A variety of fitting methods, such as polynomial regression, neural networks, and kriging models, are available for creating such meta-models [7] from sampled data sets ( $\mathbf{x}_i, \mathbf{y}_i$ ) of the design problem. In order to achieve an optimal meta-model the most suitable fitting method for

the considered design problem should be applied. There exist various statistical verification and cross validation methods [14], [15] by which the quality (or ‘representativeness’) of the different meta-models can be assessed and the most suitable method can be identified. These methods consider a (small) subset of the data set as so-called verification points, in which the error of the prediction ( $y-y^*$ ) is evaluated for a fit that is made for the data set without the verification points. The NLR fitting tool MultiFit supports user-friendly creation, assessment and comparison of fits with a wide range of multi-dimensional interpolation and approximation methods [10]. Suitable meta-models can be easily created and can be conveniently used in the further evaluation and optimization of the considered design problem.

### 3 The aircraft wing design case studies

#### 3.1 Transonic wing MDO

The wing design process is part of the Vivace wing MDO case study, which is based on a parametric aircraft wing model (Fig. 1).

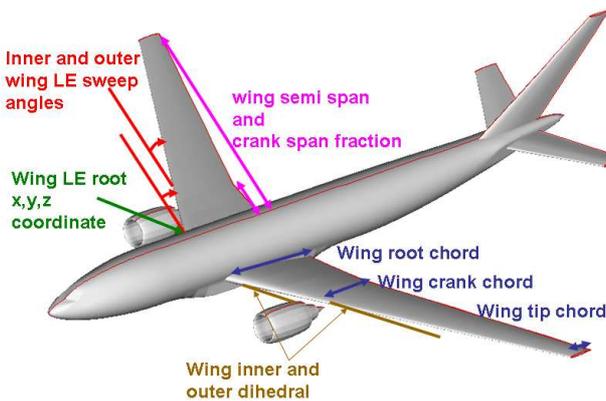


Fig. 1: The top level design parameters used in the Vivace transonic wing MDO case study.

The wing design process comprises the following consistently coupled computational analyses: geometry- and multi-model-generation, low-speed aerodynamics, engine sizing, weight book-keeping, structural

optimization, transonic aerodynamics and mission evaluation, see Fig. 2.

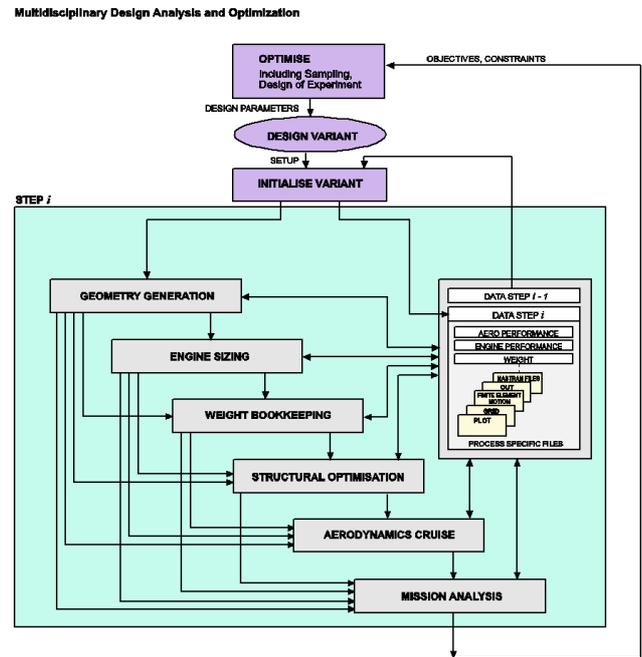


Fig. 2: Transonic wing multi-disciplinary design analysis process as used in Vivace [6].

Because the wing behavior can only be properly evaluated when taking into account the aircraft that it is part of, these wing design analyses do include other parts of the aircraft if relevant (e.g. besides the wing, also the fuselage is part of the model for the CFD cruise performance evaluation). Further details of these analyses are presented elsewhere [6] and are beyond the scope of this paper.

