Abstract

The experimental approaches are employed to investigate the wake development of finite wing with aspect ratio 3. The stream-wise and transverse velocities in the wake region are measured by the X-shaped hotwire. In the present experimental results, the wake of the finite wing in the near field could be divided into two parts in the span-wise direction. One region is with the same behaviors as those of 2-D wake and the other is affected by the wing-tip vortex. In the 2-D region, the flow filed is obviously affected by the downwash. By the comparison the downwash rates of experimental results and those by theoretical analysis, a good consistence could be found. The similarity distribution of mean velocity and root-mean-square velocity could also be observed in the region. Comparing the maximum defective velocity $U_s$, we found $U_s \propto x^{-\frac{1}{3}}$, which is compatible with what happens in the 2-D turbulent wake.

In the 3-d region, the low pressure region will form as the result of the development of the wing tip vortices to the downstream. Hence it will suck the fluid upstream and the average velocity measured will larger than that of free stream. According to the downstream evolution of $U$ contours, the development of trailing vortices could be divided into growing, saturating and decaying process.

1. Introduction

The major cause of the flight capacity limit of take-off called trailing vortices left by the airplane. The trailing vortices could travel for a long distance in the downstream. Generally speaking, the strength of the vortex system is proportional to the weight of the airplane.

Regarding the growth aviation traffic today, enhancing the capacity of take-off and landing has become an important issue. The main reason is that if the interval for take-off and landing could be reduced effectively, then the cost and time could be saved. The aspects of the trailing-vortex-related researches could be divided into two ways: (I) to enhance the knowledge of the growing and decaying of the trailing vortex by the investigation of flow field [1-7] (II) to decrease the intervals of take-off and landing by the flow control technique.[8-11].

As mentioned in the previous words, we can know that the research and control of wake flow field has become a hot topic. Especially for raising the efficiency and safety of the aviation in recent years, many ones have donated themselves in it. However, the achievement seems unsatisfied. From the review papers by Gad-el-Hak [12], we’re informed that for a shear flow field such as free shear flow and jet flow the most efficient location of control is where the instable wave originates from. Hsiao and Liu[13] also proved this point of view by their experiment. We can found by reference [12] that researches about flow control are mainly occupied by 2-D flow, which is due to the easy access to the 2-D flow field. Combining what’s mentioned above, in order to lower the strength of the wing tip vortex, to study the wake development of 3-D lifting surface is necessary. Thus, a detail experimental investigation is made to understand the influence of lifting effects on the wake development.

2. Equipments and Measurements

This experiment was carried out in the low speed wind tunnel in the National Cheng Kung University. The wind tunnel is 17 meters long and composed of guiding section, contracting section, test section,
diverging section and the driving system. The contraction ratio of the entrance and test section is 9:1. The testing is with area 90cm × 120cm and with length 30cm. The effective range of wing speed ranges from 2m/s to 35m/s and the turbulence level in the test section is under 1%. The wing model of this experiment is a rectangular wing without taper, of wing-span 200mm, and aspect ratio 3. The airfoil is NACA 0012. This model is supported by a truss. The model and its coordinate system is illustrated in Figure 1. The Reynolds number is $Re = 7 \times 10^4$ under the reference length of the wing chord. The measuring device is pitot tube and X-shaped hot wire equipped on a movable 3 D.O.F. platform. The area under measuring is divided into 8 sites counting form the rear part of the wing, as illustrated in Figure 2. The measured angles of attack are 0°, 5° and 12° respectively. The detailed average flow velocity and its root-mean-square will be compared and investigated in detail for x and y direction.

3. Results and Discussion

3.1 Division of wake flow region

Because of the larger pressure on the lower surface of wing than the upper one, the fluid beneath rolls up onto the upper side to form the wing tip vortex. For wing of low aspect ratio, this vortex could provide extra lift force. However, the existence of wing tip vortex also causes the complexity of flow separation. We divided the wing surface into two regions along span-wise direction. One region is near the wing root, where the flow separation is analogous to 2-D flow field. The other one is near the wing tip, where the flow separation causes the wing tip vortex. Hence we can infer that the characteristics of the wake show different behavior along the span-wise direction. To prove it, we measure the stream-wise and span-wise velocity at the rear part of the wing for different z location. Figure 3-a is the distribution of average velocity in x-direction for A.O.A.=0° at x/b=0.33. We can find that its velocity distribution is analogous to what happens in 2-D flow field and the strength of wake is not large such that the wake effect of the truss can be seen near the wing root. The more it approaches the wing tip, the lower the effect of 2-D wake will be. The closer the location to the tip, the lower the 2-D effect, and it almost disappear after z/c=2.10. When the A.O.A. is 5° or 12°, the velocity distribution at 0 <z/c< 1.4 still remains the characteristics of 2-D wake flow field. But the average stream-wise velocity is larger than that of free stream as illustrated in Figure 3-b and Figure 3-c. This is mainly because the wing tip vortex will suck the outer fluid and hence to accelerate it. This is also the cause to increase the non-linear lift force of a wing of low aspect ratio.

