OPTIMIZATION OF ACOUSTIC LINER DESIGN FOR AXISYMMETRIC MIXER-EJECTOR NOZZLES UNDER ESPR PROGRAM
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Abstract:
The Environmentally Compatible Propulsion System for Next Generation Supersonic Transport Research and Development Program (ESPR) is aimed at developing and demonstrating a turbofan engine for a supersonic aircraft application. This includes the development of an energy efficient exhaust system with an axisymmetric mixer-ejector for noise reduction. Noise sources generated by a mixer-ejector nozzle include internal noise generated inside the mixer-ejector and pre-merged and merged noise generated outside the ejector. The noise generated inside the ejector has significant high frequency content. The acoustic liner is therefore an essential element for the ejector to suppress the high frequency internal noise. A design process is developed and used to optimize the liner design to maximize the internal noise suppression. A polynomial equation to predict effective perceived noise level (EPNL) as a function of liner design parameters is developed for parametric and trade studies. This paper includes details of the farfield noise prediction process, liner optimization methods, and relevant results.

1. Introduction: An axisymmetric mixer-ejector exhaust nozzle, schematically shown in Figure 1, is planned for the ESPR turbofan engine. Acoustic treatment of the ejector surface becomes a viable means to reduce farfield noise. GEAE is designing appropriate liners for this application. The objective is to design bulk absorber liners for axisymmetric mixer-ejectors to achieve maximum broadband acoustic suppression in the ejector, to minimize farfield noise impact. Acoustic liners for ejector treatment were designed for scale model size (1/11.44), demonstration engine size (1/2.6), and full-scale mixer-ejectors. Optimization of these liners is performed for takeoff conditions. This paper describes the overall liner design process. The relevant results for the scale model design are presented.

2. Basic Process of Acoustic Liner Design: As illustrated in Figure 1, the important components of mixer-ejector farfield noise include noise generated inside the mixer-ejector, and the premerged and merged noise generated outside the ejector. The internal noise propagating inside the ejector has significant high frequency contributions. An acoustic liner is therefore an essential element of the ejector to suppress the
high frequency internal noise. A design process is developed to maximize the internal noise suppression utilizing prediction tools, and scale model aero acoustic data from a previously tested, representative mixer-ejector. The liner design methodology, for the prediction of farfield noise, is shown in Figure 2. Predictions can be made for Perceived Noise Level (PNL) and Effective Perceived Noise Level (EPNL) for a treated mixer-ejector. The external noise referred to here is the sum of premerged and merged noise components. The EPNL computation for a liner design requires normal impedance of the liner, acoustic suppression in the ejector, and the external noise component for the mixer-ejector. The normal impedance is acquired by using prediction models or utilizing measured data. Acoustic suppression predictions are obtained by a modal analysis method. A noise prediction model, developed for mixer-ejector systems, is used to make approximate estimations of the external noise components. Actual external noise levels are determined using the approximate predicted sound pressure level (SPL) spectral shapes and known farfield total noise for the mixer-ejector nozzle. This is usually obtained from measurement. The corresponding internal noise is then extracted from the known total noise, and the predicted external noise.

Construction of liners includes a bulk material with a perforated facesheet. Bulk material properties include its resistivity. Facesheet properties include porosity \( \sigma \), thickness \( t \), and hole diameter \( d \) for perforates, and resistivity for linear facesheets. Utilizing these properties, including the liner depth \( D \), the normal impedance is predicted, accounting for the surface flow Mach number \( M \), temperature \( T \), and overall sound pressure level (OASPL) effects.

Utilizing the normal impedance, mean flow Mach number \( M_x \) and temperature \( T_x \) conditions, and the geometry of the ejector, the acoustic suppression spectrum is predicted by a modal analysis method. Computation of farfield acoustic parameters (including EPNL) for a liner design involves prediction of the internal noise.