These analyses lead to a collection of results that represent the performance of the considered wing. One of these results, the maximum mission range for fixed take-off weight, is selected in this study as the objective for the design process. This design objective can be improved or optimized in a global optimization loop, as also is indicated in Fig. 2. However, in this study the objective is not applied directly in the optimization loop. Instead, first a set of analysis results is generated in a number of design points that were selected according to a full-factorial DOE sampling of the considered design space. For simplicity, only two of the top-level wing planform design parameters (Fig. 1) are

considered in this study: wing semi-span and outer-wing leading-edge sweep angle. From the resulting data set of analyses results, i.e. range versus semi-span and sweep, a series of meta-models (or: response surface fits) are generated with the MultiFit tool. The ‘best fits’ are selected on the basis of detailed assessment and comparison of the fits in MultiFit. Local predictive quality of the fits in the most promising region of the design domain is assessed by evaluation of the fit errors (or residuals) in subsequently two sets of verification points, i.e., in the one single data point and in the five data points with the best range values, respectively. Global predictive quality of the fits is assessed by a so-called leave-one-out cross-validation assessment, where a fit is made on the whole data set except one verification point in which the residual is evaluated, which is repeated for each of the data points. The Root Mean Squared Error (RMSE) of each of these residuals (fit errors) is considered as the global inaccuracy measure for the fit of the whole data set. The results of these assessments are shown in Table 1.

Table 1: The MultiFit assessment results: RMSE comparison for each of the methods for the three verification procedures: in the left and middle columns: RMSEs of the residual in the one and five verification points, respectively; right column: averaged RMSEs of the leave-one-out cross-validation assessment.

	Veri. 1 d1		Veri. 1 d1		Veri 1 d1
krigingcG	19.68	krigingcG	130.6	krigingcG	26.0674
krigingcE	5.194	krigingcE	50.42	krigingcE	5.3258
krigingcC	12.04	krigingcC	51.49	krigingcC	7.1808
kriginglG	18.82	kriginglG	66.98	kriginglG	25.9366
kriginglE	5.491	kriginglE	46.72	kriginglE	5.4088
kriginglC	12.64	kriginglC	49.65	kriginglC	13.3012
krigingqG	5.563	krigingqG	14.94	krigingqG	9.3439
krigingqE	4.917	krigingqE	23.15	krigingqE	5.2299
krigingqC	2.446	krigingqC	10.66	krigingqC	7.2103
rbf	14.12	rbf	369.8	rbf	106.2705
bary	51.99	bary	70.75	bary	42.6464
poly0	480.4	poly0	496.7	poly0	369.1996
poly1	88.44	poly1	136.1	poly1	152.6337
poly2	47.96	poly2	51.92	poly2	22.9701
poly3	14.27	poly3	19.31	poly3	13.7043
poly4	4.492	poly4	20.59	poly4	4.5684
poly5	6.192	poly5	59.51	poly5	4.309
poly6	8.149	poly6	219.7	poly6	6.526
ann	113.5	ann	104.7	ann	323.6689
rbfopt	23.19	rbfopt	87.57	rbfopt	220.3266

The MultiFit assessment indicates that locally, in the most promising region of the

design domain, the kriging-quadratic-cubic [16] fit (Kriging-qc) has the best predictive quality. The best global fit quality in the complete design domain, as evaluated by the leave-one-out assessment, is found with the 5<sup>th</sup> order polynomial fit (poly5). However, the MultiFit assessment also gave warnings that 5<sup>th</sup> and 6<sup>th</sup> order polynomial fits are unreliable due to the ill-conditioned least-squares matrix equation from which the polynomial coefficients were resolved. Therefore the 4<sup>th</sup> order polynomial fit (poly4) was selected as the most suitable fit for the design optimization analysis, as well as the kriging-qc fit. For reference, also the commonly used 2<sup>nd</sup> order polynomial fit (poly2) was applied in the optimization analysis in order to assess the benefits of the different fitting methods. The surfaces of these three fits are shown in Fig. 3. Note that the Poly2 fit is quite similar to the Kriging-qc fit and therefore not separately visible.

