NON-LINEAR STRUCTURES FOR STABILISATION- AND PRECESSION AXIS COUPLING OF THE ORIENTATION AND STABILISATION GYRO-SYSTEMS

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Abstract

The paper presents the results of some theoretical and experimental studies concerning a large class of non-linear systems (also known as "systems with memory"). Such systems are representing models for a few inner or outer coupling, as components of complex mechanical structures, for example gyro- stabilisators, mono-, bi- or three-axis gyro-scopic guidance heads. In fact, the bi- or three-axis gyro-systems are two or three interconnected mono-axis gyro-systems.

The model for a mono-axis gyro-system consists of two sub-systems: a linear one (which has good filter properties) and the non-linear one (described by a non-linear element without memory and the linear element with or without memory). The study's objects are the non-linear systems (see figures).

1. Non-linear system with type I static characteristics

The [1] presented method consists of some non-linear "with memory" systems modelisation, using non-linear systems (which static characteristic are "no memory relay" type) and linear elements with or without memory, interconnected by negative and/or positive feed-back.

The non-linear systems in the fig. 1.a [2] has a static characteristic as fig. 1.c shows or, particularly, if $\eta_A = \eta_0 \left(\eta^* = \eta_A / \eta_0 = 1\right)$, as fig. 1.d shows (which is the Tastin-model's static characteristic for elastic mechanical couplings). One can demonstrate, using the following equation, that the $N=N(Z)$ static characteristic (see fig. 1.b) can be described as follows:

$$N(Z) = \begin{cases} \eta_1 = \operatorname{sgn} Z, |Z| > 0, \\ \le |\eta_0|, Z = 0. \end{cases} \tag{1}$$

and, from fig. 1.a, one can write:

$$\begin{aligned} u &= N(z,\dot{z}) = z - N(Z) = \\ &= z - N(ku); z = A_z \sin \omega t. \end{aligned} \tag{2}$$

1.Case 1: $\eta_1 = \eta_0$

For $Z>0$, as fig. 1.a shows, $u = \frac{1}{k} Z > 0$, that means the equation of a straight line (zone $u>0$) which slope is $m=1$

$$u = z - N(Z) = z - \eta_1 > 0. \tag{3}$$

Similarly, for $Z<0$, one obtains another straight line's equation, (see zone $u<0$),

$$u = z + \eta_1 < 0. \tag{4}$$

For $Z=0$, that means $u = \frac{1}{k} Z = 0$, where $u = z - N(Z)$, it results $z=N(Z)$), and, using the relation (1), one obtains

$$z = N(Z) \le |\eta_1|. \tag{5}$$

so,

$$u = 0, z \le |\eta_1|. \tag{6}$$

and from (3), (4) and (6) the $u=N(z, \dot{z})$ form can be written

$$u = \mathrm{N}\,(z,\dot{z}) = \begin{cases} z + \eta_1, z < \eta_1, \\ 0, z \in [-\eta_1, \eta_1], \\ z - \eta_1, z > \eta_{1,} \end{cases} \tag{7}$$

which describes the static characteristics in fig. 1.d. [3]

2.Case 2: $\eta_1 < \eta_0$

For Z=0 it results $u = z - N(Z) = = \frac{1}{k} Z = 0$. So,

$$u = 0, z = N(Z), \dot{z} = \dot{N}(Z). \qquad (8)$$

When Z=ku>0 decreases into the value 0 ($\dot{Z} = k\dot{u} < 0$), then $N(Z)$=η_1 = const. (see fig. 1.b) and

$$u = z - N(Z) = z - \eta_1 > 0, \dot{u} = \dot{z} < 0, (12)$$

which describes the characteristic in fig. 1.c for $z > \eta_1$ (z decreasing from any value $z > \eta_1$). Obviously, when Z=ku<0 increases from any

