SUPERSONIC FLUTTER AND POST-FLUTTER CONTROL OF AN AIRCRAFT WING SECTION

Piergiovanni MARZOCCA, Liviu LIBRESCU
Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0219,
and
Walter A. SILVA,
NASA Langley Research Center, Hampton, VA 23681-2199.

Keywords: *Aeroelasticity, Lyapunov first quantity, Hopf-bifurcation*

Abstract

A number of issues related to the supersonic flutter and post-flutter of a wing section, as well to their control are addressed in this paper. In this context, the objectives of this study are: 1) to analyze the implications of nonlinear supersonic piston theory aerodynamics and structural nonlinearities on the character of the flutter instability boundary of wing sections, and 2) to implement a control capability enabling one to control both the flutter boundary and its character. This will enable one to expand the flight envelope without the occurrence of catastrophic failures of aeroelastic nature.
The study of the bifurcation behavior of the aeroelastic system in the vicinity of the flutter boundary is carried out via Lyapunov's first quantity.

1 Introduction

Depending on the nature of the flutter boundary, i.e. catastrophic or benign, if the aircraft reaches the flutter speed it can feature a catastrophic failure (unstable LCO) or can survive (stable LCO), respectively [1]. In the latter case, the failure will not occur catastrophically, but by fatigue.

At this stage, according to the flight regulations, the flutter speed should be 15% larger than the maximum speed the airplane can experience [2]. This imposed large margin of security is intended to prevent the catastrophic failure of the aircraft operating in the vicinity of the flutter speed.

This suggests the considerable importance of addressing at least two issues:

1) of including in the aeroelastic analysis the various nonlinear effects, on which basis is possible to get a better understanding of their implications upon the character of the flutter boundary [1,3], and

2) of implementing an active control capability [4,5] enabling one to increase the flutter speed and convert the catastrophic flutter boundary into a benign one. This research addresses both these issues.

2 Aeroelastic Behavior in the Vicinity of the Flutter Instability Boundary

The issue of the character of the flutter boundary, can be revealed mathematically via determination of the nature of the Hopf-Bifurcation (i.e. supercritical or subcritical, respectively). In this sense see Refs. [6,7] and, in the special case of nonlinear aeroelastic systems, Refs. [1,3].

Loosely speaking, the Hopf-bifurcation theorem [6] stipulates that if the characteristic equations of the linearized system about an equilibrium position exhibits pairs of complex conjugate eigenvalues that cross the imaginary axis as one of the control parameter varies, (in the present case this parameter is the flight speed V), then for the near-critical values of V there are limit cycles close to the equilibrium point $V_{Flutter}$.

In this study, the determination of the catastrophic/benign character of the flutter boundary and its control is carried out via determination of the sign of the Lyapunov first quantity (LFQ) [1,8], for the flutter boundary that corresponds to the purely imaginary roots of the characteristic equation. The approach used in [1] and [3] for the panel flutter, is extended in the present work to the aeroelasticity of nonlinear 2-D lifting surfaces.

The behavior of the general dynamic systems near the boundaries of the stability domains was investigated by Bautin [8] who considered only those portions of boundary of the region of stability for which the characteristic equation exhibits either one root only, or two roots, that are purely imaginary.

The LFQ corresponding to the actively controlled nonlinear flutter of wing section in a supersonic flow field is derived, and is used toward the investigation of the character of the flutter boundary.

Upon defining the expression of the Lyapunov first quantity $L(V_F)$, from the conditions $L(V_F)<0$ and $L(V_F)>0$, one can determine the benign and catastrophic portions of the flutter boundary, respectively, and implicitly, the influence played by the various nonlinearities included in the system. Herein $V_F \equiv M_F a_\infty / b\omega_\alpha$.

3 Nonlinear Model of a Wing Section Incorporating an Active Control Capability

Toward formulating the aeroelastic theory of actively controlled wing sections in a supersonic flow field both the aerodynamic and structural nonlinearities are included [3,9].

This is motivated by the fact that these nonlinearities can contribute differently to the character of the flutter boundary. Moreover, in the case when the flutter boundary is catastrophic, an active control mechanism can be implemented as to convert the flutter boundary into a benign one.