The optimum value for the range was then determined for each of the three fit functions, in the bounded design domain as indicated in the plot of Fig. 3 (28 m < span < 32 m ; 20 deg < sweep < 40 deg). These optimizations are carried out using several optimization algorithms: Matlab's constrained single-objective optimization algorithms FMINCON (Optimization Toolbox) [12] and GA (GADS Toolbox) [13], and MNSGA, an in-house developed Matlab implementation of a constrained multi-objective non-dominated sorting algorithm [17]. The gradient-based search algorithm FMINCON has a risk of converging into local optimum design points, depending on the starting point used in the optimization. Therefore this optimization is run several times, each time starting from one of the 35 design points of the data set. The GA and MNSGA algorithms are both run with these 35 design points as initial population, and with a maximum of 100 generations. Each of these three optimization algorithms finds the same optimum range value for each of the fit functions; see Fig. 3. These three optima found are slightly different from each other, indicating that in this region of the design domain the local behavior of the three fit functions is different.

Also the three optimum design points have been evaluated by the multi-disciplinary design analysis (Fig. 2), yielding the accurate range values (verification results) in these design points. These verification results are also compared to the predictions of the each of the fit functions for additional verification. The results are summarized in Table 2.

Table 2: Results of the design optimization.

Design point	Range-Analysis-verification	Range-poly2-prediction	Range-poly4-prediction	Range-krigqc-prediction
Poly2 optimum: Span: 30.5855 Sweep: 23.0082	6015.2 nm	5969.2 nm ( $\Delta=46.0$ nm)	6018.8 nm ( $\Delta=-3.6$ nm)	6005.9 nm ( $\Delta=9.3$ nm)
Poly4 optimum: Span: 30.6848 Sweep: 23.2703	6022.5 nm	5968.2 nm ( $\Delta=54.3$ nm)	6023.2 nm ( $\Delta=-0.7$ nm)	6000.3 nm ( $\Delta=22.2$ nm)
Krigqc optimum: Span: 30.5266 Sweep: 23.6278	6008.8 nm	5967.5 nm ( $\Delta=41.3$ nm)	6015.1 nm ( $\Delta=-6.3$ nm)	6009.2 nm ( $\Delta=-0.4$ nm)
RMSE:		47.5 nm	4.2 nm	13.9 nm

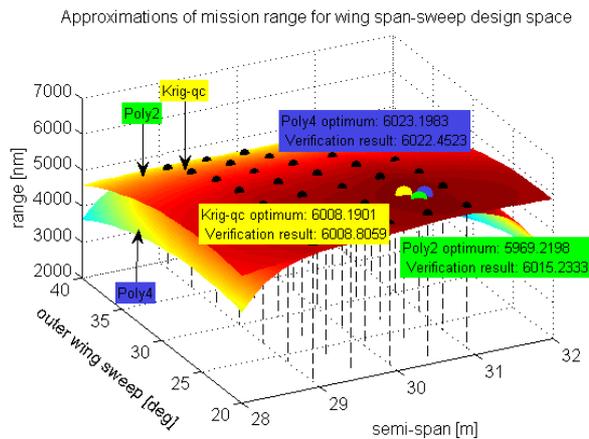


Fig. 3: The three selected response surfaces of range versus wing span and sweep angle; the color of the surface corresponds to the range value, which is also represented by the vertical axis. The black dots represent the 35 points of the data set on which the fit functions are based. The optimum range values that were found for these three response surfaces are indicated.

From these results it can be concluded that the Poly4 response surface provides the best results for this design case: the optimal design point with the best range value (6023.2 nm) is found with this fit, and was quite well confirmed (6022.5 nm) by the verification analysis performed in that design point. Also the

average accuracy of this fit in the three verification points (Table 2) is the highest (for the residuals in these points: RMSE=4.2 nm). The kriging-qc fit is slightly less accurate in these verification points (RMSE=13.9 nm), and the poly2 fit is relatively in-accurate (RMSE=47.5 nm). Because of this relatively large local fitting error of the poly2 response surface, the high range value (6015.2 nm) in the optimum design point found with this fit should be considered as a coincidence.

The multi-disciplinary design analysis computations (Fig. 2) were performed with dedicated simulation tools on specific computer architectures (i86 processor running Linux 2.6.9, and MIPS R14000 500MHz processor running SGI-Irix 6.5), and required about 1500 seconds computation time for a complete evaluation of one design point. Moreover, because of the different computers involved in the sequence of analyses, the proper data management and scheduling of the computation jobs is somewhat intricate. In contrast, the evaluation of the response surface prediction of the range value is merely a push-button operation in the Matlab environment that can be conveniently called from other programs such as optimization functions, takes only sub-second computation time and is efficiently vectorized such that even thousands of evaluations can be easily evaluated within one second on a standard PC (Pentium-4 - 2 GHz, WinXP).

