MODEL ORDER REDUCTION FOR AEROSERVOELASTICITY STUDIES BY USE OF LRSRM AND LRSM ALGORITHMS

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Abstract

The approximation of unsteady generalized aerodynamic forces from the frequency domain into the Laplace domain, acting on a Fly-By-Wire aircraft, from the frequency domain into the Laplace domain presents an important challenge in the aeroservoelasticity area. The aerodynamic forces in the reduced frequency domain have to be approximated in the Laplace domain, in order to study the effects of the control laws on the flexible aircraft structure. In this paper we present a new method for approximation of the generalized aerodynamic forces, using Chebyshev polynomials and their orthogonality properties. A comparison of this new method with Pade method used to calculate an approximation of the generalized aerodynamic forces from the frequency into the Laplace domain is presented.

This new approximation method gives excellent results with respect to the other method and is applied on the Aircraft Test Model at NASA DFRC. The order of the model is further reduced by use of LRSRM and LRSM algorithms [19]. The Aircraft Test Model ATM described in the STARS program [1] was used to validate our results.

1 Introduction

Aeroservoelasticity represents the combination of several theories regarding different aspects of aircraft dynamics. Studies of aeroservoelastic interactions on an aircraft are very complex problems to solve, but are essential for an aircraft's certification. Instabilities deriving from adverse interactions between the flexible structure, the aerodynamic forces and the control laws acting upon it can occur at any time inside the flight envelope. Therefore, it is clear that aeroservoelastic interactions are mainly studied in the research field located at the intersection of the following three disciplines: aerodynamics, aeroelasticity and servo-controls. One main aspect of aeroservoelasticity is the conversion of the unsteady generalized aerodynamic forces $Q(k, M)$ from the frequency domain into the Laplace domain $Q(s)$, where k represents the reduced frequency, M is the Mach number and s is the Laplace variable. There are basically three classical methods to approximate the unsteady generalized forces by rational functions from the frequency domain to the Laplace domain [2-6]: Least Square (LS), Matrix Padé (MP) and Minimum State (MS). To date, the approximation that yields the smallest order time-domain state-space model is the MS method [6]. All three methods use rational functions in the Padé form.

Several aeroservoelastic analysis software codes have been developed for the aerospace industry. The Analog and Digital Aeroservoelasticity Method (ADAM) was developed at the Flight Dynamics Laboratory (FDL). ADAM has been used for the non augmented X-29A and for two wind-tunnel models: 1) the FDL model (YF-17) tested in a 16 ft transonic dynamics tunnel and 2) the Forward Swept Wing (FSW) model mounted in a 5 ft subsonic wind tunnel. ISAC (Interaction of Structures, Aerodynamics, and Controls) was developed at NASA Langley Research Center, and has been used on various flight models such as DAST (Drone for Aeroelastic Structure Testing) ARW-1 (Aeroelastic Research Wing), ARW-2 and DC-10 wind-tunnel flutter models, generic X-wing feasibility studies, analyses of elastic oblique-wing aircraft, AFW (Active Flexible Wing) wind tunnel test programs, generic hypersonic vehicles and high-speed civil transports. Recently, an aeroelastic code, ZAERO, was developed at Zona Technology, and has been used for aeroservoelastic studies. STARS code was developed at NASA Dryden Flight Research Center (DFRC) [1] and has been applied on various projects at NASA DFRC: X-29A, F-18 High Alpha Research Vehicle / Thrust Vectoring Control System, B-52 / Pegasus, Generic Hypersonics, National AeroSpace Plane (NASP), SR-71 / Hypersonic Launch Vehicle, and High Speed Civil Transport. The STARS program is an efficient tool for aeroservoelastic interactions studies and has an interface with NASTRAN [8, 9], a computer program frequently used in the aeronautical industry. In this paper, the lateral dynamics of a half Aircraft Test Model (ATM) modeled in STARS was used. After performing the finite element structural modeling and the doublet lattice aerodynamic modeling on the ATM in STARS, the unsteady aerodynamic forces are calculated as functions of the reduced frequencies k and of the Mach number M. Due to the fact that $Q(k, Mach)$ can only be tabulated for a finite set of reduced frequencies, at a fixed Mach number M, it must be interpolated in the s domain in order to obtain $Q(s)$.