3.1 Nonlinear Piston Theory Aerodynamics (PTA)

As a basic ingredient towards addressing the nonlinear flutter, the nonlinear unsteady aerodynamic lift and moment should be determined. Consistent with Piston Theory Aerodynamics (PTA) [10], the pressure on the upper and lower faces of the lifting surface can be expressed as

$$p(x,t) = p_\infty \left(1 + (\kappa - 1) v_z / 2a_\infty\right)^{2\kappa/(\kappa-1)} \quad (1)$$

Herein

$$v_z = -\left(\partial w/\partial t + U_\infty \, \partial w/\partial x\right) \operatorname{sgn} z \quad (2)$$

denotes the downwash velocity normal to the lifting surface and $a_\infty^2 = \kappa \, p_\infty / \rho_\infty$, while $\operatorname{sgn} z$, assumes the values 1 or -1 for $z > 0$ and $z < 0$, respectively. In addition,

$$w(t) = h(t) + \alpha(t)(x - bx_0) \quad (3)$$

denotes the transversal displacement of the elastic surface. In addition, x_0 is the dimensionless streamwise position of the pitch axis measured from the leading edge, whereas b is the half-chord length of the airfoil; p_∞, ρ_∞, U_∞ and a_∞ are the pressure, the air density, the airflow speed and the speed of sound of the undisturbed flow, respectively; and κ is the isentropic gas coefficient. Retaining in the binomial expansions of (1), the terms up to and including $(v_z/a_\infty)^3$, yields the pressure formula for the PTA in the third-order approximation [1,3,11,12]:

$$\frac{p}{p_\infty} = 1 + \kappa \frac{v_z}{a_\infty}\gamma + \frac{\kappa(\kappa+1)}{4}\left(\frac{v_z}{a_\infty}\gamma\right)^2 + \frac{\kappa(\kappa+1)}{12}\left(\frac{v_z}{a_\infty}\gamma\right)^3 \quad (4)$$

Herein the aerodynamic correction factor [11] $\gamma \equiv M_\infty \big/ \sqrt{M_\infty^2 - 1}$, enables one to extend the validity of the PTA to the entire low supersonic-hypersonic flight speed regime. As mentioned in [3,11], the validity of Eq. (4) is satisfactory even for $M \geq 2$. It is also important to remark that, PTA in general and Eq. (4), in particular, are applicable as long as the transformations through compression and expansion may be considered as isentropic, that is as long as the shock losses would be insignificant (low intensity waves). On the other hand, a more general formula for the pressure, obtained from the theory of oblique shock waves (SWT), and valid over the entire supersonic/hypersonic range, can be applied [3,11].

3.2 Nonlinear aeroelastic model

Upon denoting the dimensionless time and speed parameters, $\tau = U_\infty t/b$ and $V = U_\infty / b\omega_\alpha$, the aeroelastic governing equations of an actively controlled wing section elastically constrained by the linear translational and the nonlinear torsional springs (Fig. 1), and featuring plunging ξ and twisting α degrees of freedom, exposed to a supersonic/ hypersonic flow field are [13]:

$$\xi''(\tau) + \chi_\alpha \alpha''(\tau) + 2\zeta_h(\varpi/V)\xi'(\tau) + (\varpi/V)^2 \xi(\tau) = l_a(\tau) \tag{5}$$

$$(\chi_\alpha/r_\alpha^2)\xi''(\tau) + \alpha''(\tau) + (2\zeta_\alpha/V)\alpha'(\tau) + 1/V^2\, \alpha(\tau) + \delta_S\, 1/V^2\, B\alpha^3(\tau) = m_a(\tau) - \psi_1/V^2\, \alpha(\tau) - \delta_C\, \psi_2/V^2\, \alpha^3(\tau) \tag{6}$$

Herein $l_a\left(\equiv L_a b/mU_\infty^2\right)$ and $m_a\left(\equiv M_a b^2/I_\alpha U_\infty^2\right)$ denote the dimensionless aerodynamic lift and moment, respectively. These are expressed as:

$$l_a(\tau) = -\gamma/(12\mu M_\infty)\{12\alpha(\tau) + M_\infty^2(1+\kappa) \times \gamma^2\alpha^3(\tau) + 12[\xi'(\tau) + (1-x_0)\alpha'(\tau)]\} \tag{7}$$

$$m_a(\tau) = -(\gamma/12\mu M_\infty r_\alpha^2)\{12(1-x_0)\alpha(\tau) + \delta_A M_\infty^2(1-x_0)(1+\kappa)\gamma^2\alpha^3(\tau) + 4[(3(1-x_0)\xi'(\tau) + (4-6x_0+3x_0^2)\alpha'(\tau))]\} \tag{8}$$