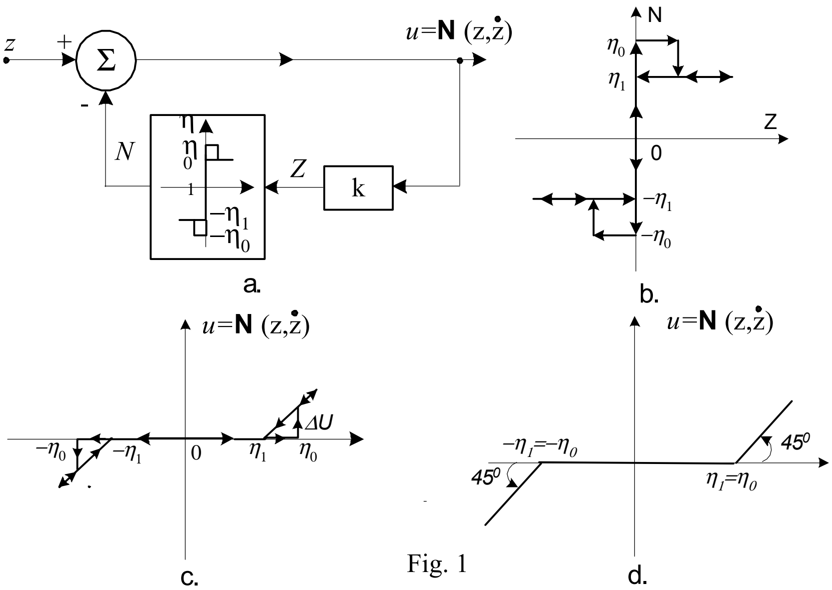


Fig. 1

For $z \in [-\eta_1, \eta_1]$ the non-linearity in fig. 1.b is the same in the Case 1 $(\eta_1 = \eta_0)$. So, the equations (5) and (6) remain also valid, $N(Z) < |\eta_1|$ and

$$u = 0, z \le |\eta_1|. \qquad (9)$$

From fig. 1.b, for Z=0, if $N(Z) \in [\eta_1, \eta_0]$ and $N(Z)$ is continue increasing, that means $\dot{N}(Z) > 0$, the (8) equations are leading to

$$u = 0, z = N(Z) \in [\eta_1, \eta_0], \dot{z} > 0, \qquad (10)$$

which are describing the characteristic in fig. 1.c (for $N(Z) \in [\eta_1, \eta_0]$ and $\dot{z} > 0$).

For Z=0, if $N(Z) \in [-\eta_0, -\eta_1]$ and $N(Z)$ decreasing $\left(\dot{N}(Z) < 0\right)$, one can write, using (8) equation:

$$u = 0, z = N(Z) \in [-\eta_0, -\eta_1], \dot{z} < 0. \quad (11)$$

negative value to 0 $\left(\dot{Z} = k\dot{u} > 0\right)$, therefore $N(Z) = -\eta_1$ = const. and

$$u = z - N(Z) = z + \eta_1 > 0, \dot{u} = \dot{z} > 0 \, (13)$$

which describes the characteristic in fig. 1.c for an increasing z, $z < -\eta_1$.

The step at Z=0_+ of $N(Z)$, from $N(Z) = \eta_0$ to $N(Z) = \eta_1, (\Delta N(Z) = \eta_1 - \eta_0 < 0)$, produces a step for u from $u = \frac{1}{k} Z = 0$ to a positive value, therefore

$$\Delta u = \Delta z - \Delta N(Z) = -\Delta N(Z) = \eta_0 - \eta_1 > 0. (14)$$

Because of the straight line's slope m, which value is m=1, ($m = \frac{\Delta u}{\Delta z} = \frac{\Delta u}{\eta_0 - \eta_1} = 1$), the step for u develops until it intercepts the straight line $u = z - \eta_1$. For $z = -\eta_0, \Delta u = \eta_1 - \eta_0 < 0$, system's behaviour is similar.

