NONLINEAR MODEL MATCHING FLIGHT CONTROL SYSTEM FOR LIFT/Cruise FAN AIRCRAFT

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Abstract
Up to the present various flight control systems have been proposed for linear aircraft motion on some equilibrium point such as steady flight, but very little has been done for nonlinear one on dynamic maneuver. Originally aircraft motion is expressed with six-degree-of-freedom nonlinear equations, especially the equations become more complicated for some special VTOL aircraft which change the form because the thrusts act like variable vectors.

To solve the problems, we propose a nonlinear model matching method. This method will give a hint to solve other complicated nonlinear control problems.

1 Introduction
Recently many advanced middle size airplanes have been developed for transportation of the passengers and the freight. Generally airplanes need vast airports [1], especially the long run ways. Also it seems that it costs a vast sum of money to construct and manage an airport. Then it is useful if a VTOL [2] and middle size aircraft, which does not need the airports, could be developed.

The VTOL and middle size aircraft suggested in this study has the flat body, no main plains, and the four duct fans which angles can be adequately controlled. It hovers from the apron near a coast line, speeds up above sea surface making the best of the ground effect by the four duct fans, turns the duct fans backward gradually and flies like a normal airplane with the lift arisen by the flat body. It is named “Lift/Cruise Fan Aircraft (L/CFA) [3]”.

In the past time NASA had tried to develop a Lift/Cruise Fan Type VTOL Aircraft X-22A [4] which was very similar to above aircraft, but the plan was suspended for some reason. At present time some Tilt Rotor Type VTOL aircraft like BA609 [5] have been already developed, but this type VTOL aircraft has the small payload.

And so far linear control laws like PI control law with gain scheduler have been used for normal airplanes. But in the case of above VTOL aircraft, the equations of aircraft motion [6] include several terms of the products and the trigonometric functions with respect to the state-space variables. That is, this controlled system is complicated nonlinear one. Perhaps the enough control performance can not be obtained by linear control law such as PI controller. Especially in the equations there exist several terms of the products of the thrust and the trigonometric function of the duct fan angles. It means that the control law can not be determined uniquely, because both the thrusts and the duct fan angles may be considered as the main control inputs for L/CFA. Then we have a question, whoever or whatever turns the duct fans backward gradually on taking off and landing and makes the aircraft go forward? We think the answer is only an expert pilot at present time.

To solve the above problems, we propose a nonlinear model matching method [7] and attempt to apply it for the maneuver of L/CFA. This method will give a hint to solve other complicated nonlinear control problems.
2 Longitudinal Nonlinear Equations of Aircraft Motion

In this section, the longitudinal nonlinear equations of aircraft motion [6] for L/CFA are shown as the controlled system. At first the nonlinear ones are constructed in continuous time, secondly they are transformed to the discrete form [7].

2.1 Longitudinal Motion of L/CFA

The body axes of L/CFA can be set as in Fig. 1.

![Fig. 1. Body Axes of L/CFA](image)

Then the longitudinal nonlinear equations of L/CFA motion are expressed as follows:

\[
\dot{\mathbf{m}}(\mathbf{U} + \mathbf{QW} - \mathbf{VR} + g \sin \Theta) = \mathbf{X} \tag{1}
\]

\[
\dot{\mathbf{m}}(\mathbf{W} + \mathbf{PV} - \mathbf{UQ} - g \cos \Theta \cos \Phi) = \mathbf{Z} \tag{2}
\]

\[
\dot{\mathbf{Q}} \mathbf{I}_y + \mathbf{PR}(\mathbf{I}_x - \mathbf{I}_y) + (\mathbf{P}^2 - \mathbf{R}^2) \mathbf{J}_{xz} = \mathbf{M} \tag{3}
\]

where \( \mathbf{m} \): aircraft mass [kg], \( g \): gravity acceleration [9.8 m/s\(^2\)], \( \mathbf{I}_x, \mathbf{I}_y, \mathbf{I}_z \): moments of inertia about each body axis [kg \( \cdot \) m\(^2\)], \( \mathbf{J}_{xz} \): product of inertia [kg \( \cdot \) m\(^2\)], \( \mathbf{X} \): thrust and aerodynamic force in direction of X body axis [N], \( \mathbf{Z} \): thrust and aerodynamic force in direction of Z body axis [N], \( \mathbf{M} \): aerodynamic moment about Y body axis [N \( \cdot \) m].

Also the longitudinal model of L/CFA motion [3] can be considered such as Fig. 2.