### 3.2 Multi-objective optimization of winglets

To demonstrate both the power and the flexibility of the response surface approach presented in this paper, this section deals with a more extensive design problem in a similar way as the previous transonic wing design problem. While, for the sake of simplicity, the previous case was limited to only two design parameters and one objective, this second design case concerns a generalized transport aircraft winglet design study, involving 9 geometric design parameters (Fig. 4) and 3 separate objective functions that should be minimized simultaneously. These objectives are based on the difference between the behavior of the

aircraft with - and without winglets. The objective functions represent the drag coefficient increments ( $\Delta C_{D1}$  and  $\Delta C_{D2}$  respectively) due to the winglets for two different points in the flight envelope, and the bending moment increment ( $\Delta M_{X2}$ ) due to the winglets in the second aforementioned flight envelope point. For efficiency of representation, in the following text the 3 objectives  $\Delta C_{D1}$ ,  $\Delta C_{D2}$  and  $\Delta M_{X2}$  will be denoted as  $y_1$ ,  $y_2$ ,  $y_3$ , respectively, and the 9 design parameters as given in Fig. 4 will be denoted as  $x_1$  to  $x_9$ .

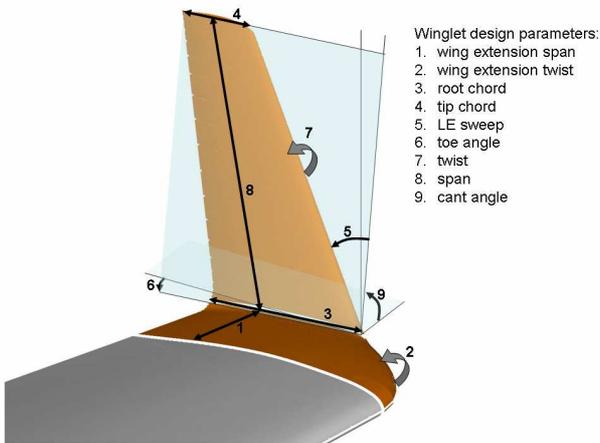


Fig. 4: The 9 geometric winglet design parameters.

Just like in the previous wing design case, the approach is to sample the considered design space of the winglet, which is spanned by the 9 design variables. An important difference here, however, is the high dimension of this design space. A full-factorial sampling in this case would lead to a prohibitively large number of design points. For example 3 values for each parameter would result in  $3^9 = 19683$  design points. Considering that each design analysis would take about 10000 seconds, and a maximum of 4 analyses could be run in parallel (due to hardware, software and license limitations) it would require at least 49 million seconds (nearly 14000 hours) of throughput time. Therefore a much coarser sampling is applied, using a ‘space-filling’ latin-hypercube method (Matlab’s LHSDESIGN function). A first set of 126 design points was created, which was aimed at having ample data points available for creating at least the (9 dimensional)  $2^{nd}$  order polynomial fit (having 55 coefficients).

These design points were submitted to the design analysis process, which consists of geometry generation, static aerodynamic loads, aero-elastic analysis and atmospheric turbulence loads (PSD) (Fig. 5).

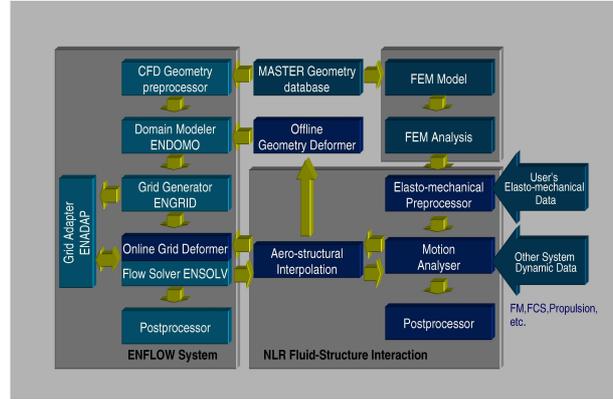


Fig. 5: The aerodynamic and aero-elastic analysis process for the winglet design study.