For Z>0 it results $N(Z)=\eta_1=\text{const.}$ (see fig. 1.b) and, for an increasing z $\left(z>\eta_1, \dot{z}>0\right)$, the characteristic's equation becomes

$$u = z - \eta_1 > 0. \qquad (15)$$

One can observe that the static characteristic in figure 1.c, for the system in fig. 1.a, is described as

$$u = \mathrm{N}\,(z,\dot{z}) = \begin{cases} 0 & ,z \in [-\eta_1, \eta_0), \dot{z} > 0, \\ z-\eta_1, z > \eta_0, \dot{z} > 0, \\ z+\eta_1, z < -\eta_1, \dot{z} > 0, \\ 0 & ,z \in (-\eta_0, \eta_1], \dot{z} < 0, \\ z+\eta_1, z < -\eta_0, \dot{z} < 0, \\ z-\eta_1, z > \eta_1. \end{cases} \qquad (17)$$

2.Non-linear systems with type II static characteristics

About the non-linear system in fig.2, one can demonstrate that its static characteristic is the one in fig. 2.b, or the one in fig.2.c.

The system in fig. 2.a. is described by the equations:

$$u = z - N(Z), z = A_z \sin \omega t; \qquad (18)$$

$$N(Z) = \begin{cases} \eta \operatorname{sgn} Z, |Z| > 0, \\ \leq |\eta|, Z = 0; \end{cases} \qquad (19)$$

$$T\dot{Z} + Z = ku. \qquad (20)$$

1)When Z=0=const., $\dot{Z}=0$ and according to (20), u=0, that means (see (18)) that $z = N(Z) >$ or < 0.

1.1) z=N(Z)>0 and $\dot{z} = \dot{N}(Z) > 0 \Rightarrow$ $\Rightarrow u = 0$ for z=N(Z)=const.$\leq \eta$ (see the characteristic's slice, for $\dot{z} > 0$, between 0 and η, in fig. 2.b);

1.2) z=N(Z)>0 and $\dot{z} = \dot{N}(Z) = 0 \Rightarrow$ $\Rightarrow u = 0$ for z=N(Z)=const.$\,\eta$;

1.3) z=N(Z)>0 and $\dot{z} = \dot{N}(Z) < 0 \Rightarrow$ $\Rightarrow u = 0$ for z=N(Z)$\leq z_i$ (see the characteristic's slice, for $\dot{z} < 0$, between 0 and z_i, in fig. 2.b);

1.4) z=N(Z)<0 and $\dot{z} = \dot{N}(Z) > 0 \Rightarrow$ $\Rightarrow u = 0$ for $-z_i < z = N(Z) < 0$ (see the characteristic's slice, for $\dot{z} > 0$, between $-z_i$ and 0, in fig. 2.b);

1.5) z=N(Z)<0 and $\dot{z} = \dot{N}(Z) > 0 \Rightarrow$ $\Rightarrow u = 0$ for z=N(Z)=const. $\geq -z_i$;

1.6) z=N(Z)<0 and $\dot{z} = \dot{N}(Z) < 0 \Rightarrow$ $\Rightarrow u = 0$ for z=N(Z)$\geq -\eta_i$ (see the characteristic's slice, for $\dot{z} < 0$, between $-\eta$ and 0, in fig. 2.b).

2) When $|Z| > 0$, according to (19), $N(Z) = \eta = \text{const.}$ and the equation (18) becomes

$$u = z - \eta >< 0. \qquad (21)$$

A similar behaviour is for Z<0, $N(Z) = -\eta = \text{const.}$ and (18) becomes

$$u = z + \eta >< 0. \qquad (22)$$

2.1) Z>0 and z increases or decreases $(z > \eta)$, the static characteristic's equation becomes

$$u = z - \eta > 0. \qquad (23)$$

that means the u>0 zone of the straight line which contains the point $z = \eta$ and has the slope m=1;

2.2) Z>0 and z decreases $(z < \eta)$ the static characteristic's equation becomes

$$u = z - \eta < 0. \qquad (24)$$

that means the u<0 zone of the straight line $u = z - \eta$.