![Figure 2. Process of estimating farfield noise due to acoustically treated ejector](image-url)
component in the ejector and its radiation to the farfield. In this process, known farfield acoustic data for mixer-ejectors with hardwall and with a known acoustic treatment are utilized. The predicted acoustic suppression inside an ejector is usually different from the actual measured internal component of $\Delta$PWL (difference of internal sound power level, PWL, between hardwall and treated configurations). This is especially true at lower frequencies. This is due to the simplified assumptions made in the modal analysis model for suppression prediction. A frequency dependent correction factor, calculated from the predicted acoustic suppression and $\Delta$PWL for a test case, is utilized to minimize the prediction uncertainties.

Figure 2 illustrates the process of farfield noise estimation. The physical properties of the liner design (input 1 in the figure) and the flow and acoustic environment on the treatment surface of the mixer-ejector M, T, and OASPL, (input 3 in the figure) are used to estimate the normal impedance of the treatment. The acoustic suppression is then evaluated utilizing the normal impedance, the mean flow parameters ($M_x$ and $T_x$, as input 4), and the physical dimensions of the treated ejector (input 2). The acoustic suppression reduces the internal noise in the ejector and the reduced noise is radiated to the farfield. Farfield noise is a function of the emission angle $\theta$ (defined in Figure 1).

Figure 2 also schematically illustrates the radiated internal noise for treated and hardwall ejector configurations. The external noise is assumed to be the same for hardwall and treated ejector configurations. The internal noise difference between the hardwall and treated configurations (shown as shaded area) is the effect of treatment. This internal noise reduction is the performance of the liner. The total noise is the sum of internal and the external noise components. The total noise reduction due to the liner, shown by shaded area, is the effectiveness of the liner. It is important to realize the difference between the liner performance and liner effectiveness. While the liner performance depends only on the liner design, ejector dimensions, and flow and acoustic environment in the ejector, the liner effectiveness depends on these parameters and also the external noise components. With a best performing liner, the effectiveness can be poor if the external noise is relatively high compared to the internal component.

PWL, PNL, and EPNL are constructed utilizing the spectral sound pressure levels from various emission angle locations for internal, external, and total noise components. Maximum acoustic suppression in terms of minimum SPL and PWL is achieved when the liner impedance is optimum. However, the objective is to minimize EPNL rather than SPL or PWL. EPNL is calculated using weighted noise levels by frequency dependant annoyance factors. If a liner design achieves optimum impedance at frequencies critical to EPNL it may not be necessary to achieve optimum impedance at frequencies with low annoyance factors.

The nozzle parameters, critical frequencies, and various liner design parameters for the three nozzle scales, considered in the current effort, are listed in Table 1. These parameters are generated on the basis of the mixer-ejector design (see Figure 1) and the flow conditions for the engine. Dynamic pressures measured on the internal ejector surface from the representative existing data are utilized to derive the internal OASPLs. Maximum frequencies and peak Noy frequencies for different scales are established on the basis of linear scale factors with respect to full-scale engine. It should be noted that the maximum frequency for full-scale engine for EPNL evaluation is 10 kHz.

Measured acoustic data for a mixer-ejector with hardwall and with a known acoustic treatment is essential for optimum liner design. Measured farfield acoustic data from the representative axisymmetric mixer-ejector are adjusted for the ESPR mixer-ejector geometric and flow parameters. This adjustment is made for SPL at all angles. For example, the results of this adjustment for $90^\circ$ and the resultant PWL are shown in Figure 3. Absolute SPL and PWL derived in this manner may not be accurate for the ESPR design. However, the relative noise levels between the noise components and the noise benefit due to treatment are expected to be realistic. Overall this representative linear design would provide 2.4 EPNdB total noise reduction for the ESPR application without any optimization. This data is then used to optimize the ESPR mixer-ejector liner.
Table 1. Geometric, flow, and acoustic parameters for ESPR Mixer-ejector Nozzles at Takeoff conditions.