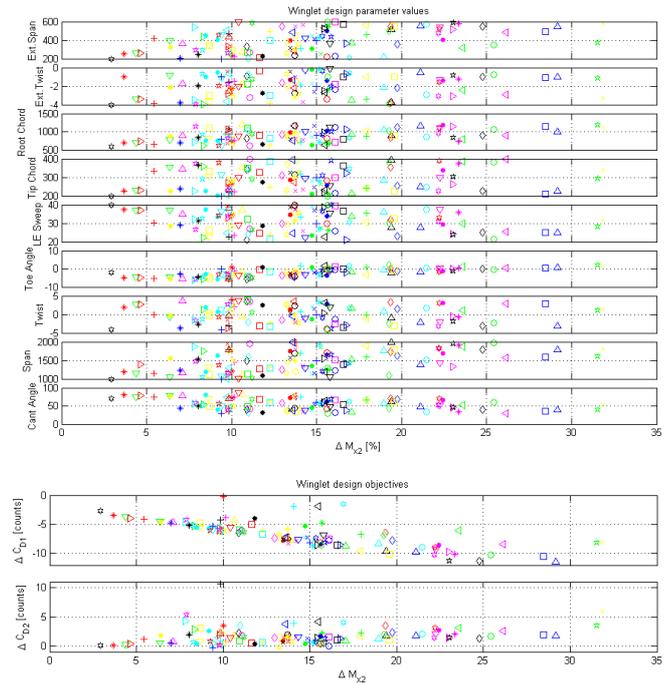


Fig. 6: The 104 design points and their analysis results as contained in the data set. The upper 9 graphs give the 9 design parameter values for each of these 104 design points. The lower two graphs give the two drag objective functions ( $\Delta C_{D1}$  and  $\Delta C_{D2}$ ) on the vertical axis. The third objective ( $\Delta M_{X2}$ ) is used as horizontal axis in each of the 11 graphs. Note that each design point is represented in each of the graphs by the same marker symbol.

From the analyses results the objective function values were derived for each of the design points, where it should be noted that 22 out of the 126 analysis runs failed due to numerical simulation problems, resulting in 104 successfully evaluated design points. These 104 design points are shown in Fig. 6.

Just like in the previous wing MDO study, also this data set of 104 design results was further processed with the MultiFit tool into the ‘best-fit’ meta-models (or: response surface fits) for each of the three design objectives. From extensive cross-validation assessments, like the ones described for the previous design study, it appeared that the kriging-linear-cubic (Kriging-lc) method provides the most suitable fits for each of the three objectives. These three fits were then applied in several optimization analyses using the same optimization algorithms as in the previous study.

Firstly, a straight-forward multi-objective optimization of the three objectives with the MNSGA program was performed with a population of 1000 individuals and 1000 generations, where the initial population was randomly created. The search domain was bounded to the central 70% of the 9 dimensional design space hypercube that was used for the sampling of the 126 design points. The edges of these two concentric hypercubes have lengths  $\Delta x_i^{\text{search-space}} = 0.7 \Delta x_i^{\text{design-space}}$ . This is illustrated for the two dimensional case in Fig. 7 below.

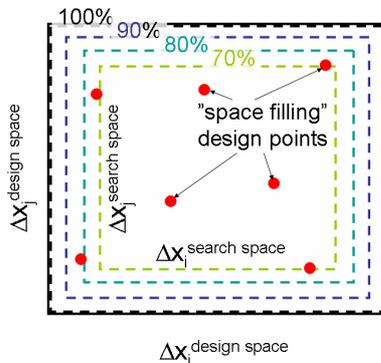


Fig. 7: Illustration in two dimensions of the definition of the 70%, 80%, 90% and 100% search spaces.

The rather restrictive 70% search space was used to avoid extrapolation outside the 9D ‘convex hull’ (or ‘cloud’) of the 104 design points, in order to keep a reasonable accuracy of the fit functions. In the larger search space, there will be lower probability of having sampled design points near the boundaries, i.e. higher probability of fit extrapolation.