Figure 4 shows the contour diagram of $U_{xy} = \sqrt{u^2 + v^2}$ for A.O.A. =12° at x/b=0.33. We found that at 1.54 <z/c< 2.10 there exists an obvious strong contour of high speed velocity. Its distribution type is quite analogous to the known vortex. Thus this kind of fluid structure could be taken as the main characteristics of the trailing vortex.

Since the wing tip vortex is caused by the rolling fluid from the underneath of wing tip, we can infer that the variation of velocity near the wing tip must be stronger. Figure 5 shows v/U distribution for different z/c location at x/b=0.33. It can be seen that the out-boarding velocity at y/c ≥0 is positive , at y/c ≤ 0 is negative , and this value gets smaller along the reference line elongating through y/c=0. This means the wake has the tendency to expand to the outside region. What’s more interesting is from wing root to tip the out-boarding velocity begins with increasing and ends up with rapid decreasing. The highest positive v value occurs at z/c=1.54 on the upper surface and the highest negative v value occurs at z/c=1.68 on the lower surface. Figure 6 describes the comparison of max/min v at x/b=0.33, which obviously shows us this phenomenon. Figure 7 shows contour diagram of A.O.A. =5° and 12° at x/c=1.0, which more obviously points out that there exists a strong circular contour distribution. Comparing to Figure 4, the location where the max out-boarding velocity occurs is just at the center of the vortex. This is because the place where the trailing vortex exists will form a low-pressure region and attracts the outside fluid.

3.2 The influence of downwash on wake flow in the downstream

According to modern aerodynamic theory, we know that the major difference between 3-D and 2-D wing is downwash effect on the latter, which cause the effective A.O.A. smaller. Based on the fact that the downwash affects the resultant velocity, we can deduce that the wake in the downstream is also affected by the same effect. To study this effect, Figure 8 shows The development diagram of the stream-wise velocity along the downstream direction.
for $\alpha=0^\circ, 5^\circ$, and $12^\circ$ at $z/c=0.98$. For $\alpha=0^\circ$, the wake development is beyond the influence of downwash. However, for $\alpha=5^\circ$ and $12^\circ$ the center of wake gets lower along the downstream direction. The larger the A.O.A. is, the more obvious the phenomenon will be. This is due to the larger pressure difference between upper and lower wing surface. Define $y_{\text{wake}}$ and $\frac{dy_{\text{wake}}}{dt}$ to be the $y$ coordinate of the max defective velocity of wake and the descent rate of wake respectively, and illustrate $\frac{dy_{\text{wake}}}{dt}$ on Figure 9. From it, it can be seen that the wake is apt to descent with the descent rate.

Through the analysis of experimental data, the descent rate of $5$ and $12$ degree are as follows

$$\frac{dy_{\text{wake}}}{dt} \approx -0.0571 \quad \text{at } \alpha=5$$

$$\frac{dy_{\text{wake}}}{dt} \approx -0.12 \quad \text{at } \alpha=12$$

$$\frac{dy_{\text{wake}}}{dt} \approx 2.1 \quad \text{at } \alpha=5$$

$$\frac{dy_{\text{wake}}}{dt} \approx 12 \quad \text{at } \alpha=12$$

So we can say that the descent rate of wake is almost proportional to A.O.A.

In order to make a comparison with the theoretical value, here we let the $x$ and $y$ velocity of wake to be $U_s$ and $V_{\text{wash}}$ which is the induced velocity by downwash. We can write,

$$\frac{dy_{\text{wake}}}{dt} = -\frac{V_{\text{wash}}}{U_s} = -\frac{V_{\text{wash}}}{U_c}$$

From theory of aerodynamics,

$$\frac{V_{\text{wash}}}{U_c} = \frac{C_l}{\pi k} = -\frac{2 \pi r + kr^2}{\pi kr}$$

where $k=2$ (16)

and the theoretical value is,

$$\frac{dy_{\text{wake}}}{dt} = -0.0598 \quad \text{at } \alpha=5, \text{theory}$$

$$\frac{dy_{\text{wake}}}{dt} = -0.1489 \quad \text{at } \alpha=12, \text{theory}$$

It shows good agreement between theoretical and experimental values.

