**Abstract**

The design of modern helicopter subfloors is called to accomplish structural functions under crashworthiness requirements in order to assure the safety of occupants.

The present study proposes a numerical methodology to optimise the shape of a helicopter subfloor intersection element made in composite material. In particular, in the first phase of the work, using LSTC LS-Dyna, a numerical material model for composite structures is validated referring to experimental tests carried out using composite cylindrical specimens. In order to reduce the total computational time, the optimisation is carried out using Radial Basis Functions, an established response surface technique. The shape of the stabilizers, which form the intersection elements, is properly parameterised and the domain of interest is defined. The Design of Experiments required by the response surfaces is then obtained via Finite Element analyses within the previously defined domain of interest.

Four distinct configurations are optimised considering different levels of mean force. One of these optimized configurations is then selected for further finite element investigation. Results are presented and discussed.

**1 Introduction**

The design of subfloor [1-4] is central for the crashworthiness of modern helicopters and, therefore, several efforts are nowadays devoted to improve its performances and to propose new design methods and manufacturing technologies.

In particular, an increasing interest has been growing around composite materials, which represent an advantageous alternative to metallic materials (basically Aluminium alloy) customarily used in subfloor manufacturing. In fact, composite materials exhibit very good absorption capabilities with controlled and stable crush forces, very high stiffness-to-weight and strength-to-weight ratios. In addition, composite materials make it possible to build lightweight structures (that is one of the main aims of aircraft industries) and, therefore, crashworthiness improvements should be achieved without remarkable penalties in weight.

The definition of new design procedures to develop new high-efficiency and crashworthy structures in short time period is mandatory. In particular, in this work, an optimization procedure has been applied to find the optimal shape of a typical helicopter subfloor intersection element made in Carbon Fibre Reinforced Plastic (CFRP) woven material.

The intersection elements connect the longitudinal beams and the bulkheads of the subfloor frame, as shown in Figure 1, and are responsible of the main crash loads on the helicopter floor, as well as of its overall energy absorption capabilities.

The crash behaviour of the intersection elements is investigated by means of a widely diffused explicit nonlinear Finite Element (FE) commercial code, that has been shown to be particularly effective for the analysis of crash events: LSTC LS-Dyna [5].

The numerical model of the composite material is validated referring to experimental
data obtained by vertical impact tests on composite cylindrical shells carried out at the Dipartimento di Ingegneria Aerospaziale (DIA) of the Politecnico di Milano, Italy.

A preliminary study is performed to point out the most promising shape design variables and their domain. Hence, the shape of the intersection elements is parameterised referring to the geometry of the upper and lower edges of the stabilizers and using a hyper-elliptic function.

Since an optimization procedure directly based on the use of nonlinear FE analyses is extremely time-consuming, it was decided to use a global approximation method [6-8]. Once the optimisation runs have been concluded, the performances of the optimized configurations are discussed with regard to the related crashworthiness capabilities by considering both the load-shortening curves and the deformed shape evolutions.

2 Numerical Model of the Intersection Element

One of the most difficult tasks in numerical investigations on composite structures is the definition of suitable and reliable material models able to correctly account the crash behaviour and the progressive damage of the material. And, as a matter of fact, this task is fundamental for any further numerical investigation that involves composite structures. For these reasons, the numerical model of the material used in the present work has been validated referring to experimental tests carried out using cylindrical shells.

2.1 Validation of the Material Model

The tested cylindrical shells have a nominal height of 300 mm and a diameter of 70 mm and are made of the same material used for the subfloor: a Carbon Fibre Reinforced Plastic (CFRP) woven. The main properties of the material are reported in Table 1. A single staking sequence is considered during the experimental tests: [0°/90°/90°/0°].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$ [kg/m³]</td>
<td>1445</td>
</tr>
<tr>
<td>Ply thickness $s$ [mm]</td>
<td>0.28–0.31</td>
</tr>
<tr>
<td>Elastic Modulus $E_{11}$ [MPa]</td>
<td>62600</td>
</tr>
<tr>
<td>Elastic Modulus $E_{22}$ [MPa]</td>
<td>60550</td>
</tr>
<tr>
<td>Poisson’s Ratio $\nu$</td>
<td>0.048</td>
</tr>
<tr>
<td>Shear modulus $G$ [MPa]</td>
<td>5500</td>
</tr>
</tbody>
</table>

Tab. 1. Mechanical properties of the CFRP woven.