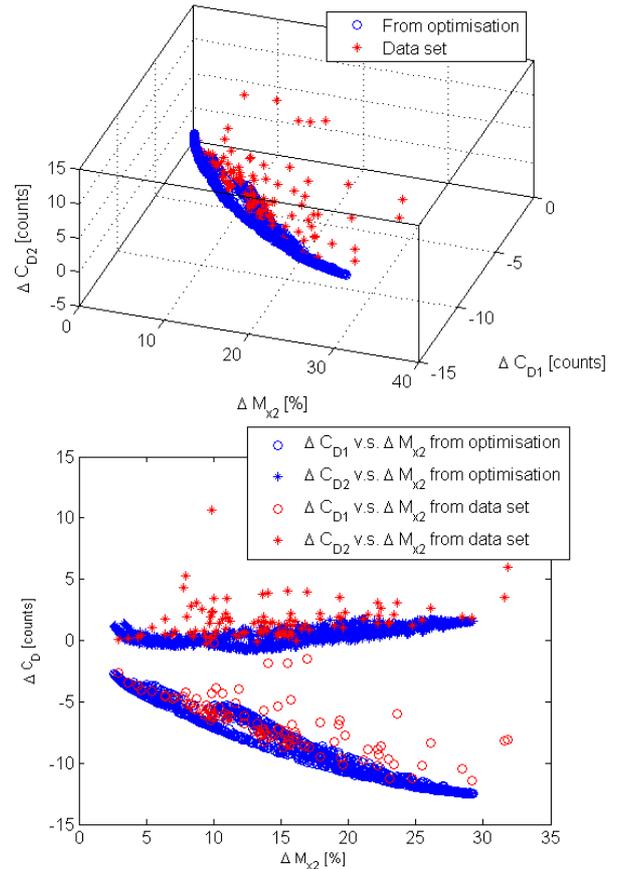


Fig. 8: Resulting set of non-dominated design points of the three-objective MNSGA optimization in the 70% search space of the winglet design. Note that these results are presented in objective space. Each point in objective space corresponds to a point in the 9D design space according to the transformation of the (fitted) objective functions ( $y^* = f^*(x)$ ). The upper graph shows the 3D plot of these results. The lower graph shows the 2D projections of these results.

The multi-objective search algorithm in MNSGA looks for design points for which each of the objectives have minimal values according to the Pareto optimal definition [17]. This leads

to a set of so-called non-dominated design points, i.e. design points for which each of the objectives could not be further improved without worsening of the other objectives. This set of non-dominated design points is shown in Fig. 8.

Besides this straight-forward three-objective optimization, also more specific search runs were performed with MNSGA where one or two of the objective functions were transformed into constraint functions. In these runs the objective(s)  $y_i$  were minimized and inequality constraint function(s)  $y_j < \lambda_{j,k}$  were applied ( $i, j, k \in \{1, 2, 3\}; i \neq j$ ). Separate runs with three different constraint values  $\lambda_{j,k}$  for each  $y_j$  were performed, resulting in 37 MNSGA runs in total (i.e., one run with no constraints,  $3 \times 3 = 9$  runs with one constraint, and  $3 \times 3 \times 3 = 27$  runs with two constraints). The constraint values  $\lambda_{j,k}$  used in these runs are given in the first column of Table 3 below. These constrained runs searched more specifically for those design points that just fulfilled the considered constraints, and the corresponding sets of non-dominated design points were found. The 27 runs with two constraints (i.e., constrained-single-objective optimizations) were also done with the two single-objective optimization algorithms GA and FMINCON. The GA runs were performed with a population of 104 individuals, and 100 generations, using the points in the data set as initial population. Starting point for the FMINCON runs was always the optimum point found in the GA run. The results of all these  $3 \times 27 = 81$  single objective runs are given in Table 3. It should be noted that these single objective runs were rather computationally expensive, in particular the FMINCON runs of which some took up to about 30 minutes (Pentium-4 - 2 GHz, WinXP).

Table 3: Results of the 27 constrained-single-objective optimization runs with each of the 3 optimizers.