<table>
<thead>
<tr>
<th>Nozzle Parameters:</th>
<th>Target-Size Engine</th>
<th>Demonstrator Engine</th>
<th>Scale Model Nozzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$ – Area of Primary Flow Passage at Throat (Mixer Exit) - m$^2$ (Square Inches)</td>
<td>1.616 (2504.92)</td>
<td>0.239057 (370.55)</td>
<td>12348x10$^{-6}$ (19.14)</td>
</tr>
<tr>
<td>Linear Scale Factor</td>
<td>1.0</td>
<td>0.385</td>
<td>0.0874</td>
</tr>
<tr>
<td>$D_E$ - Ejector Diameter – m (Inches)</td>
<td>2.688 (105.82)</td>
<td>1.034 (40.7)</td>
<td>235x10$^{-3}$ (9.25)</td>
</tr>
<tr>
<td>$L_E$ - Ejector Length – m (Inches)</td>
<td>5.0908 (200.4)</td>
<td>1.958 (77.09)</td>
<td>445x10$^{-3}$ (17.52)</td>
</tr>
<tr>
<td>$A_s$ – Area of Secondary Flow Passage at Mixer Exit, - m$^2$ (Square Inches)</td>
<td>4.040068 (6262.18)</td>
<td>0.5976432 (926.376)</td>
<td>30870x10$^{-6}$ (47.85)</td>
</tr>
<tr>
<td>Mixer Perimeter at the Exit – m (Inches)</td>
<td>28.2568 (1112.47)</td>
<td>10.868 (427.87)</td>
<td>2470x10$^{-3}$ (97.24)</td>
</tr>
<tr>
<td>$L_t$ - Treatment Length – m (Inches)</td>
<td>4.8048 (189.16)</td>
<td>1.848 (72.754)</td>
<td>420x10$^{-3}$ (16.535)</td>
</tr>
</tbody>
</table>

| Liner Parameters: | | | |
| Peak Noy Frequency - Hz. | 3150 | 8190 | 36036 |
| Maximum Frequency - Hz. | 10000 | 26000 | 114400 |
| OASPL, dB- Takeoff | 174.5 | 171.0 | 166.0 |
| Displacement Thickness, $\delta^*$ - m (Inches) | 5.08x10$^{-3}$ (0.20) | 3.81x10$^{-3}$ (0.15) | 1.27x10$^{-3}$ (0.05) |

| Flow Parameters: | | | |
| M – Flow Mach Number at Liner Surface | 0.65 | | |
| $M_x$ – Mixed Flow Mach Number in the Ejector | 0.75 | | |
| $W_x/W_p$ – Pumping (Secondary Flow Rate/Primary Flow Rate) | 1.06 | | |
| $T_p$ – Primary Stream Total Temperature - deg. K (deg. R) | 840 (1512) | | |
| NPR - Primary Stream Total Pressure Ratio | 2.5 | | |
| $V_{max}$ – Mixed Velocity in the Ejector m/s (ft/s) | 345 (1131.9) | | |
| Sideline Distance for Takeoff m (ft) | 540.8 (1774.4) | | |
| $M_F$ – Flight Mach Number | 0.30 | | |

The normal impedance of the ejector treatment is predicted utilizing the liner design parameters and the bulk resistivity. The acoustic suppression spectrum corresponding to the liner impedance is predicted utilizing a cylindrical modal analysis method. Predicted impedance and acoustic suppression spectra are plotted in Figure 4. The predicted acoustic suppression data is curve fitted and also plotted in Figure 4. The internal components of PWL for hardwall and treated mixer-ejector configurations, shown in Figure 3 b, are used to compute the measured acoustic suppression (internal component of $\Delta$PWL). Figure 5 shows the curve fitted acoustic suppression and the corresponding internal component of $\Delta$PWL spectra. Ideally, the spectral level of acoustic suppression predicted for the ejector, would be the same as the internal component of $\Delta$PWL. As can be seen in Figure 5, this is not the case. A frequency dependent factor, $\gamma(f)$, termed the acoustic suppression transfer factor is developed to equalize the predicted acoustic suppression at each frequency $f$ with the internal component of $\Delta$PWL. The correlated acoustic suppression transfer factor $\gamma(f)$ is also shown in Figure 5.