The tested cylindrical shells are modelled with four-node shell elements. An integration point is defined for each ply throughout the shell thickness. After a sensitivity analysis on the element size, carried out to avoid mesh effects on the numerical results, the characteristic length of the elements has been fixed to 3 mm. Thus, the FE model consists of 3916 shell elements. Boundary conditions, impact velocity and impacting-mass were carefully considered and reproduced according to the experimental tests. In particular, two rigid walls were used: one fixed and the other reproducing an impacting mass of 110 kg with initial impact velocity of 10 m/s.

Accordingly to the real shape of the experimental specimens, an initial trigger is realised by selectively decreasing of about one-fifth the thickness of some elements in the first two shell rings.

The material model used in this work is the *MAT_58 of LSTC LS-Dyna [5,9] and is developed specifically for laminated composite material. Basically, it is a damage model developed around the idea that damages introduce micro-cracks and cavities into materials and that these defects primarily cause
merely stiffness degradation with rather small permanent deformation unless material undergoes rather high loading and is not close to deterioration. A non-smooth failure surface is assumed and, in order to allow an almost uncoupled failure, all failure criteria are taken to be independent.

The numerical results show a good correlation both in terms of load and absorbed energy values, as reported in Table 2, as well as in terms of the front crash and deformed shape evolution, as reported in Figure 2.

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Load</td>
<td>27.0</td>
<td>28.2</td>
<td>26.2</td>
</tr>
<tr>
<td>Mean Load</td>
<td>21.1</td>
<td>24.0</td>
<td>20.6</td>
</tr>
<tr>
<td>Residual Height</td>
<td>145</td>
<td>152</td>
<td>144</td>
</tr>
<tr>
<td>Absorbed Energy</td>
<td>3571</td>
<td>3522</td>
<td>3439</td>
</tr>
</tbody>
</table>

Tab. 2. Numerical-experimental correlation.

A good correlation is also obtained in terms of the load-shortening curves, reported in Figure 3. In particular, the relative errors, with regard to the absorbed energy, are within 5%: 3.7% and 2.4% when Test 1 and Test 2 are considered, respectively.

2.2 Baseline Configuration of the Intersection Element

The baseline configuration of the intersection elements used for further optimisations comes out from the geometry of a helicopter subfloor, such as the one show in Figure 1.

The intersection element is 200 mm high and consists of four stabilizers connected by an adhesive film to the lateral and longitudinal panels of the subfloor frame. The vertical panels are supposed of four [0°/90°/90°/0°] layers. The same stacking sequence is used for all the four stabilizers.

A detailed FE model of the intersection element is developed referring to the results obtained on the cylindrical specimens. In particular, the final model of the intersection elements consists of four-node shells (Belytsckho-Tsay formulation) with a reference length of 3 mm.

Adhesive connections between the parts of the intersection elements are modelled by using contact algorithms tuned so as to reproduce the stiffness of the adhesive junctions. Indeed, preliminary simulations have been performed to test the model of this junction starting from the mechanical property of the adhesive.

As previously done for the cylindrical specimens, the impact conditions are imposed by means of two rigid walls; the first one fixed, while the second one reproduces an impacting mass with a prescribed vertical velocity of 10 m/s.
3 Optimization Process

As previously mentioned, the definition of an optimization procedure using directly nonlinear FE analyses would be extremely time-consuming. Thus, FE analyses are replaced with a set of response surfaces to estimate the value of the objective function and of the constraints during the optimization process.

The response surfaces are defined using Radial Basis Functions (RBF) [10-12] and considering a number of sample points inside the optimization domain to which correspond a well-defined shape. A detailed finite element analysis has been performed to evaluate the crash behaviour of each sample point.