![Fig. 2. Longitudinal Model of L/CFA](image)

Where \( T_F \) and \( T_R \): thrusts of the front and rear duct fans [N], \( T_{Fu} \) and \( T_{Ru} \): horizontal elements of \( T_F \) and \( T_R \) [N], \( T_{Fw} \) and \( T_{Rw} \): vertical elements of \( T_F \) and \( T_R \) [N], \( \Theta_F \) and \( \Theta_R \): angles of the front and rear duct fans [deg].

Then \( X, Z \) and \( M \) [6] in the right hands of Eqs. (1), (2) & (4) may be given as follows:

\[
X = \frac{1}{2} \rho V_F^2 S (C_x + C_{x69} \Theta_F + C_{x69} \Theta_R) + T_F \sin \Theta_F \cos \Theta + T_R \sin \Theta_R \cos \Theta \tag{5}
\]

\[
Z = \frac{1}{2} \rho V_F^2 S (C_z + C_{z69} \Theta_F + C_{z69} \Theta_R) - T_F \cos \Theta_F \cos \Theta - T_R \cos \Theta_R \cos \Theta \tag{6}
\]

\[
M = \frac{1}{2} \rho V_F^2 S C \Theta + \frac{1}{4} \rho V_F^2 S C \Theta \Theta + T_F \cos \Theta_F \ell - T_R \cos \Theta_R \ell \tag{7}
\]

Moreover the lateral-directional state variables in Eqs. (1)- (4) can be given as \( V = P = R = \Phi = 0 \), as a result the following equations can be obtained.

[Longitudinal force]

\[
\dot{U} = -QW - g \sin \Theta - \frac{1}{2m} \rho V_F^2 S (C_x + C_{x69} \Theta_F + C_{x69} \Theta_R) + \frac{1}{m} T_F \sin \Theta_F \cos \Theta + \frac{1}{m} T_R \sin \Theta_R \cos \Theta \tag{8}
\]

[Vertical force]

\[
\dot{V} = UQ + g \cos \Theta + \frac{1}{2m} \rho V_F^2 S (C_z + C_{z69} \Theta_F + C_{z69} \Theta_R) - \frac{1}{m} T_F \cos \Theta_F \cos \Theta - \frac{1}{m} T_R \cos \Theta_R \cos \Theta \tag{9}
\]

[Pitch rate]

\[
\dot{\Phi} = Q \tag{10}
\]
[Pitching moment]

\[
\dot{Q} = \frac{1}{2l_y} \rho V^2_T \overline{C}_m (C_m + C_{m6\theta} \theta_F + C_{m6R} \theta_R) \\
+ \frac{1}{4l_y} \rho V^2_T \overline{C}_m^2 m_Q Q \\
+ \frac{1}{l_y} T_F \cos \theta_F \ell - \frac{1}{l_y} T_R \cos \theta_R \ell
\] (11)

Now we have a problem for the control inputs. As you see, the products of the thrust and the trigonometric function of the duct fan angles exist in the above equations, the control law can not be determined uniquely. Because both thrust and duct fan angle can be considered as the control inputs. Then consider the front and rear duct fan angles as the state variables, the following first order systems are added to the above equations.

\[
\dot{\theta}_F = -\frac{1}{T} \theta_F + \frac{1}{T} u_F \\
\dot{\theta}_R = -\frac{1}{T} \theta_R + \frac{1}{T} u_R
\] (12, 13)

Where T: time constant of duct fan actuator, u_F: and u_R: piloted front and rear duct fan angles.

2.2 Discrete Time Nonlinear Equation of L/CFA

Normally we have no strict transform method for nonlinear equation between continuous and discrete. Then apply following first order approximation [7] to the differentiation terms of state variables in left hands of Eqs.(8)-(13). \( \Delta \) is the sampling time [s].

\[\dot{x}(t) = [x_i(k+1) - x_i(k)]/\Delta \] (14)

The discrete time nonlinear state-space equation can be given as follows:

\[x(k+1) = F(x) + B(x)u(k), y(k) = C x(k)\] (15)

where \( x(k) = [U(k), W(k), \theta(k), Q(k), \theta_F(k), \theta_R(k)]^T \): state variable vector, \( u(t) = [T_F(t), T_R(t), U_F(t), U_R(t)]^T \): input vector, \( y(t) = [W(t), \theta(t), \theta_F(t), \theta_R(t)]^T \): output vector and

