Abstract

Multidisciplinary Design Optimization (MDO) is becoming a popular technique on various design phases nowadays. However, when the ranges of design space are not proper to the feasible domain, obtaining the results is taken for a long time or dissatisfied results are often brought. Therefore, recognizing and narrowing the design space in the early stage of MDO are required.

This paper describes discriminant analysis to recognize and narrow-down the design space for MDO study and describes an application of MDO to an engine conceptual design which consists of engine performance, HPT cooling, HPT passage analysis and HPT structure analysis.

1 Engine Conceptual Design

The goal of turbofan engine conceptual
design is to define engine cycle parameters, such as Overall Pressure Ratio (OPR), Turbine Inlet Temperature (TIT), Bypass Ratio (BPR) and component configurations such as flow passages, stage numbers and so on at the early phase of aircraft engine development.

The conceptual design is the first step of engine design, so it is important to evaluate the engine performance, weight, noise, cost and system feasibility. As there are a lot of flexibility and no details (ex. disk shape, blade shape, flow passage, and so on) at this phase, some simple and light tools are usually adequate and heavy tools (CFD, FEM, etc.) are not required. The schematic of design parameter flow in the conceptual design phase is shown in Figure 1. The flowchart shows that the key design characteristics--engine performance, component aerodynamics, structure stress, and so on--are interacting complicatedly, and an optimization of the design parameters is not an easy task. Therefore, various attempts to adapt multidisciplinary design optimization techniques to the conceptual design has been made.

2 Difficulties in MDO

MDO, however, often encounters difficulties in the application to real design. When one run an optimization algorithm that might be typically one of many non-linear programming (NLP) algorithms or heuristic algorithms such as Genetic Algorithm, it is very important to find “feasible domain(s) of design parameters” prior to the optimization task. Here, the “feasible domain(s)” is a/some domain(s) in a design parameter space that satisfies all of given design constraints. If initial searching point happens to be outside of the feasible domain(s), the optimization task will often fail. Therefore, it is very important to find the feasible domain(s) before a MDO algorithm is run. This is, of course, the case for the turbofan engine conceptual designs.

3 MDO Approach

As mentioned above, the design parameter space where MDO must be carried out is usually much "larger" than feasible domain(s). Therefore, it is beneficial to place a pre-process to find the feasible domain(s) prior to MDO calculation, in order to narrow down the design space in advance that should be investigated. The flow of this approach is shown in Figure 2.

1. Setting Initial Design Space
   The initial design spaces are set extensively to catch an optimum point in the spaces. (The feasible domain(s) is not recognized at this time.)

2. Design of Experiments (DOE) sampling
   A sampling by DOE calculation is performed. In this study, Latin Hyper Cubic (LHC) Method is used.

3. Confirm the number of feasible cases
   If the number of feasible cases is adequate, go to MDO calculation. If not, proceed to the next step.
Discriminant analysis is performed on the DOE sampling results. Discriminant variable is obtained.

The design space is narrowed down based on the discriminant variable to establish feasible domain, and return to the step 2.

3.1 Discriminant Analysis

Discriminant analysis was applied to find the feasible domain. Discriminant analysis is one of well-known multivariate analysis method in statistical analysis, and it evaluates the boundary between a group into some sub groups if such kind of potential structure exists in parameter group concerned (Figure 3).

The parameter space of optimization problem is divided into two regions: feasible domain and infeasible domain. The following iterative procedure would be available to establish feasible domain, using discriminant analysis technique one can predict what combination of design parameters would produce feasible design.

Discriminant analysis can find the discriminant variable $\xi$. Usually the variable is written as equation (1). However, in this study, higher order terms of design parameters are also considered in discriminant variable and it is written as equation (2).

$$\xi = \sum_{i=1}^{n} a_i x_i$$  \hspace{1cm} (1)

$$\xi = \sum_{i=1}^{n} a_i x_i + \sum_{j=1}^{n} \sum_{k=1}^{n} a_{jk} x_j x_k$$  \hspace{1cm} (2)

where $a_i/a_{jk}$ is coefficient of discriminant variable, $x_i/x_{jk}$ is design parameters and $n$ is the number of design parameters.