All of these codes use two main classical methods for aerodynamic force approximations from the frequency domain (aeroelasticity) into the Laplace domain (aeroservoelasticity), Least Square (LS) and Minimum State (MS). In this paper we describe a new interpolation method that uses the Chebyshev polynomials, and its results. We further present a detailed survey on the other methods existing in the literature.

The aerodynamic forces dependence on s may be written as an irrational function even for simple cases such as two-dimensional potential incompressible flows on an airplane wing profile. During the 1950's, Theodorsen [11] proved that $Q(s)$ could be expressed by use of Hankel's functions. A few years later, Wagner found the first rational approximation [11] for $Q(s)$. Another approach used the approximations of unsteady aerodynamic forces by Padé polynomials. This approach was based on a fractional approximation of the form $P(s)/R(s)$, where P and R are two polynomials in s, for every term of the unsteady forces matrix. In this way, every pole of $R(s)$ showed a new state, called augmented state, in the final linear invariant aeroservoelastic system. In case where the initial square matrix has the N dimension, and where a Padé approximation of M order is used, then N (N+M) augmented states will be introduced.

The number of augmented states was reduced by Roger [4]. In his formulation, only NxM modes were introduced, where N is the number of initial modes. Roger's method is based on the fact that the aerodynamic lag terms remain the same for each element of the unsteady aerodynamic forces matrix. This method is called Least Squares (LS) and is used in computer aeroservoelastic codes such as STARS and ADAM.

Another method derived from the LS method was proposed by Vepa [5]. This method uses the same denominators for every column of the aerodynamic matrix Q, and is called Matrix Padé (MP).

Various improvements were done for the two methods presented above, LS and MP. One such type of improvement is that one could impose different conditions (restrictions) to these approximations to pass through certain points. Generally, the approximations are imposed to be exact in zero and in two other chosen points. The first point could be chosen to represent the estimated flutter frequency and the second point to represent the gust frequency. The improved methods have been renamed: ELS method (Extended Least-Squares) [2, 12] and EMMP method (Extended Modified Matrix-Padé) [13]. Later, Karpel [14] proposed a completely different approach in order to solve the above approximations. His goal was to find a linear invariant system in the time domain and so he decided to integrate this information directly into the equation representing the unsteady aerodynamic force values by adding a term similar to the transfer function of a linear system. Because he wanted to find a linear system of reasonable dimensions, he wrote the approximation under the MS (Minimum State) form. The advantage of this method over Roger's method is that it provides an excellent approximation with a smaller number of augmented states.

All of the methods described above allow the approximation of unsteady aerodynamic forces for one Mach number at a time. A valid approximation for a range of Mach numbers would be very useful for military Fly-By-Wire aircraft, where the Mach number varies rapidly during high speed maneuvers, and where aeroservoelastic interactions are extremely important. Poirion [15, 16] constructed an approximation allowing the calculation of the unsteady aerodynamic forces for a range of Mach numbers and for a range of reduced frequencies. He used several MS approximations, obtained for several fixed Mach numbers, and a spline interpolation method for Mach number dependence. Thus, he obtained formulae which allow the unsteady aerodynamic forces to be computed for any couple (k, $Mach$), where k is the reduced frequency and $Mach$ is the Mach number.

The approximation methods should simultaneously satisfy two opposed criteria: an excellent (exact) approximation, which can be obtained by increasing the number of lag terms, and a linear invariant system in the time domain of a very small dimension (with the smallest possible number of lag terms). For the time being, there is no method adequately satisfying both criteria. In two recent papers, Botez and Cotoi [17, 18] proposed a new approach based on a precise Padé approximation. The authors used four order reduction methods for the last term of the approximation, term which could be seen as a transfer function of a linear system. The approximation error for this new method is 12-40 times less than for the MS method for the same number of augmented states and is dependent on the choice of the order reduction method. However, this method remains very expensive in terms of computing time. In this paper, we present a new method that uses Chebyshev polynomials to produce approximations for $Q(s)$.