\[
C = [c_1^T, c_2^T, c_3^T, c_4^T]^T \\
c_1 = [0 1 0 0 0 0 ] \\
c_2 = [0 0 1 0 0 0 ] \\
c_3 = [0 0 0 0 1 0 ] \\
c_4 = [0 0 0 0 0 1 ] \\
F(x) = [f_1(k), f_2(k), ..., f_6(k)]^T \\
f_1(k) = U(k) - \Delta Q(k)W(k) - \Delta g \sin \theta(k) \\
\frac{\Delta}{2m} \rho V^2_T[C_x + C_{s6\theta} \theta_F(k) + C_{s6R} \theta_R(k)] \\
f_2(k) = W(k) + \Delta U(k)Q(k) + \Delta g \cos \theta(k) \\
\frac{\Delta}{2m} \rho V^2_T[C_x + C_{s6\theta} \theta_F(k) + C_{s6R} \theta_R(k)] \\
f_3(k) = \theta_F(k) + \Delta \theta_F(k) \\
f_4(k) = Q(k) + \frac{\Delta}{2l_y} \rho V^2_T \overline{C}_m (C_m + C_{m6\theta} \theta_F(k) + C_{m6R} \theta_R(k)) \\
\frac{\Delta}{4l_y} \rho V^2_T \overline{C}_m^2 m_Q Q(k) \\
f_5(k) = \theta_R(k) - \frac{\Delta}{T} \theta_R(k) \\
f_6(k) = \theta_R(k) - \frac{\Delta}{T} \theta_R(k) \\
B(x) = [b_1^T(x), b_2^T(x), ..., b_6^T(x)]^T \\
b_1(x) = [b_{11}(x) b_{12}(x) 0 0 ] \\
b_{11}(x) = \frac{\Delta}{m} \sin \theta_F(k) \cos \theta(k) \\
b_{12}(x) = \frac{\Delta}{m} \sin \theta_R(k) \cos \theta(k) \\
b_2(x) = [b_{21}(x) b_{22}(x) 0 0 ] \\
b_{21}(x) = -\frac{\Delta}{m} \cos \theta_F(k) \cos \theta(k) \\
b_{22}(x) = -\frac{\Delta}{m} \cos \theta_R(k) \cos \theta(k) \\
b_3(x) = [0 0 0 0 0 ] \\
b_4(x) = [b_{41}(x) b_{42}(x) 0 0 ] \\
b_{41}(x) = \frac{\Delta}{l_y} \cos \theta_F(k) \ell \\
b_{42}(x) = -\frac{\Delta}{l_y} \cos \theta_R(k) \ell \\
b_5(x) = [0 0 \frac{\Delta}{T} 0 0 ] \\
b_6(x) = [0 0 \frac{\Delta}{T} 0 0 ]
3 SYNTHESIS OF NONLINEAR MODEL MATCHING CONTROL SYSTEM

In this section a design method using our proposed nonlinear model matching method [7] for nonlinear system is generally described.

3.1 Formulation of the Problem

Consider the following nonlinear system as a controlled system.

(System $\Sigma$)
\[
\begin{align*}
    x(k+1) &= F(x) + B(x)u(k), \quad x(0) = x_0 \quad (16) \\
    y(k) &= Cx(k) \quad (17)
\end{align*}
\]

where
\[
F(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T, \quad f_i(x) : \mathbb{R}^n \to \mathbb{R}
\]
\[
B(x) = [b_1(x), b_2(x), \ldots, b_n(x)]^T, \quad b_i(x) : \mathbb{R}^n \to \mathbb{R}^p
\]
\[
C = [c_1, c_2, \ldots, c_p]^T \in \mathbb{R}^{p \times n}
\]
\[
x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T \in \mathbb{R}^n
\]
\[
y(k) = [y_1(k), y_2(k), \ldots, y_p(k)]^T \in \mathbb{R}^p
\]
\[
u(k) = [u_1(k), u_2(k), \ldots, u_p(k)]^T \in \mathbb{R}^p
\]
\[
x(k) \in \mathbb{R}^n, \quad y(k) \in \mathbb{R}^n, \quad u(k) \in \mathbb{R}^p
\]

On the other hand, consider the following equation as a reference model which the system designer sets arbitrarily.

(Reference System $\Sigma_M$)
\[
\begin{align*}
    x_M(k+1) &= A_M x_M(k) + B_M u_M(k) \quad (18) \\
    y_M(k) &= C_M x_M(k) \quad (19)
\end{align*}
\]

where
\[
x_M(k) = [x_{M1}(k), x_{M2}(k), \ldots, x_{Mn}(k)]^T
\]
\[
y_M(k) = [y_{M1}(k), y_{M2}(k), \ldots, y_{Mp}(k)]^T
\]
\[
u_M(k) = [u_{M1}(k), u_{M2}(k), \ldots, u_{Mp}(k)]^T
\]
\[
C_M = [c_{M1}, c_{M2}, \ldots, c_{Mp}]^T
\]
\[
x_M(k) \in \mathbb{R}^m, \quad u_M(k) \in \mathbb{R}^p, \quad y_M(k) \in \mathbb{R}^p
\]

The objective of this study is to design a model matching control system which forces the output of the state vector nonlinear system $y(k)$ to match the reference model output $y_M(k)$.