Discriminant variable can be generated from results of the sampling for which group membership is known. Then the group--feasible or infeasible--to which a combination of the
design parameters belongs can be evaluated by the discriminant variable.

3.2 Tool for Sampling and Optimization

The DOE sampling by LHC and the optimization were performed on iSIGHT of Engineous Software, Inc. The software can handle not only several optimizations but also DOEs very easily.

4. Analysis Models

This study focuses on the High Pressure Turbine (HPT) module with the engine performance, keeping the design parameters of the other components constant, as sample of MDO/discriminant-analysis application.

The turbofan engine analysis model used in this study consists of four sub-models; Engine Performance Model, HPT Flow Passage Model, HPT Cooling Flow Model and a HPT Structure Model. They are not special tool for the MDO studies and can be used independently. Figure 4 shows typical parameter flows that flow into/out of each sub-model. All sub-models except the structure model (highly) interacts. In this study, ten (10) design parameters and seventeen (17) constraints were chosen.

4.1 Engine Performance Model

Engine performance model used in this study is a one-dimensional engine cycle code similar to DYNGEN [1]. In this case, the HPT efficiency calculated in the HPT Flow Passage Model and cooling flow rates calculated in the HPT Cooling Flow Model are only used by this model as input parameters. Other cycle parameters kept constant during this optimization.

4.2 HPT Cooling Flow Model

HPT Cooling Flow Model consists of the so-called similarity design tools. The HPT cooling flow rates are calculated from cooling type, blade material, gas temperature and HPT flow rate. The cooling type and blade material are selected by designer, and gas temperature
and HPT flow rate are calculated from Engine Performance Model.

### 4.3 HPT Flow Passage Model

HPT Flow Passage Model is a one-dimensional Mean-line model. HPT inlet and exit flow rate, total temperature, total pressure and HPT total power come from Engine Performance Model, cooling flow rates are determined in HPT Cooling Flow Model. And this model gives HPT efficiency back to Engine Performance Model and gives inter-stage gas flow properties to HPT Structure Model and HPT Cooling Model. HPT Structure Model also receives the number of blades from this model.

### 4.4 HPT Structure Model

Stresses at blade roots, dovetails and disks are calculated in this model. So-called similarity designs were used for blade root and dovetail stress calculation and Manson’s method [2] was used for the disk stress calculation. Redline speed, which is an output of this model, calculated by the stresses and the rotor speed. Disk shapes, which are determined by a partial optimization in this model, are also outputted from this model.

## 5 Results

### 5.1 Results of Discriminant Analysis

Initial LHC sampling in the initial design space resulted in the feasible cases of no more than 3% of the sampling cases, 71% of infeasible cases and 25% of un-converged cases (see Table 1). Then discriminant analysis to the initial sampling results was performed. Figure 5 shows the results. In this figure, coordinate Y indicates the value of discriminant variable and coordinate X indicates the order of penalty which represents a deviation from the feasible domain. The boundary between feasible domain and infeasible domain cannot be clearly defined on the plot.

In the next step, the design parameters were divided into two groups and the discriminant analyses were performed again to each group. Figure 6 shows the results. The boundary between feasible domain and infeasible domain are displayed more clearly than the previous analysis. Coefficients of discriminant variable were obtained in this analysis.