Using a global approximation method a complete separation between the optimization algorithm and the modelled system is achieved. This aspect is here proficiently used to perform different optimizations changing constraint values without requiring any further time-consuming FE analyses.

Furthermore, the choice of an appropriate shape parameterisation allows to describe the shape of the stabilizers by means of continuous design variables. Consequently a standard Quasi-Newton method is proficiently used to run the optimization searches.

3.1 Optimization Domain

The shape of the stabilisers is parameterised so that it only depends on the geometry of the upper and lower cross-section edges. The upper and lower edges are defined, in the reference xy-plane, using the following hyper-elliptical function [13]:

\[
\left(\frac{x}{a}\right)^n + \left(\frac{y}{a}\right)^n = 1
\]

where \(a\) is the length of the generalised semi-axis and \(n\) is the exponent of the hyper-ellipse.

Accordingly, the optimisation domain is made depending on four distinct design variables:

- the first one, \(a_1\), describes the semi-axis of the upper edge of the stabilizers. It ranges from 25.0 mm to 50.0 mm
- similarly, the second one, \(a_2\), describes the semi-axis of the lower upper edge of the stabilizers. It ranges from 25.0 mm to 50.0 mm
- \(n_1\) represents the exponent of the upper edge and ranges from 0.5 to 5.0
- \(n_2\) represents the exponent of the lower edge and ranges from 0.5 to 5.0

Furthermore, a constraint was imposed on the first two design variables to have an upper cross-section smaller than the lower one; accordingly:

\[
a_1 \leq a_2
\]

Fig. 4. Shapes of the stabilizer as a function of the design variables.
The adopted parameterisation allows to describe a wide range of different shapes for the stabiliser as shown in Figure 4. In fact, the cross-section of the stabilizers is circular when \( n=2 \) and square when \( n=1 \). When \( n_1=n_2=2 \) and \( a_1=a_2=r \), the stabilisers form a cylinder with radius equals to \( r \), while, when \( n_1=n_2=1 \), the stabilisers form a tube with square section having the edges which form 45 deg angles with the lateral panels.

### 3.2 Allocation of the Sample Points

The sample points, which will be used to built the global approximation, are chosen inside the optimisation domain using a classical approach: the Latin hypercube [7,14].

Essentially, the matrix that defines the position of each sample point inside the interest domain is randomly arranged to construct the Design of Experiments. Different Designs of Experiments are generated and the best one is selected using an algorithm based on the MaxMin criterion: the aim of the algorithm is to maximise the minimum distance between any pairs of the sample points.

The classical Latin hypercube method is modified in order to handle the presence of constraints and discrete variables. Thus, it is possible to consider irregular and non-convex domains.

As there are no established and general enough rules to correctly predict the total number of sample points necessary to obtain a desired level of accuracy, it is important to be able to add new sample points to an existing Design of Experiments. A fitting technique is then developed within the Latin hypercube framework.

The described methodology is used in this work to generate a first set of 30 points in the optimisation domain, as depicted in Figure 5(a). Two different symbols are associated to each sample point in the \( a_1a_2 \)-plane: a circle and a diamond. The size of the symbols, namely of the circle and of the diamond, is proportional to the value of \( n_1 \) and \( n_2 \), respectively.

![Fig. 5. Initial (a) and fitted (b) Design of Experiments.](image)

The initial set of the sample points is then fitted with two other sets, each one consisting of 20 points. Figure 5(b) shows the first set of fitting points.

As desired, also all these new points satisfy the constraint on the design variable, eq. (2).

### 3.3 Definition of the Global Approximations

During the optimization process, the weight of the intersection elements is calculated using analytical formulae while the crash behaviour is estimated using different response surfaces defined overall the optimisation domain. Since three crash parameters are considered, three distinct response surface are built using the RBF technique:
the first peak force level,
the ultimate force level, defined as the force returned by the absorption device when a final shortening of 150 mm is achieved, and
the energy absorbed at a final shortening of 150 mm.