And here the model matching is defined as follows:

Definition:
For the following condition of System $\Sigma$ and Reference Model $\Sigma_M$
\[
x_0 = 0, \quad x_M(0) = 0, \quad F(x_0) = 0, \quad B(x_0) = 0
\]
when
\[
y_M(k) = y(k) \quad \text{for} \quad k \geq 0
\]
can be achieved, it is called that System $\Sigma$ can be model-matched to Reference Model $\Sigma_M$.

3.2 Nonlinear Model Matching

In this subsection, for System $\Sigma$ and Reference System $\Sigma_M$, the dynamic model matching control system based on Hirschorn’s algorithm [8] extended with Silverman’s structure algorithm [9] is proposed. For System $\Sigma$ and Reference System $\Sigma_M$, perform the following procedure.

[Step 1] Consider the time shift signals of the output $y_i(t)$ and left-multiply the both sides of Eqs. (17) & (19) by $z$, the following equations are obtained
\[
z y_M(k) = c_{i1} F(x) + c_{i2} B(x) u(k)
\]
\[
z y_M(k) = c_{i1} A_M x_M(k) + c_{i2} B_M u_M(k)
\]

where $z$ is the time-shift operator.

Next formally replace the above equations with as follows:
\[
z^{[i]} y_M(k) = c_{i1} A_M x_M(k) + c_{i2} B_M u_M(k)
\]
where the left "1" of subindex "11" means the 1st output, the right "1" means the 1st power of $z$. And generally $c_{11}$, $c_{M1}$, $A_M$, $B_M$, $u_M$ and $y_M$ are polynomial functions and matrices with respect to the state variable.

In the above equations, when if $D_{ai}(x) \neq 0$, replace $f_i$, $C_M$, $A_M$ and $B_M$ with $f_i$, $C_M$, $A_M$ and $B_M$ and go to the next step. When $D_{ai}(x) = 0$, the following equations are obtained by repeating the time shift
\[
z^{[i]} y_M(k) = c_{i1} A_M x_M(k) + c_{i2} B_M u_M(k)
\]
where $Da_i(x) \neq 0$ and it is assumed that "j" which satisfies the above equations exists. Likewise, replace the subindices "1" of $f_i$, $C_M$ and $A_M$ in the above equations with "1" and go to the next step.
[Step 2] Do the same procedure as Step 1 for the output \( y_2(k) \), the following equations are obtained

\[
\begin{align*}
z^{f_21}y_2(k) &= Ca_2(x) + Da_2(x)u(k) \\
z^{f_21}y_{M2}(k) &= CaM_2(x)x_M + DaM_2(x)u_M(k)
\end{align*}
\]

[Step 3] When if \( Da_2(x) \neq \alpha_21(x)Da_1(x); \alpha_21(x) \neq 0 \), replace the subindex "2j" with "2" and do the same procedure from Step 2 for \( y_3(k) \). Where, \( \alpha_21(x) \) is a polynomial function with respect to \( x(t) \).

When if \( Da_2(x)=\alpha_21(x)Da_1(x) \), consider the new outputs[10] as follows:

\[
\begin{align*}
-z^{j}y_f(1)(k) + z^{f_1j}y_2(k) \\
-z^{j}y_{M1}(k) + z^{f_21}y_{M2}(k)
\end{align*}
\]

and do the same procedure from Step 2.

[Step 4] By repeating the above procedure to the outputs \( y_f(k) \) and \( y_{Mj}(k) \), the following equations can be obtained

\[
\begin{align*}
Na(z, x)y(k) &= Ca(x) + Da(x) u(k) \\
Na(z, x)y_M(k) &= Ca_M(x)x_M + Da_M(x)u_M(k)
\end{align*}
\]

where \( Na(z, x) \) is a lower triangular matrix in which the diagonal entries are \( z^{-1} \), and \( Ca(x) \) and \( Da(x) \) are respectively

\[
\begin{align*}
Ca(x) &= [Ca_1(x), Ca_2(x), \ldots, Ca_p(x)]^T \\
Da(x) &= [Da_1^T(x), Da_2^T(x), \ldots, Da_p^T(x)]^T
\end{align*}
\]

\[
\begin{align*}
Ca_M(x) &= [Ca_{M1}(x), Ca_{M2}(x), \ldots, Ca_{Mp}(x)]^T \\
Da_M(x) &= [Da_{M1}(x), Da_{M2}(x), \ldots, Da_{Mp}(x)]^T.
\end{align*}
\]

Then it is clear that \( Na(z, x) \) is a lower triangular matrix because of the procedure in Step 3 which the relations between \( y_f(k) \) and \( u(k) \) can be obtained with the time shift form of \( y_f(k) \).