<table>
<thead>
<tr>
<th>Sampling Number</th>
<th>Feasible Ratio</th>
<th>Infeasible Ratio</th>
<th>Un-converging Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL</td>
<td>3%</td>
<td>71%</td>
<td>25%</td>
</tr>
<tr>
<td>2nd</td>
<td>8%</td>
<td>86%</td>
<td>6%</td>
</tr>
<tr>
<td>3rd</td>
<td>43%</td>
<td>54%</td>
<td>3%</td>
</tr>
<tr>
<td>4th</td>
<td>55%</td>
<td>44%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 1. Feasible ratio of each sampling
In designing parameter 1, 5, 7, and 9, the values of the discriminant variable are shown in Figure 7. In design parameter 3, 4, 8 and 10, there are no remarkable trends. These trends serve as the feasible domain, so that the value becomes small. In design parameter 6, the value serves as the infeasible domain, so that the value becomes small. In design parameter 2, near both ends of initial design range, the value of the discriminant variable serves as infeasible domain and the value near middle serves as feasible domain. In design parameter 3, 4, 8 and 10, there are no remarkable trends. These trends are almost consistent with the sense of experts. Then the design ranges of each design parameter except the parameters, which have no remarkable trends, were revised/narrowed-down by the indications.

As it was confirmed that the discriminant analysis is effective in the evaluation of each domain, the discriminant analysis was repeated three (3) times in order to narrow down the design space after each samplings in this study. Consequently, the crop of the sampling was improved: the feasible ratio was increased from 3% to 55%, the infeasible ratio was decreased from 71% to 44% and un-converging...
ratio was decrease from 25% to 1% with the sampling number (Table 1).

The distributions of the feasible and infeasible cases after the third discriminant analysis are shown in Figure 8. The boundary between feasible domain and infeasible domain are much clearer than the initial sampling.

5.2 Results of HPT MDO

Two kinds of MDO, in which the HPT tip-speed is treated in a different way, were performed. In the first case, the HPT tip-speed was considered as one of the variable parameters in the optimization. In the second case, the HPT tip-speed was handled as a constant parameter and optimization was performed for each HPT tip-speed. In both cases, the objective parameter was Thrust Specific Fuel Consumption (TSFC) of the engine.

In the first case, the results of the optimization in which HPT tip-speed was handled as a variable parameter for the optimization are shown in Figure 9-(a). We obtained the optimum point that is at +40m/s.

In the second case, the optimizations were performed for BASE (+0), +25, +50, +75 and +100m/s delta HPT tip-speeds. Trends of TSFC, HPT efficiency, HPT cooling airflow rate and rotor speed margin with HPT tip-speed are plotted in Figure 9. In this optimization, the derived disk shapes of each tip-speed are also obtained and they are shown in Figure 10.

The relationship between HPT tip-speeds and TSFC, which is the objective parameter, is shown in Figure 9-(a). TSFC is getting better from base up to +50m/s tip-speeds and getting worse over 50m/s. And it is confirmed that the optimum point of the first case, +40m/s point, consists with the TSFC trend and the optimum point is the smallest/optimum TSFC in two kinds of MDO.

The relationship between HPT tip-speeds and HPT efficiency is shown in Figure 9-(b). The trend is the reverse of the TSFC. The efficiency is getting better from BASE to +50m/s and getting worse over +50m/s as the tip-speed becomes higher.

In Figure 9-(c), it is shown that the cooling flow rate increases as the tip-speed increases.

Figure 9-(d) shows the rotor speed margin change with HPT tip-speed. The rotor speed margin is reducing as the HPT tip-speed becomes higher and at delta HPT tip-speed of 100m/s, the rotor speed margin is about –20% worse from BASE tip-speeds.

6 Conclusion

This paper has discussed discriminant analysis applied to MDO, and shown the application of MDO to aircraft engine conceptual design consists of four disciplines; engine performance, HPT aerodynamics, HPT cooling and HPT structure.

In this study, we confirmed followings;
1) Discriminant analysis made the crops of the sampling improved.
2) Discriminant analysis gave indications, which were almost consistent with the sense of experts, to narrow-down the design space.

Discriminant analysis is promising technique for multidisciplinary design optimization to recognize the design space and to reduce its difficulties.

Reference


Figure 9. Optimum point of TSFC and trends of TSFC, HPT efficiency, HPT cooling flow rate and rotor speed margin with HPT tip-speed

Figure 10. HPT passage and disk shape