The accuracy of the response surfaces is evaluated by means of two accuracy indices: the Average Percentage Error (APE) and the Maximum Absolute Error (RMAE).

The first estimator of accuracy is a global measure of the average percentage error and is defined as:

$$APE = \frac{\sum_{i=1}^{M} |F(\bar{x}_i) - f(\bar{x}_i)|}{\sum_{i=1}^{M} |F(\bar{x}_i)|} \cdot 100$$

(1)

Since an overall accuracy does not necessarily imply a good local accuracy, the second estimator of accuracy is related to the maximum local errors computed as:

$$RMAE = \max_{i=1,...,M} \|F(\bar{x}_i) - f(\bar{x}_i)\|$$

$$\sqrt{\frac{1}{2} \sum_{i=1}^{M} [F(\bar{x}_i) - f(\bar{x}_i)]^2}$$

(2)

where M is the total number of points, $\bar{x}_i$, which are used to evaluate the accuracy of the approximation, $F(\bar{x}_i)$ is the exact value of the function to be approximated and $f(\bar{x}_i)$ is its approximated value.

In order to investigate the influence that the number of sample points reflect on the obtained accuracy level, it was decided to build the response surfaces in three subsequently steps. In the first one, the response surface are made using only the first set of 30 sample points, i.e. using the Design of Experiments of Figure 5(a).

The estimators of accuracy are then calculated for all the remaining 40 sample points. In the second and third steps, the response surfaces are re-built adding new sample points to the original set and calculating the estimators of accuracy for the remaining ones.

The obtained values of accuracy are reported in Table 3 for each approximation, while the total number of sample points is progressively increased.

<table>
<thead>
<tr>
<th>Sample points</th>
<th>APE</th>
<th>RMAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>First peak force</td>
<td>30</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>7.1</td>
</tr>
<tr>
<td>Ultimate force</td>
<td>30</td>
<td>12.1</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>9.2</td>
</tr>
<tr>
<td>Absorbed energy</td>
<td>30</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Tab. 3. Accuracy of the obtained response surfaces.

As an example of the approximations the values of the absorbed energy are shown in Figures 6-8 as a function of the exponents of the upper and lower edges. Three different values of the semi-axes have been considered, namely 30 mm, 40 mm and 50 mm.

Fig. 6. Absorbed energy changing the exponents of the upper and lower edges – $a_1=a_2=30$ mm.
5 Optimization Results

As discussed in previous works [15], the design of crashworthy subfloors is obtained introducing absorbing devices between the cabin-floor and the fuselage skin. The physical characteristics of these devices are selected with regard to the energy to be absorbed and so that the value of force/deceleration on the cabin crew remains within the tolerance limits of human body. Indeed, if these devices are not properly designed, the strength and the stiffness of the whole subfloor might result inadequate to redistribute the loads. As a consequence, the capabilities of the subfloor to absorb energy are reduced. In these cases, additional weight could be required to redistribute the impact loads overall the subfloor structure. Thus, the positions and the characteristics of the absorption elements must be evaluated as soon as possible during the design phases.

In this scenario, it becomes extremely important to have wide libraries of optimized intersection elements that can be used to base the design of the subfloor. In this way, supposing that the height of the subfloor has been fixed, it would be easy to select the characteristics of each energy absorption device once its mean force is known. As a consequence, the availability of optimal absorption elements will bring to the design of light and high-efficiency subfloors.

The research of optimal intersection elements is then the main objective of this work. It could be state that an optimal absorption device is an element that provides the desired mean force level with the lowest weight and the highest ratio between its mean and maximum force.

5.1 Formulation of the Optimization Problem

Accordingly with the observations previously made and recalling that absorption devices made in composite materials are in general characterized by very high ratios between mean and maximum force, the optimization problem can be formulated as:

\[
\text{minimize } W(\bar{x})
\]

subjected to:

\[
\begin{align*}
& a_1 > a_2 \\
& F_{\text{MEAN}} \approx \overline{F}_{\text{MEAN}}
\end{align*}
\] (4)

where \(W(\bar{x})\) is the weight of the intersection elements, while \(\bar{x}\) is the vector of the design variables.

