Model Reduction for GPS/SINS Integrated Navigation system Using Krylov Subspace Methods

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Abstract

Kalman filter is extensively using in integrated navigation system-GPS/SINS. The main factor in determining the computation time of Kalman filter is the dimension *n* of the model state vector. Krylov subspace methods based on the moment matching is efficiently way to reduce order of the system. The paper presents the methodology to model order reduction of the state formulas of the GPS/INS using the Krylov subspace methods. Result shows it is an exciting way to economize the computational time of the Kalman Filter.

1 Introduction

Numerical simulation of dynamical systems is a powerful tool for the analysis of complex phenomena. However many systems are very complex, and therefore hard to analyze numerically, as this involves computation and storage matrices. If the complexity exceeds a certain amount, the computations become hard, due to time and memory limitations, and error propagation in computation.

This is where model reduction comes into play. Model reduction consists of replacing the original system with one of a much smaller dimension. The goal of model reduction is to obtain an approximate model of much lower complexity of the physical system to be analyzed. This approximate model should have the following desirable properties[4]:

• The reduced system must be an accurate epresentation of the original one for the analysis performed.

• The cost of generating the reduced model must be much smaller than the cost of performing the analysis using the original model.

Kalman filter is extensively using in integrated navigation system, such as Global Positioning System and Strapdown Inertial Navigation System, for short GPS/SINS. The main factor in determining the computation time of Kalman filter is the dimension *n* of the model state vector. The number of computations per iteration is on the order of n^3. Any reduction in the number of states will significantly decrease the computation time. The paper first presents the GPS/SINS integrated navigation system, then presents the reduced-order model--- a moment matching model reduction methodology based on projection on Krylov subspaces is presented.. Through the simulation the reduced-order model also can provide batter parameters.

2 GPS/SINS model

SINS and GPS have different model based

on the different requires. Here, GPS and SINS are integrated with velocity and position. The difference between the velocity and position of the GPS and SINS is as observation, which is used to estimate the SINS error by Kalman filter an to revise the error of SINS. The integrated model of GPS/SINS has different types. The integrated model are as the follows.

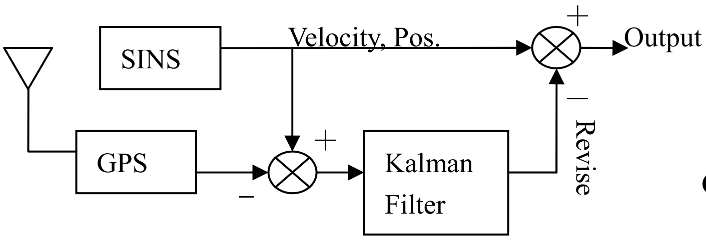

Fig.1 open loop GPS/SINS system

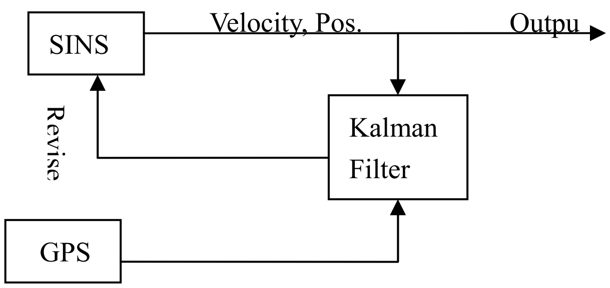

Fig.2 Closed loop GPS/SINS system

2.1 System States Formulas

To the SINS, navigation reference frame is East, North and Up geography coordinate system.GPS/SINS integrated navigation high order dynamic model as follows:

$$\dot{X}(t) = F(t)X(t) + G(t)W(t) \qquad (2..1.1)$$

X(t)—state variables.

$$X = [\phi_E, \phi_N, \phi_U, \delta V_E, \delta V_N, \delta L, \delta\lambda, \varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}, \varepsilon_{rx}, \varepsilon_{ry}, \varepsilon_{rz}, \nabla_x, \nabla_y]^T$$

w(t)—system noises matrix

$$W = [\omega_{gx}, \omega_{gy}, \omega_{gz}, \omega_{rx}, \omega_{ry}, \omega_{rz}, \omega_{ax}, \omega_{ay}]^T$$

F(t)—dynamic matrix

$$F = \begin{bmatrix} F_N & F_s \\ 0 & F_M \end{bmatrix}_{15\times15}$$

$$Fs = \begin{bmatrix} C_b{}^n & C_b{}^n & 0_{3\times2} \\ 0_{2\times3} & 0_{2\times3} & (C_b{}^n)_{2\times2} \\ 0_{2\times3} & 0_{2\times3} & 0_{2\times2} \end{bmatrix};$$

$$F_M = diag(0,0,0,-\frac{1}{T_{rx}},-\frac{1}{T_{ry}},-\frac{1}{T_{rz}},-\frac{1}{T_{ax}},-\frac{1}{T_{ay}});$$

G(t) —coefficient matrix

$$G = \begin{bmatrix} C_b{}^n & 0_{3\times3} & 0_{3\times2} \\ 0_{7\times3} & 0_{7\times3} & 0_{7\times2} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times2} \\ 0_{2\times3} & 0_{2\times3} & 0_{2\times2} \end{bmatrix}_{15\times8}$$

$C_b{}^n$ is the attitude conversion matrix form the carrier coordinate to the navigation coordinate.