Now that the farfield acoustic data for the ESPR design is constructed and the appropriate acoustic suppression transfer factor is established, they are utilized to execute the liner optimization.

3. Liner Design Optimization: Five (5) physical liner parameters are to be optimized to minimize EPNL (EPNL of internal noise component) or EPNLT (EPNL of total noise). A general process would require the calculation of EPNL by varying each parameter in steps, keeping the other parameters fixed. Assuming one needs 8 steps for each parameter, the process would require $8^5$ (32768) calculations of EPNL. The computer code
Figure 3. Total and extracted external and internal (a) SPL and (b) PWL spectra for scale model mixer-ejector for hardwall and treated configurations for an arbitrary ejector treatment, Sideline distance=1774.4 ft., $A_8=2505 \text{ in}^2$, NPR=2.5, $T_t = 1512^\circ R$, $M_F = 0.3$.

used for this process needs about 5 minutes per case. This operation would require 2730.7 hours. This is not a practical situation. The process can be improved by varying the parameters judiciously. Suppose one can accomplish the optimization process with five variations of each parameter. In this case 3125 ($5^5$) calculations of EPNL, requiring 260.4 hours of execution time,
Figure 4. Predicted normal impedance spectra for the liner used in the axisymmetric ejector and the predicted acoustic suppression spectrum, $R=12.5$ Rayls/cm, $D=0.4''$, $\sigma=37\%$, $t=0.025''$, $d=0.045''$, $T=1008^\circ R$, $M=0.65$, $OASPL=166$ dB, $M_x=0.75$, $T_x=1467^\circ R$

Figure 5. Predicted acoustic suppression, computed $\Delta$PWL and linear frequency factor $\gamma(f)$ for the liner used in the axisymmetric ejector, $R=12.5$ Rayls/cm, $D=0.4''$, $\sigma=37\%$, $t=0.025''$, $d=0.045''$, $T=1008^\circ R$, $M=0.65$, OASPL=166 dB, $M_x=0.75$, $T_x=1467^\circ R$
would optimize the liner design. Even 260.4 hours for liner optimization seems unreasonable. Thus, an iterative method has been developed and utilized for the liner optimization process, which took significantly less time.

The iterative liner optimization process is illustrated in Figure 6. In this process all the parameters other than the first one (X₁) are arbitrarily set. The first parameter is then varied to determine the value, which minimizes the corresponding dependant variable Y (i.e., EPNL or EPNLT in this case). This process is repeated for the second parameter (X₂) with the optimum value of X₁ and arbitrary values of the rest. Similarly, the other parameters are optimized using the optimum values of the parameters optimized ahead of them. The whole process is repeated until the two subsequent optimum values for each parameter become the same. The range of variation of each parameter was chosen to span the practical mechanical design limitations of a liner. The step size for each parameter was chosen to be sufficiently small such that any further reduction in step size would result in negligible differences in resultant EPNL benefits.

3.1 Liner Optimization for ESPR Scale Model Mixer-Ejector: The iterative process for liner optimization, schematically illustrated in Figure 6, is shown in Figure 7 for the scale model ejector treatment. It required three iterations (denoted by j) to arrive at the optimum values for all five liner parameters to achieve minimum EPNL. The optimum liner is 0.4” deep with a bulk of resistivity 80 Rayls/cm. The optimum values for facesheet parameters obtained by the optimization process are unrealistic from mechanical design considerations. Thus, these parameters are set to practical values as close as possible to the optimum. These are the perforated facesheet porosity of 45%, thickness of 0.01”, and hole diameter of 0.02”. The acoustic suppression observed in the farfield for this optimum liner is shown in Figure 8, which corresponds to Figure 3 for the liner before optimization. The result is a minimum internal EPNL of 75.7 dB and a corresponding total EPNL of 79.3 dB. Compared to hardwall ejector (internal EPNL=88.7 dB & total EPNL=89.7 dB) suppressions of 13.0 internal EPNdB and 10.3 total EPNdB are predicted with this optimum liner. This is a significant improvement over the liner effectiveness before optimization, which resulted in only 2.4 EPNdB in total noise suppression.