2 Aircraft equations of motion

Flexible aircraft equations of motion, where no external forces are included, may be written in the time-domain as follows:

$$\tilde{\mathbf{M}}\ddot{\eta} + \tilde{\mathbf{C}}\dot{\eta} + \tilde{\mathbf{K}}\eta + q_{dyn}Q(k, Mach)\eta = 0 \qquad (1)$$

where $q_{dyn} = 0.5\rho V^2$ is the dynamic pressure with ρ as the air density and V as the true airspeed ; η is the generalized variable defined as $\boldsymbol{q} = \mathbf{\Phi}\eta$ where $\boldsymbol{q}$ is the displacement vector and $\mathbf{\Phi}$ is the matrix containing the eigenvectors of the following free-vibration problem:

$$\mathbf{M}\,\ddot{q} + \mathbf{K}\,q = 0 \qquad (2)$$

The following transformations are used in equation (1):

$$\tilde{\mathbf{M}} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi},\ \tilde{\mathbf{C}} = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi},\ \tilde{\mathbf{K}} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} \\ Q(k, Mach) = \mathbf{\Phi}^T A_e(k)\mathbf{\Phi} \qquad (3)$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$ are the generalized mass, stiffness and damping matrices; k, the reduced frequency, is written as $k = \omega b/V$ where ω is the natural frequency and b is the wing semi-chord length. $A_e(k)$ is the aerodynamic influence coefficient matrix for a given Mach number M and a set of reduced frequency values k. The Laplace transformation is further applied to equation (1), and we obtain:

$$[\tilde{\mathbf{M}}s^2 + \tilde{\mathbf{C}}s + \tilde{\mathbf{K}}]\eta(s) + q_{dyn}Q(s)\eta(s) = 0 \qquad (4)$$

The approximation of the unsteady generalized aerodynamic forces is essential for the control analysis of our system. Due to the fact that $Q(k, Mach)$ can only be tabulated for a finite set of reduced frequencies, at a fixed Mach number M, these unsteady generalized aerodynamic forces must be interpolated in the s domain in order to obtain $Q(s)$. In this paper we describe an interpolation method using the Chebyshev polynomials and its results.

Matrix $Q(s)$ may be written under the following form, which is used in the Minimum State MS method of approximation: $Q(s) = D + C\,(sI\text{-}A)^{-1} B$. This matrix is further replaced with a reduced order matrix of the following form: $\hat{Q}(s) = \hat{D} + \hat{C}(sI - \hat{A})^{-1}\hat{B}$. These types of methods use the following steps:

- T is a square matrix of n x n dimensions which is inversable so that the space of projection is given by the first k columns of T.
- The quadruplet $(T^{-1}AT, T^{-1}B, CT, D)$ is equivalent to (A, B, C, D).
- Define $S_B = T(:,1:k)$ and $S_C = T^{-t}(:,1:k)$, then $(S_C^t A S_B, S_C^t B, C S_B, D)$ is the reduced order system $(S_C^t S_B = I_k)$. Various choices give various results, and among these choices we chose the algorithms Low Rank Square Root Method (LRSRM) and Low Rank Schur Method (LRSM) described in the following two sections.

<u>Low Rank Square Root Method (LRSRM)</u>

- Calculate the approximation of $X_B = Z_B Z_B^T$ et $X_C = Z_C Z_C^T$ where X_B is the observability grammian and X_C is the controllability grammian and the Singular Value Decomposition (SVD) for $Z_C^T Z_B = U_{C0} \Sigma_0 U_{B0}^T$.
- Define the reduced matrices: $U_C = U_{C0(:,1:k)}$, $\Sigma_0 = \Sigma_{0(:,1:k)}$, $U_B = U_{B0(:,1:k)}$
- Define the transformation matrices: $S_B := Z_B U_B \Sigma^{-1/2}$ et $S_C := Z_C U_C \Sigma^{-1/2}$
- The reduced system is then defined by the following matrices: $\hat{A} := S_C^t A S_B$, $\hat{B} := S_C^t B, \hat{C} := C S_B$, $\hat{D} := D$.