2.2 System Measuring Formulas

SINS position signals are as follows:

$$\begin{cases} \lambda_I = \lambda_t + \delta\lambda \\ L_I = L_t + \delta L \end{cases} \qquad (2.2.1)$$

GPS position signals are as follows:

$$\begin{cases} \lambda_G = \lambda_t + \dfrac{N_E}{R_N \cos L} \\ L_G = L_t - \dfrac{N_N}{R_M} \end{cases} \qquad (2.2.2)$$

SINS velocity signals are as follows:

$$\begin{cases} V_{IN} = V_N + \delta V_N \\ V_{IE} = V_E + \delta V_E \end{cases} \qquad (2.2.3)$$

GPS velocity signals are as follows:

$$\begin{cases} V_{GN} = V_N - M_N \\ V_{GE} = V_E - M_E \end{cases} \quad (2.2.4)$$

Combined the formulas of (2.2.1)～(2.2.4), we can get the position and velocity measuring formula of the GPS/SINS.

$$Z(t) = \begin{bmatrix} Z_p(t) \\ Z_v(t) \end{bmatrix} = \begin{bmatrix} H_p(t) \\ H_v(t) \end{bmatrix} \times X(t) + \begin{bmatrix} V_p(t) \\ V_v(t) \end{bmatrix}; \quad (2.2.5)$$

3 Krylov Subspace Model Reduction

Depending on the properties of the original system that are retained in the reduced model, there are different model reduction methodologies. Hence, there are techniques based in directly identifying and preserving certain modes of interest or based on the singular value decomposition (SVD), such as balanced truncation [7], Hankel norm approximation [4], etc., focusing on the observability and controllability properties of the system. Another family of model reduction techniques, on which this paper builds on, is the moment matching methods [1], [5], [6]. The property of interest here is the leading coefficients of a power series expansion of the transfer function of the reduced system around a user-defined point that have to match those of the original system transfer function.

3.1 Krylov Subspace

Krylov subspace: a j^{th} dimensional Krylov subspace corresponding to some matrix G and vector g is denoted κ_j (G,g) and is defined as

$\kappa_j(G,g) = \mathrm{span}\{g, Gg, G^2g, \ldots, G^{j-1}g\}$.

IF V is a orthogonal matrix and is transformed from Krylov subspace $\kappa m(A^{-1}E, A^{-1}B)$, the m-order moment of model reduction transfer function of the GPS/SINS integrated navigation through V is matching of the original transfer function. The V can generate through Arnoldi algorithm.

Arnoldi algorithm:

```
Input: A,b,q
 Output: V, v_{q+1} ,H
[V, v_{q+1} ,H]=arnoldi(A,b,q)
{
  v1=b/||b||
  for I=1:j
     {
        h_ij = w^T v_i
        w=w-h_ij v_i
        h_{j+1,j}=||w||
        if(h_{j+1,j}≠0)v_{j+1}=w/h_{j+1,j}}
  }
```

3.2 GPS/SINS Reduction Model

Using the Krylov subspace methods, the GPS/SINS integrated formulas (2.1.1) and (2.1.5) reduction model is as follows. The dimensions of the GPS/SINS are from 15 reduction to 9.

$$\dot{\tilde{X}}(t) = \tilde{F}(t)\tilde{X}(t) + \tilde{G}(t)\tilde{W}(t)$$

$$\tilde{Z}(t) = \begin{bmatrix} \tilde{H}_p(t) \\ \tilde{H}_v(t) \end{bmatrix} \times \tilde{X}(t) + \begin{bmatrix} \tilde{V}_p(t) \\ \tilde{V}_v(t) \end{bmatrix}$$

4 Simulation

Using the Krylov subspace methods, about the attitude angular error and velocity error of the model reduction of GPS/SINS Integrated Navigation system are as Fig.3 and Fig.4.

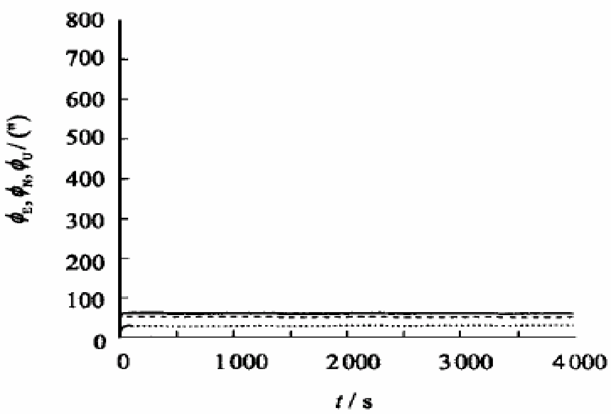

fig.3 attitude angular error

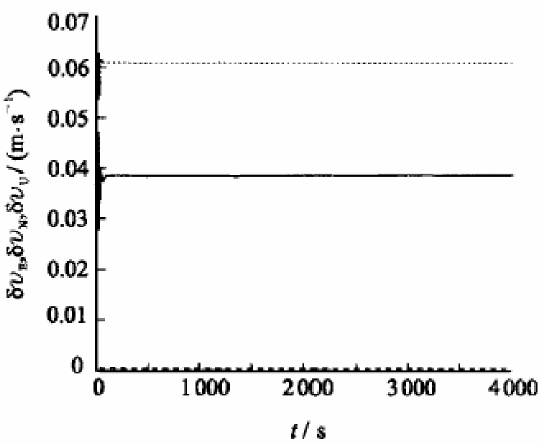

fig.4 velocity error

5 Conclusion

In this paper, an approach to model reduction in GPS/SINS based on Krylov subspace has been presented. Through the Krylov subspace methods, the model reduction model can provide the be up to the mustard navigation precision. The computational time of the kalman filter is reduced form 15^3 to the 9^3. Time reduced approximately 21%. This is an exciting result to the application.

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