SOO case (objective and constraints)	MNSGA result	GA result	FMINCON result
Min(y1);y2<1.5;y3<15	y1 = -9.236	y1 = -8.0900	y1 = -9.2579
Min(y1);y2<0.5;y3<15	y1 = -8.879	y1 = -7.9062	y1 = -8.9102
Min(y1);y2<0;y3<15	y1 = -7.980	y1 = -6.2619	y1 = -7.3622
Min(y1);y2<1.5;y3<10	y1 = -7.145	y1 = -6.1327	y1 = -6.5086
Min(y1);y2<0.5;y3<10	y1 = -7.061	y1 = -6.4018	y1 = -7.1193

Min(y1);y2<0;y3<10	y1 = -7.015	y1 = -6.2620	y1 = -7.0914
Min(y1);y2<1.5;y3<5	y1 = -4.617	y1 = -4.0052	y1 = -4.4829
Min(y1);y2<0.5;y3<5	y1 = -4.629	y1 = -4.2378	y1 = -4.6670
Min(y1);y2<0;y3<5	y1 = -4.205	y1 = -4.0531	y1 = -4.1000
Min(y2);y1<-8;y3<15	y2 = 0.030	y2 = 0.6500	y2 = 0.0317
Min(y2);y1<-5;y3<15	y2 = -0.784	y2 = 0.6500	y2 = -0.3638
Min(y2);y1<-3;y3<15	y2 = -0.784	y2 = 0.6500	y2 = -0.3701
Min(y2);y1<-8;y3<10	y2 = 0.401	y2 = 0.6500	y2 = 0.5275
Min(y2);y1<-5;y3<10	y2 = -0.462	y2 = 2.1387	y2 = -0.3690
Min(y2);y1<-3;y3<10	y2 = -0.473	y2 = 2.5328	y2 = -0.2318
Min(y2);y1<-8;y3<5	y2 = 0.154	y2 = 0.6500	y2 = 0.2111
Min(y2);y1<-5;y3<5	y2 = 0.139	y2 = 0.9609	y2 = 0.2921
Min(y2);y1<-3;y3<5	y2 = -0.319	y2 = 1.6885	y2 = -0.0287
Min(y3);y1<-8;y2<1.5	y3 = 11.976	y3 = 14.9810	y3 = 11.8913
Min(y3);y1<-5;y2<1.5	y3 = 5.671	y3 = 11.8238	y3 = 5.8026
Min(y3);y1<-3;y2<1.5	y3 = 2.870	y3 = 14.2869	y3 = 2.8642
Min(y3);y1<-8;y2<0.5	y3 = 12.339	y3 = 14.3771	y3 = 12.0676
Min(y3);y1<-5;y2<0.5	y3 = 5.720	y3 = 13.9039	y3 = 5.6304
Min(y3);y1<-3;y2<0.5	y3 = 3.040	y3 = 13.9039	y3 = 3.0344
Min(y3);y1<-8;y2<0	y3 = 15.324	y3 = 16.1005	y3 = 14.6918
Min(y3);y1<-5;y2<0	y3 = 6.579	y3 = 10.0450	y3 = 5.8316
Min(y3);y1<-3;y2<0	y3 = 3.485	y3 = 10.0450	y3 = 3.8906

All solutions (i.e., sets of non-dominated design points) of each of the 91 (37+27+27) optimization runs were now put together in one large set of more than 10000 different design points. For this set the so-called Pareto ranking (i.e., the order in which the design points are dominated) was determined, and the best (Pareto rank 1, or non-dominated) design points were selected. It appeared that many of these non-dominated designs were found on the boundaries of the 70% search space. Therefore it was decided to perform additional MNSGA search runs in increased search domains of 80%, 90% and 100% of the design space, respectively. Again 1000 individuals and 1000 generations were used in these runs. The sets of non-dominated design points resulting from these runs were added to all the non-dominated designs points of the 70% search space, resulting in a total set of nearly 16000 points. For this set again the non-dominated design points were determined. Still very many (thousands) rank 1 points remained, and a further selection ('filtering') of interesting design points was applied. Firstly, the points that originated from the 70% search runs were selected because these are likely to have the smallest fitting error. This resulted in 161 selected points. From the other points, which originated from the 80%, 90% and 100% runs, first a selection was made on the basis of the distance of a point to its neighboring points in

objective space. 89 points with the largest distances were selected, such that together with the 70% points, there remained 250 best points. From these points, the points with very high values for one or two of the objectives were removed. Subsequently the points that were likely to have large fitting errors were removed, i.e. the points closest to the design domain boundaries. Finally the 7 most suitable Pareto design points remained, as shown in Fig. 9 below.