<u>Low Rank Schur Method (LRSM)</u>

- Calculation of Z_B and Z_C by use of the LR-Smith algorithm.
- The QR factorization of Z_B and Z_C is written as follows: $Z_B := Q_{B1} R_B$ and $Z_C := Q_{C1} R_C$.
- Define the Singular Values Decomposition (SVD) of: $R_B Z_B^T Z_C R_C^T = Q_{B2} D Q_{C2}^T$.
- Define the following matrices: $Q_B := Q_{B1} Q_{B2}$, $Q_C := Q_{C1} Q_{C2}$.
- Define the Schur factorization: $D Q_C^T Q_B := P_B T_B P_B^T$, $D^T Q_B^T Q_C := P_c T_c P_c^T$
- Define the matrices: $V_B := Q_B P_{B(:,1:k)}$, $V_C := Q_C P_{C(:,1:k)}$
- Define the Singular Value Decomposition (SVD) for the product $V_C^T V_B := U_C \Sigma U_B^T$.
- Define the transformation matrices: $S_B := Z_B U_B \Sigma^{-1/2}$ et $S_C := Z_C U_C \Sigma^{-1/2}$.
- The reduced system is then defined by the following matrices: $\hat{A} := S_C^t A S_B$, $\hat{B} := S_C^t B, \hat{C} := C S_B$, $\hat{D} := D$.

These algorithms were detailed by Penzl [19] and we applied them on the ATM model in the STARS [1] program and the new obtained results were further analyzed.

3 Chebyshev polynomials theory

3.1 Chebyshev polynomials of the First Kind

These polynomials are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential equation [10] and are denoted as $T_n(x)$. They are used as an approximation to a least squares fit, and are closely connected with trigonometric multiple-angle equations. Chebyshev polynomials of the first kind are implemented in Mathematica as ChebyshevT [n, x], and are normalized so that $T_n(1) = 1$.

3.2 Continuous functions represented using Chebyshev polynomials

Any continuous function may be expressed by use of Chebyshev polynomials as follows :

$$f(x)=\frac{1}{2}c_0+\sum_{j=1}^{\infty}c_jT_j(x) \qquad (5)$$

where the Chebyshev polynomials have the following form:

$$T_j(x)=\cos(j\arccos(x)) \qquad (6)$$

and the coefficients c_j used in equation (5) are expressed as follows :

$$c_j=\frac{2}{\pi}\int_{-1}^{1}\frac{f(x)T_j(x)}{\sqrt{1-x^2}}dx \text{ where } j=1,2,\ldots \qquad (7)$$

3.3 Orthogonality of Chebyshev polynomials

In our new approximation method for unsteady aerodynamic forces, we have used the Chebyshev polynomials because they have a specific orthogonality property. This interesting property allows us to keep the approximation's error within a predetermined bandwidth, and may further be expressed as:

$$\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}T_r(x)T_s(x)dx=\begin{cases}0,\ r\neq s\\ \pi,\ r=s=0\\ \frac{\pi}{2},\ r=s\neq 0\end{cases} \qquad (8)$$

3.4 Recurrence formulae and the solution of Chebyshev polynomials

The following recurrence relationships have been used in the Chebyshev polynomials new approximation method:

$$\begin{cases}T_0(x)=1\\ T_1(x)=x\\ T_{r+1}(x)=2xT_r(x)-T_{r-1}(x)\end{cases} \qquad (9)$$

Next, we impose the following condition to find the Chebyshev polynomials solution :

$$T_r(x)=0 \qquad (10)$$

where r specifies the rank of the Chebyshev polynomial. Equation (10) gives the following solution :

$$x=\cos\frac{(2j+1)\pi}{2r} \qquad (11)$$

Thus, the expression:

$$\tilde{Q}_r(x)=\frac{1}{2^{r-1}}T_r(x) \qquad (12)$$

will oscillate with an extreme amplitude within the interval [-1, 1].

3.5 Extreme amplitudes

$T_r(x)$ is a function defined by cosines, which lets us conclude that between two solutions of this function we will find an extreme of |1| amplitude exactly in the middle of the interval, specifically at :

$$x=\cos\frac{j\pi}{r}\quad ;\quad j=0,1,\ldots,r \qquad (13)$$

4 Methodology for the Chebyshev approximation method

In order to develop our approximation method, we used the predefined functions using Chebyshev polynomials expressed in equations (6) which have already been implemented in the Maple's kernel, in Matlab.