### 3.3 2-D region

As shown in Figure 3, the flow field approaches the 2-D flow field in the region of $0.7 \leq z/c \leq 1.4$ and the distribution of the average stream-wise velocity within $0.7 \leq z/c \leq 1.4$ seems analogous. $\frac{V_{\text{wake}}}{x}$ vs. $U-U_s$ at various stream-wise location with $z/c=0.98$ are shown in Figure 10, where $s$ is the half-width of wake and $U_s$ is the maximum defective velocity. We could find a good similarity between them. Thus the wake flow field in the 2-D region of lifting surface conforms to that of general 2-D wake flow field. According to turbulence theory, the max defective velocity $U_s$ of the turbulence wake satisfies:

$$U_s \approx x^{-\frac{3}{2}}$$

Figure 11 is the comparison between log$U_s$ and log$X$ for different A.O.A. at $z/c=0.98$. It shows that the slope of log$U_s$ w.r.t. log$X$ is about $-1/2$, which seems close to the theoretical value. Figure 12 is the distribution of $u'$ and $v'$.

### 3.4 The development of trailing vortex

We can find from Figure 4 that an obvious vortex structure near the wing tip at $z/c=1.54$, which is the so-called trailing vortex. Figure 13 is the development of this vortex in the downstream. The covering range of it is getting larger but its strength is getting weak after $x/b=2.33$. In addition, within the trailing vortex-affected region the stream-wise velocity seems to accelerate due to the attraction from the low pressure of the vortex. So the velocity in this region may exceed that of free stream somewhere. And from this figure we can observe that the variation of trailing vortex conforms to the general mode of a vortex, that is, growing, saturating and decaying.

### 4. Conclusion

Combining what’s mentioned above, it is obvious that the wake development under A.O.A. effect could be divided into two regions due to the existence of wing tip vortex. One the region dominated by characteristics of 2-D wake, and the other by that of 3-D wake.

The 2-D region behaves as the following

1. For case of $\alpha=5^\circ$ and $\alpha=12^\circ$. The 2-D-dominated range is $0 \leq z/c \leq 1.4$.

2. For case of $\alpha=5^\circ$ and $\alpha=12^\circ$. The 2-D region is obviously affected by the downwash and the descent rate of wake $\frac{dy_{\text{wake}}}{dt}$ is compatible with the theoretical value. It shows that the descent rate of wake is governed by the same mechanism as the downwash does.

3. As the mean stream-wise velocity, r.m.s.
stream-wise velocity and r.m.s. transverse velocity are normalized by proper scale in the 2-D region, we could find good compatibility. The max defective velocity \(U_s\) in the wake is proportional to \(x^{-\frac{1}{2}}\). It shows the characteristics in this region obeys that of turbulence wake flow.

4. The wake flow region dominated by wing tip vortex has the following characteristics:
   (a) Because of the existing of wing tip vortex, the wake on the upper surface at \(z/c=1.54\) and \(z/c=1.68\) forms large out-going velocity out-boarding velocity.
   (b) the development of trailing vortices could be divided into growing, saturating and decaying process.

Reference

Figure 3-b $\alpha=5^\circ$, $x/b=0.33$, $U/U_\infty$ distribution at different wing span location

Figure 3-c $\alpha=12^\circ$, $x/b=0.33$, $U/U_\infty$ distribution at different wing span location

Figure 4 $\alpha=12^\circ$, Contour of $\sqrt{U^2+V^2}/U_\infty$

Figure 5 $v/U$ distribution for different $z/c$ location at $x/b=0.33$

Figure 6 Comparison of max/ min $v$ at $x/b=0.33$

Figure 7 Contour diagram of $U$, A.O.A. = $5^\circ$ and $12^\circ$ at $x/c=1.0$

Figure 8 The development diagram of the stream-wise velocity along the downstream direction for $\alpha =0^\circ, 5^\circ, and 12^\circ$ at $z/c=0.98$
Figure 9  $\frac{dy_{wake}}{dz}$ values at different spanwise location

Figure 10 Distribution of U downstream at z/c= 0.98

Figure 11 Comparisons between logUs and logX for different A.O.A. at z/c= 0.98

Figure 12 $\frac{u'}{U_s}$ and $\frac{v'}{U_s}$ distribution for different x/b location at z/c= 0.98,

$\alpha = 5^\circ$
Figure 13 Contours of $U'/U$ at various span-wise locations