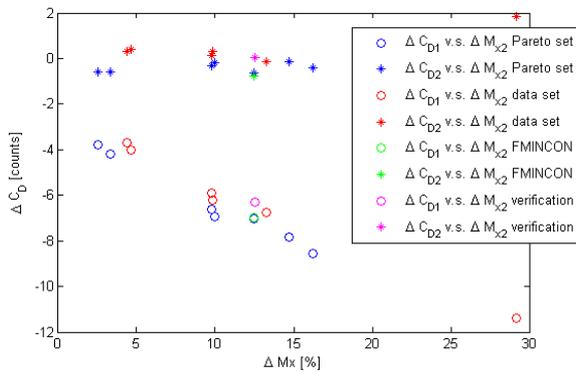


Fig. 9: The objective values for the 7 best design points from the Pareto set and the 5 best design points from the data set, and for the final point found with FMINCON and its verification results.

The design point around  $y_3$  (i.e.,  $\Delta M_{X2}$ )  $\approx$  12.5% was then further improved by using it as starting point in 3 additional FMINCON runs, in which subsequently each of the 3 objectives was further minimized constraining the other objectives to their value in the starting point. One final best design point was then selected, which is given in the table 4 below.

Table 4: Resulting optimum design points.

Closest data set point ( $y_1, y_2, y_3$ )	(-6.76, -0.13, 13.25)
Final best point ( $y_1, y_2, y_3$ )	(-7.00, -0.78, 12.5)
Verification of final best point ( $y_1, y_2, y_3$ )	(-6.30, 0.02, 12.5)

For verification, the final best design point was also evaluated by the design analysis. The objective values as predicted by the meta-models (Kriging-1c fits) showed quite good correspondence with this verification result for  $y_3$ , but were less accurate for  $y_1$  and  $y_2$ ; see table 4. Although these accuracies in this final design point were not very satisfactory, it was

concluded from additional MutiFit evaluations that the other fitting methods would not provide any significantly better local accuracy in this design point. Moreover, the limited accuracy for  $y_1$  and  $y_2$  eventually is due to the small data set, which is very sparse for the 9 dimensional design space.

The verification result in the final best design point, when compared to the best of the 104 design points of the original data set (Table 4), shows a clear improvement for  $y_3$ , but slightly worse values for  $y_1$  and  $y_2$ . It should be noted though that this verification result is Pareto optimal when compared to the points of the data set. Moreover, it provides a quite different optimal design alternative as it is located in a quite different region of the design space.

## Conclusions and discussion

The response surface optimization approach for aircraft design presented in this paper is flexible and applicable to a variety of design problems. A key benefit of this approach is that large numbers of interesting (Pareto optimal) design points can be found relatively quickly and easily at the cost of only few computationally expensive analyses, whilst a reasonable control of the accuracy is maintained. For high-dimensional design problems the visualization, assessment and selection of the most interesting design points requires special attention. Representation of the results in parameter space as well as in objective space provides valuable information for design decisions, where involvement of design specialists is required.

The accuracy of the objective function values as predicted by the fits is an aspect of this approach that needs special attention. Several ways to deal with the accuracy aspect were demonstrated: Use as much as possible information that is available, e.g. proper DOE, as many as possible data points, a priori knowledge of the underlying functions; use different fitting methods and determine the best fit; carefully define appropriate validity domains

for the fits (e.g. avoid extrapolation); try to account for fitting errors in the fit prediction by incorporating residual estimations.

The FMINCON optimization runs in the winglet study required quite many (several hundreds) evaluations of the objective and constraint (response surface) functions. These evaluations are performed sequentially, so cannot be effectively vectorized. Although these functions are quickly evaluated (less than one second), computing time does become an issue if many FMINCON runs are required. This issue can be dealt with by incorporating the gradients of the objective and constraint functions in the FMINCON optimization run. Current investigation is ongoing to include these gradient functions by means of the derivatives of the considered response surface functions.

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