These functions (*chebpade* and *chebyshev*) allowed the construction of a polynomial interpolation for the unsteady generalized aerodynamic forces, acting on the Aircraft Test Model (ATM) for 14 values of reduced frequencies *k*, and 10 values of Mach number. The elements forming the matrices of the unsteady generalized aerodynamic forces calculated by the Doublet Lattice Method DLM in STARS were denoted by $Q(i,j)$ with $i = 0\ldots8$ and $j = 0\ldots8$ for the first eight elastic modes.

The approximation by means of this method is obtained using a similar path to the one used for the Padé method. For each element of the unsteady aerodynamic forces matrix we have determined a power series development in the following form, by use of the "*chebyshev*" function:

$$Q_{ij}(s) = \frac{1}{2}c_0^{(ij)} + \sum_{n=1}^{\infty} c_n^{(ij)} T_n^{(ij)}(s) \qquad (14)$$

where $c_n^{(ij)} = \frac{2}{\pi}\int_{-1}^{1}\frac{Q_{ij}(s)T_n^{(ij)}}{\sqrt{(1-s^2)}}ds$ for every n = 0,1,…

Next, by use of the "*chebpade*" function, we have found an approximation by rational fractions in the following form:

$$\hat{Q}_{ij}(s) = \frac{\sum_{n=0}^{M} a_n^{(ij)} T_n^{(ij)}(s)}{1+\sum_{n=1}^{P} b_n^{(ij)} T_n^{(ij)}(s)} \qquad (15)$$

where $M = P + 2$.

This new form is very useful since it integrates the orthogonality properties of Chebyshev polynomials and allows us to vary the degree of the nominator and the denominator, in order to obtain a very good approximation that best satisfies our desire for a small number of lag terms.

We have compared the results found by means of our Chebyshev approximation method with the results given by another classical interpolation method such as Padé. These results were expressed in terms of the approximation error.

The Padé method uses a parameter identification solution in order to determine a polynomial fractional form which identifies an orthogonal polynomial interpolation. This fractional form is the key aspect of this method, due to the fact that it allows the order reduction system.

We can see in Figures 1 and 2 that our new approximation method gives the best approximation error on an interval [0, 1] chosen in the proximity of each approximation point. This shows the effect of the Chebyshev polynomials properties given in equation (4). Due to these properties, we were able to impose a bandwidth for the error convergence on the approximation for each element of the unsteady generalized aerodynamic forces matrices. Both figures show the overall approximation error by Chebyshev and Padé. The aircraft is the Aircraft Test Model ATM generated in STARS code at the Mach number M = 0.5 and for 14 reduced frequencies k = [0.0100 0.1000 0.2000 0.3030 0.4000 0.5000 0.5882 0.6250 0.6667 0.7143 0.7692 0.8333 0.9091 1.0000]. Some differences at both ends of the approximation interval may be seen in the figures.

We used only a few different values of the polynomial approximation order by the Padé method and the Chebyshev polynomial fractions method (polynomial order should be equivalent for both methods) in order to calculate the overall approximation error - which was found to be much smaller for the Chebyshev polynomial method with respect to the overall approximation error given by Padé.