Because of the z decreasement u becomes negative ($u_i = z_i - \eta$), see fig. 2.a, or see equation (20), Z decreases non-periodic, from a positive value to a negative one $Z_i \approx ku_i$; the $\dot{Z}$ value depends of T and

$N(Z_i)=-\eta$. The transition from Z=0_+ to $Z=0_-$ is sudden, through a step from $N(Z)=\eta$ to $N(Z)=-\eta$, that means that the same value u_i of u verifies the equation

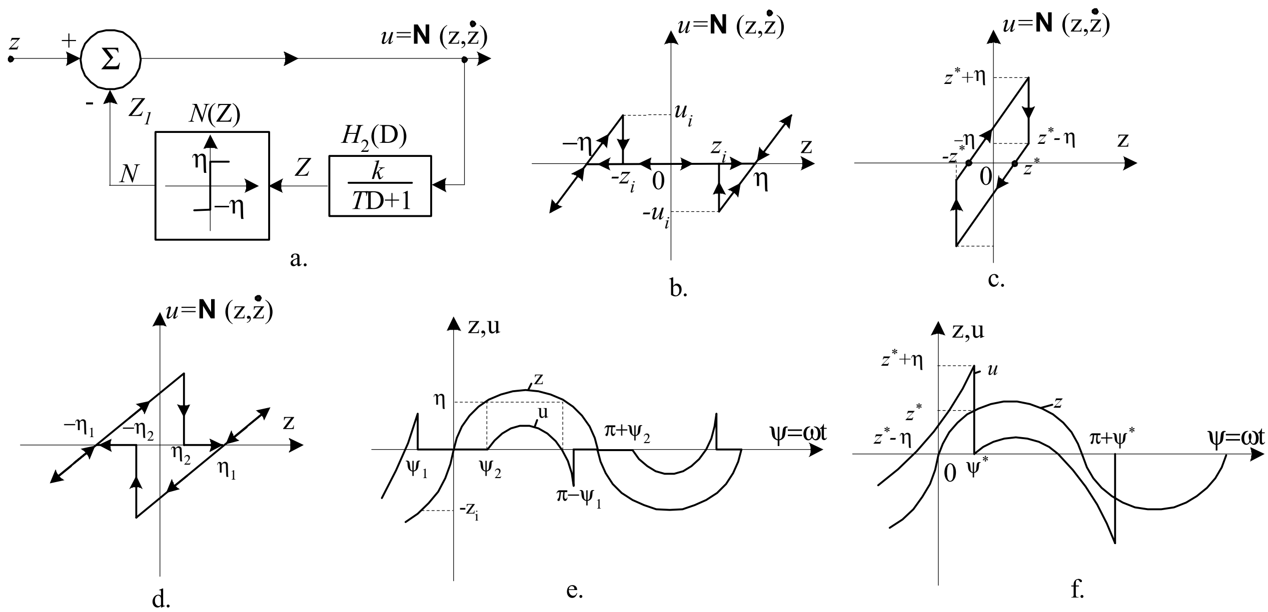


Fig. 2

$$u=z+\eta>0. \qquad (25)$$

The u-step, from the negative value u_i (on the (24) straight line $u_i=z_i-\eta$) to the positive value u_i' (on the (25) straight line $u_i'=z_i+\eta$) passes through 0 value ($u(z_i)=0$). Subsequently, Z increases from the negative value to Z=0 and the system's behaviour is the one presented at 1); $\Delta u=0-u_i=0_- -u_i>0$. One can also affirm that the step is reduced from one straight line to another ($u=z-\eta$ to u=0), when $z=z_i$. A similar conclusion can be obtained studying the z<0 zone of the static characteristics in fig.2.b.

The non-linear system's output signal $u(t)$, for $z=A_z\sin\omega t$, is represented in fig 2.e [4] .