The first constraint in eq. (6) forces the optimization process to identify a final configuration for which the semi-axis of the lower edge, \(a_1\), is non-minor of the upper one, \(a_2\). The second constraint in eq. (6) imposes that...
the mean force of the final configuration is almost equal to the desired one.

5.2 Results

It was then decided to perform four distinct optimizations considering the following level of required mean force: 15 kN, 20 kN, 25 kN and 30 kN. The results of these optimizations are summarized in Table 4, as returned by the response surfaces already defined and validated.

<table>
<thead>
<tr>
<th>Mean Force [kN]</th>
<th>Mass [kg]</th>
<th>Design Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired</td>
<td>Final</td>
<td>$a_1$ [mm]</td>
</tr>
<tr>
<td>15.0</td>
<td>15.3</td>
<td>0.286</td>
</tr>
<tr>
<td>20.0</td>
<td>20.2</td>
<td>0.248</td>
</tr>
<tr>
<td>25.0</td>
<td>25.4</td>
<td>0.251</td>
</tr>
<tr>
<td>30.0</td>
<td>30.2</td>
<td>0.271</td>
</tr>
</tbody>
</table>

Tab. 4. Optimization results.

The performances obtained by the different optimizations indicate that the efficiency of the absorber, in terms of the ratio between the mean force and the weight, increases together with the required mean force level, as shown in Figure 9.

Among the previous identified optimal configurations, the one with mean force of 25 kN has been selected for further FE investigations. The results obtained by the global approximation are then compared to those obtained by a FE analysis in Table 5.

<table>
<thead>
<tr>
<th>Mass [kg]</th>
<th>Mean Force [kN]</th>
<th>Absorbed Energy [kJ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBF</td>
<td>FEM</td>
<td>RBF</td>
</tr>
<tr>
<td>0.251</td>
<td>25.37</td>
<td>25.79</td>
</tr>
</tbody>
</table>

Tab. 5. Third optimized configuration, RBF and FEM analysis.
6 Conclusions

Modern helicopter subfloors are called to accomplish structural functions under crashworthiness requirements to assure the safety of occupants. The development of numerical procedures and analysis methods able to drive the design of high-efficiency crashworthy structures in short time period could represent the key for a new generation of successful helicopter subfloors. Indeed, the increasing interest in composite materials requires new validated numerical models able to correctly predict their crash behaviour and their damage mechanisms.

In this work, a material model available within Ls-Dyna has been validated referring to experimental data on cylindrical specimens, providing good numerical-experimental correlations. Hence, the material model has been used for the optimization of a typical intersection element of helicopter subfloors.

For this purpose, an optimization procedure based on Quasi-Newton algorithms and Radial Basis Functions, used as a global approximation technique, has been employed together with high fidelity Finite Element analyses. An appropriate parameterisation of the shape and an accurate choice of the sample points inside the optimization domain (Design of Experiment) also allowed a rather effective formulation of the optimisation problem.

Since the proposed optimisation procedure has the merit not to require new Finite Element analyses once the global approximation has been defined, four distinct optimizations have been performed changing the desired level of mean force or, equivalently, of absorbed energy.

The results obtained with the Radial Basis Functions are then validated by means of further finite element analyses.

The performances obtained by the different optimizations indicate that the efficiency of the absorber, in terms of the ratio between the mean force and the weight, increases together with the required mean force level. This trade-off should be proficiently considered in the preliminary phases of the subfloor design with respect to the need to achieve a good load distribution on the floor during crash events. In this within, the data collected from the optimizations should be used to find an appropriate configuration (number, characteristics and positions) of the absorption elements inside the subfloor.

Acknowledgement

The authors would like to express their appreciation to Prof. Giuseppe Sala of the Politecnico di Milano for the support he provided in the manufacturing of the cylindrical shells and for the interesting discussions on the behaviour of composite materials.

References