Using the above relation the following theorem can be obtained.

<Theorem>
If the following condition is satisfied

\[
\text{rank}(Da(x)) = p \quad \text{for} \quad \forall x(k) \in R^n
\]

System \( \Sigma \) can be model-matched to Reference System \( \Sigma_M \) by the control law \( u(k) \) as

\[
u(k) = Da(x)^{-1}[ - Ca(x) + CaM(x)x_M(k) \\
+ DaM(x)u_M(k) ]
\]

(Proof) Define the output error \( e(k) \) as

\[
e(k) = y_M(k) - y(k)
\]

the following relation can be obtained using Eqs.(20)--(23)

\[
Na(z, x)e(k) = 0
\]

Where notice the form of \( Na(z, x) \), especially the diagonal entries which have the stable polynomials, for the condition : \( x(0)=0, \ x_M(0)=0, \ F(x_0)=0, \ B(x_0)=0, \) the following relation can be obtained

\[
y(k) = y_M(k), \quad \text{for} \quad k \geq 0
\]

and the model matching can be achieved. Also because of Eq.(24), for the arbitrary initial values, the following relation can be obtained

\[
y(k) \rightarrow y_M(k), \quad \text{for} \quad k \rightarrow \infty
\]

[Comment] As a result, by replacing \( Na(z, x) \) with an interactor matrix [10] of a system, we can understand that this method is a extension of the linear model matching control system proposed by Wolovich [11].

### 4 Application to Lift/Cruise Fan Aircraft

In this section, we attempt to apply the proposed method to the flight control system for the L/CFA, and investigate the feasibility by numerical simulations. The data [4] of L/CFA, flight condition [12] and reference models are given as follows. Yet many data of another aircraft are included in them.

#### [Data of L/CFA]

m: 5195 [kg], \( I_z: 178457 \) [kg·m²], S: 39.56 [m²], \( \overline{C} : 4.89 \) [m], \( \ell : 4.5 \) [m], \( \Delta : 0.05 \) [s]

#### [Flight Condition]

Hovering at altitude 6,000 [m], \( \rho : 0.5495 \) [kg/m³], \( C_X: -0.0325 \), \( C_Z: -0.851 \), \( C_{m_0}: 0.0373 \), \( C_X \Theta : 1.6 \), \( C_Z \Theta : 2.5 \), \( C_{m_0}: 3.4 \), \( C_X \Theta : 4.3 \), \( C_{m_0}: 2.5 \), \( C_m \Theta : 6.1 \) and \( C_{m_0}: 6.0 \).

#### [Reference Model]

2nd order transfer functions with damping ratio \( \zeta = 0.9 \) and natural frequency \( \omega_n = 5.2 \) [rad/s] are given for each output. The reference inputs are given as \( u_M(k) = \{0.1 \text{[m/s]}, 0\text{[deg]}, 0\text{[deg]}, 0\text{[deg]}\} \) from the beginning to 1 sec, \( u_M(k) = \{1 \text{[m/s]}, 0\text{[deg]}, 10\text{[deg]}, 10\text{[deg]}\} \) from 1 sec to 5 sec, \( u_M(k) = \{-1, 0, 10, 10\} \) from 5 sec to 10 sec.
The results show that each output perfectly matched to the reference model outputs and the smooth vertical translation keeping the pitch angle 0 deg could be accomplished. Especially the front and rear duct fans are gradually turned backward, the forward velocity approximately 60 km/h could be obtained. Also the thrusts of front and rear duct fans are made within the range of available power.

5 Conclusions
In this paper, we suggested a new type of L/CFA and attempted to apply our nonlinear model matching control method for its maneuver. And we
showed the feasibility of proposed method with numerical simulations. However it is necessary to consider the following premises or assumptions. 1 The inverse system is stable. But normally an inverse system of nonlinear plant cannot be defined and its stability cannot be proved. 2 All state variables of the aircraft can be measured. 3 The plant is an M inputs and M outputs system. 4 The plant parameters [3,13,14] such as non-dimensional aerodynamic derivatives are fixed. 5 The effect of disturbance [15,16] such as gust is not considered. These are remained as the theme which should be improved.

References