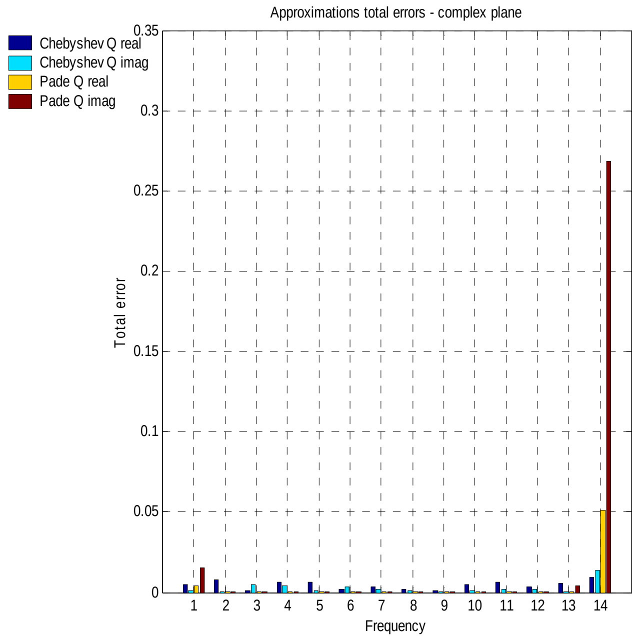

Fig. 1. The Approximations Total Errors for [16, 14] Model Order

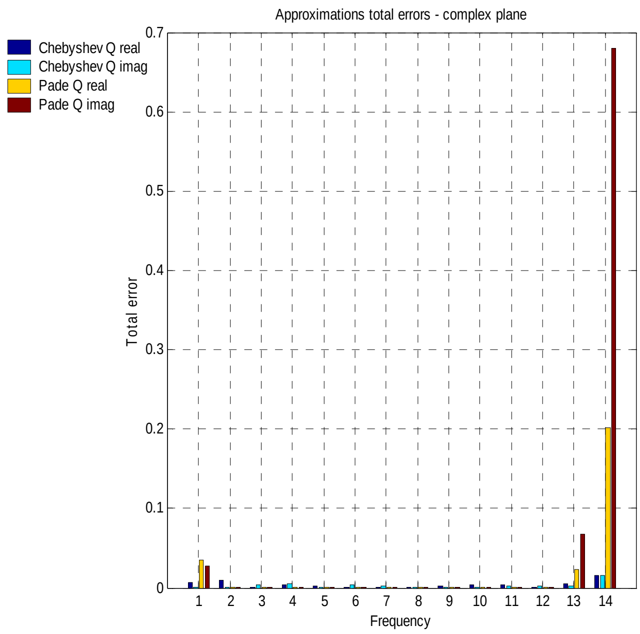

Fig. 2. The Approximations Total Errors for [15, 13] Model Order

More specifically, Padé method gives a small error near the approximation point (in our examples, the middle of the [0, 1] interval, which is 0.5) and an increased error towards each end of the [0, 1] interval. The Chebyshev approximation method demonstrates an almost constant value of the error all along the approximation interval. The threshold of this error could be imposed from the beginning of the calculations in order to find the unsteady generalized aerodynamic forces approximation matrices.

In the above diagrams, we can compare the overall approximation error, calculated for the whole unsteady generalized aerodynamic forces matrix at Mach number M = 0.5, for the methods described.

Since the system we had to approximate was rather a very large one (64 unsteady generalized aerodynamic forces for each of the 14 frequencies for each of the 9 numbers of Mach), one solution to achieve our goal was the use of LRSRM and LRSM order reduction methods that were implemented in the Lyapack Toolbox in Matlab. These two methods provided similar results, preserving the system ...cteristics.

5 Conclusions

The Chebyshev approximation method provides the smallest error in comparison with other methods' given errors. However, due to the fact that the Chebyshev polynomials had to be generated using the data provided from the ATM, which involve quite large differences between the values of the elements contained in the unsteady generalized aerodynamic forces matrices (1e+10), some restraints regarding the threshold of the approximation error had to be imposed. Therefore, for smaller elements we have imposed an error value of 1e-4 and for larger elements an error value of 1e-2. Without these restraints, the Chebyshev polynomials cannot be generated. When the approximation order for the Chebyshev method is increased, then the overall error will decrease even faster than by use of the Padé method. This method will be useful for further aeroservoelastic interaction studies in the aerospace industry. We could observe that by using this method in an

Open Loop, we were able to find excellent approximated values for the flutter speed and the frequencies at which flutter occurs.

One of the most important achievements of our new method, if not the most important, is the fact that the computation time for the Open Loop case is up to 3 times shorter than in the *Pk*-Padé method and up to 30 times shorter than in the LS case, even for an increased approximation order. Furthermore, the use of LRSRM and LRSM order reduction methods in the Open Loop case cut in half the total computational time, and made possible a reasonable use of computer's resources.

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