For $z\in[-z_i,\eta]$, u=0 (see fig.2.b), $\dot z>0$. Subsequently, in fig. 2.c u=0 for $\varphi\in[\varphi_1,\varphi_2]\cup$ $\cup[\pi-\varphi_1,\pi+\varphi_2]$. For z>$\eta$, u>0 (see fig. 2.b), and in fig 2.c $\varphi\in[\varphi_2,\pi-\varphi_2]$ for z<η $\left(\dot z<0\right)$, $u=z-\eta<0$ (for $[z_i,\eta]$), $\varphi\in[\pi-\varphi_2,\pi-\varphi_1]$ etc. The phase angles are

$$\varphi_1=-\arcsin\frac{z_i}{A_z};\varphi_2=\arcsin\frac{\eta}{A_z}. \qquad (26)$$

If the $N(Z)$ step from η to $-\eta$ (or in reverse order) is realized for a value $z_i=-z^*<0$ which u reaches faster following the $u=z+\eta$ straight line then following the u=0 line (relay type regime), than the system's (fig.2.a) static characteristic is the one in fig.2.c. In this case the $u(t)$ output signal is the one in fig.2.f [4], where

$$\varphi^*=\varphi_1=\varphi_2=\arcsin\frac{z^*}{A_z}; \qquad (27)$$

for $z=z^*=z(\varphi^*)=A_z{}^*\sin\varphi^*$ the step is realized from the $u=z+\eta$ line to the $u=z-\eta$ line (see fig.2.c and 2.f).

Furthermore, one can deduce the φ_1 and φ_2 sliding regions phase arguments equations. The equation (20) becomes

$$T\dot{Z}+Z=k(z-Z_1), z=A_z\sin\omega t=A_z\sin\varphi, Z_1=\pm\eta \qquad (28)$$

or, in complex,

$$Z(s)=\frac{kA_z}{(s^2+\omega^2)(Ts+1)}-\frac{kZ_1(s)}{Ts+1}+\frac{k}{Ts+1}C, \qquad (29)$$

C- integration constant.

Because Z=0 for u=0, according to fig. 2.e, $Z(\varphi_2)=u(\varphi_2)=0$ and $Z(\pi-\varphi_1)=$ $=u(\pi-\varphi_1)=0$.

Also, $u(\varphi_2)=z-\eta=A_z\sin\varphi_2-\eta=0$. From one of these three equations one obtains the C- value and from the others, applying the inverse Laplace operator- the transient equations

$$1+\lambda[\sin(\varphi_1-\theta)+\sin(\varphi_2-\theta)\exp((\varphi_2-\varphi_1-\pi)/\omega T)]-\exp((\varphi_2-\varphi_1-\pi)/\omega T)=0, \qquad (30)$$

$$\varphi_2=\arcsin\frac{1}{A},$$

where $A=\frac{A_z}{\eta}$ and $\lambda=A\cos\theta$.

Introducing $\varphi_1=\varphi_2=\varphi^*$ in the first (30)- equation, one obtains

$$\sin(\varphi^*-\theta)=\frac{1}{\lambda}\frac{e^{-\frac{\pi}{\omega T}}-1}{e^{-\frac{\pi}{\omega T}}+1}=-\frac{1}{\lambda}\text{th}\frac{\pi}{2\omega T}$$

or (31)

$$\varphi^*=\theta-\frac{1}{\lambda}\text{th}\frac{\pi}{2\omega T}.$$

$A=\frac{1}{\sin\varphi^*}$, where φ^* is given by (31). Finally, one obtains [4]

$$A^*=\left\{1+\text{ctg}^2(\theta)\left[1+\frac{1}{\cos^2\theta}\text{th}\frac{\pi}{2\omega T}\right]^2\right\}^{1/2}. \qquad (32)$$