3.2 Polynomial Expressions for Total & Internal EPNL as Functions of Liner Design Parameters: DoE (Design of Experiment) tools were used to develop polynomial expressions of internal and total EPNL as functions of liner

Figure 6. Process map for an iteration procedure for liner design optimization.
together to formulate two equations, one for internal EPNL and the other for total EPNL. These expressions are of the following form:

\[
\text{EPNL} = a_0 + \sum_{i=1}^{n_{\text{opt}}} a_i x_i + \sum_{i=1}^{n_{\text{opt}}} \sum_{j=1}^{n_{\text{opt}}} a_{ij} x_i x_j
\]

In the expression, \(a\) represents the polynomial coefficients, and \(x\) represents the liner parameters. The polynomial expressions are utilized to generate contour and surface plots for internal and total EPNL. A response surface plot for internal EPNL, generated using the polynomial expression, is shown in Figure 9. This plot is generated by holding the facesheet parameters fixed at the practical mechanical design values established in section 3.1. The minimum EPNL and the corresponding bulk resistivity and liner depth, observed in this figure, are the same as those derived in section 3.1. This plot can be used for weight trade studies to determine the reduction of liner effectiveness if weight reductions were required and achieved by using liners with lower depth.
Internal EPNL contours as a function of bulk resistivity and facesheet porosity are plotted in Figure 10 with liner depth, facesheet thickness and hole diameter, fixed at the values, established in section 3.1. Strong impact of facesheet porosity on EPNL is evident in this figure. A minimum EPNL of about 75.0 dB is possible if higher facesheet porosity values become practical. This is about 0.8 dB lower than the optimum established in section 3.1 with the practical mechanical constraints imposed. Significant noise benefit can be achieved if facesheet material can be improved to withstand the load requirements with 75% porosity. A trade between EPNL and facesheet porosity can be made utilizing the results of this figure.

Figure 11 is another internal EPNL contour plot as a function of facesheet porosity and thickness with
Figure 9. Contour plots for internal EPNL, Fixed Facesheet Parameters: Hole Diameter $d=0.02''$, Thickness=0.01'', Porosity=0.45.

Figure 10. Contour plots for internal EPNL, Fixed Liner Parameters: Depth=0.40'' Facesheet Hole Diameter $d=0.02''$, Facesheet Thickness=0.01''.

It is evident from this plot that a lower EPNL is achievable with a lower facesheet thickness and with higher facesheet porosity. Again, trade studies can be made for these two parameters.

**4.0 CONCLUDING REMARKS:** An optimum liner is designed for a scale model axisymmetric mixer-ejector nozzle for the ESPR application. Data for the actual ESPR mixer-ejector configuration was not available. Thus, the acoustic input is constructed utilizing data from a representative mixer-ejector configuration. The accuracy of the absolute predicted noise levels is somewhat uncertain. Even though the absolute levels may not be accurate, the relative internal noise between hardwall and treated configurations is expected to be reasonable. Total EPNL however depends on the internal as well as the external noise components. The predicted external noise component is most uncertain due to the design differences between the representative mixer-ejector used in this analysis and the ESPR nozzle. However, the optimum liner design, based on internal noise minimization, is unlikely to be changed when actual ESPR scale-model mixer-ejector data is used in the same design process.

The iterative process used in the liner design accurately provides the optimum parameters as a function of multiple independent variables. Some liner design parameters are set different from their optimum values on the basis of mechanical design considerations. A total 10.3 EPNdB reduction is predicted with the optimum liner. This is a significant improvement over the liner before optimization, which resulted in only 2.4 EPNdB in total noise suppression.

Response surface modeling using DoE methods is a powerful tool for multiple independent variable function optimizations. It also provides useful parametric results, which could be used for liner design trade studies.