The static characteristic in fig. 2.d is obtained from the one in the fig. 2.b for $\eta_1=\eta$ and $\eta_2=-z_i>0$ (the case when the step is realized at $z_i<0$ value); $A=\frac{A_z}{\eta_1}$; $\lambda=A\cos\theta$ and the (26)- equations become

$$\varphi_1=\arcsin\frac{\eta_2}{A_z}; \varphi_2=\arcsin\frac{\eta_1}{A_z}=\arcsin\frac{1}{A}. \qquad (33)$$

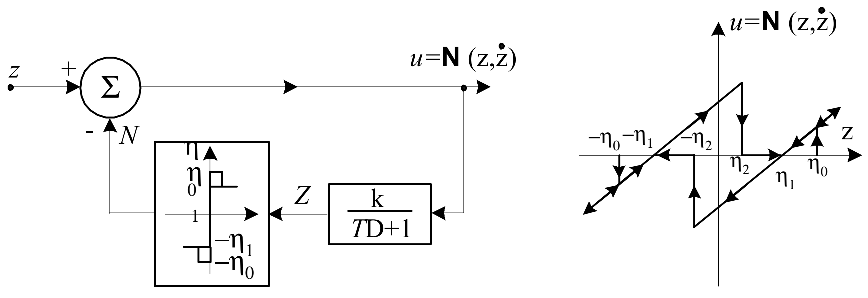
The (30) equation become

$$1-\lambda[\sin(\varphi_1+\theta)-\sin(\varphi_2-\theta)\exp((\varphi_2-\varphi_1-\pi)/\omega T)]-\exp((\varphi_2-\varphi_1)/\omega T)=0, \qquad (34)$$

$$\varphi_2=\arcsin\frac{1}{A}.$$

3.Non-linear systems with type III static characteristics

One further demonstrates that the system in fig. 3.a [5] realize the static characteristic in fig. 3.b. For $\eta\in[-\eta_1,\eta_1]$ and for $\eta_0=\eta_1$ the system is identical with the one in fig. 2.a and its static characteristic lies in fig. 2.d, the periodic regimes are sliding-type; the phase-angles φ_1 and φ_2 are the (34) transient equation roots. Superposing the system's in fig. 1.a characteristic, in fig. 1.c (or system in fig. 3.a for $\omega T=0$) and the characteristic in fig. 2.d, one obtains the characteristic in fig. 3.b. It can



a. b.

Fig. 3

The minimum A^* value which generates relay–type periodic regimes is obtained from the second (30)- equation for $\varphi^*=\varphi_2$; it results be explained as for the system in fig. 2.a, adding the other ones for the intervals $[-\eta_0, -\eta_1]$ and $[\eta_1, \eta_0]$. For these intervals, according to (1) equation, it results [1]

$$|\eta_1| \le N(Z) \le |\eta_0|, Z = \text{const.} = 0. \qquad (35)$$

Subsequently, $\dot{Z} = 0$ and (20) becomes

$$Z = k \cdot u. \qquad (36)$$

As a conclusion, for these intervals the system in fig. 3.a behaves like the system in fig. 1.a, which is modeling the characteristic in fig. 1.c.

For the relay-type regimes, the static characteristic in fig. 3.b modifies; this one becomes like the one in fig. 2.c, where $\eta = \eta_1$ and $z^* = \eta_0$. In this case, $\varphi^* = \arcsin\left(\frac{\eta_0}{A_z}\right)$ is calculate with (31). The minimum amplifying of the relay-type regimes is A^*.The model for a mono-axis gyro-system consists of two sub-systems: a linear one (which has good filter properties) and the non-linear one (described by a non-linear element without memory and the linear element with or without memory).

Conclusions

The paper presents the results of some theoretical and experimental studies concerning a large class of non-linear and linear couplings, as applications for complex gyroscopic systems for aerospace stabilization and guidance. The structural models, consisting of linear and/or non-linear elements, with or without memory, could be studied by computer simulation; it can also provide some important further conclusions concerning the stability and quality of the non-linear dynamic regimes.

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