Together with the parameters defined in Refs. [4,9,13,14], we also define the parameter B that represents the nonlinear restoring moment, defined as ratio of the linear and of nonlinear stiffness coefficients. As a result, B constitutes a measure of the degree of the nonlinearity of the system, where $B<0$ corresponds to soft structural nonlinearities, while $B>0$ to hard structural nonlinearities; ψ_1 and ψ_2 denote the normalized linear and nonlinear control gains respectively; $\delta_S, \delta_A, \delta_C$ are tracing quantities identifying the structural, aerodynamic and non-linear control terms, respectively.

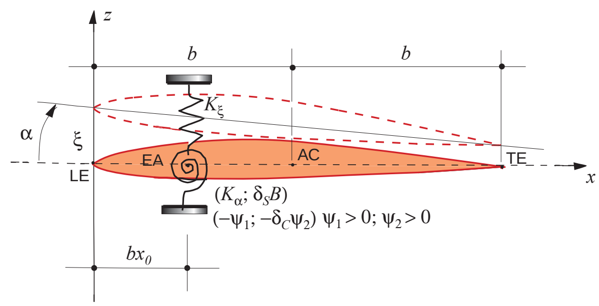


Fig. 1. Characteristics of the wing section

The numerical simulations, unless otherwise stated, involve the case of an airfoil whose geometric parameters are:
$\mu = 50; \chi_\alpha = 0.25; \varpi = 1.0; r_\alpha = 0.5; \zeta_\alpha = \zeta_\xi = 0;$
$x_0 = 0.5; \omega_\xi = \omega_\alpha = 60\text{Hz}; b = 1.5\text{m}; \psi_1 = \psi_2 = 0;$
$\delta_A = \delta_S = \delta_C = 1; B = 1$.

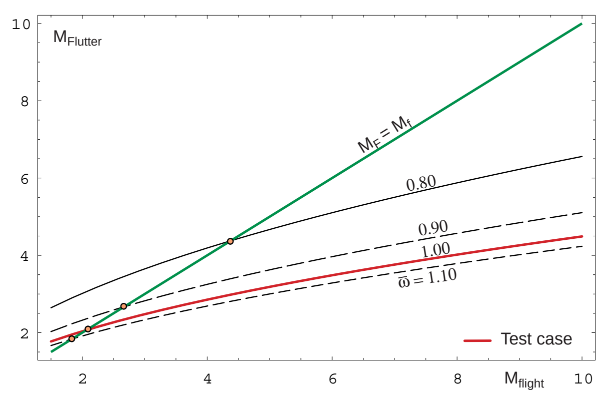


Fig. 2. Mach flutter vs. Mach flight. Influence of the frequency ratio $\varpi(\equiv\omega_\xi/\omega_\alpha)$.

In addition, the flow field characteristics are: $\gamma = 1; a_\infty = 340\text{m/s};\ \rho_\infty = 1.225\text{kg/m}^3; \kappa = 1.4$. The graphs depicting the influence of some geometrical parameters of the wing section on the flutter instability boundary as a function of the flight Mach number are displayed in Figs. 2 and 3. The trend of these plots is in excellent agreement with the results provided in Bisplinghoff *et al*. [13]. In Fig. 4, the dependence of the Mach flutter ($M_{Flutter}$) as a function of the Mach flight (M_{flight}) for selected values of the linear feedback gain ψ_1 is presented.

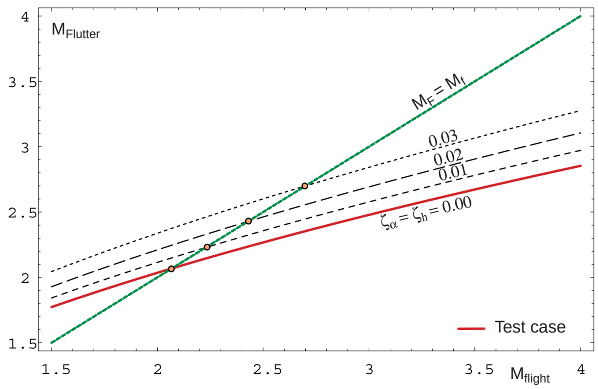


Fig. 3. Mach flutter vs. Mach flight. Influence of the structural damping coefficients ζ_ξ and ζ_α.

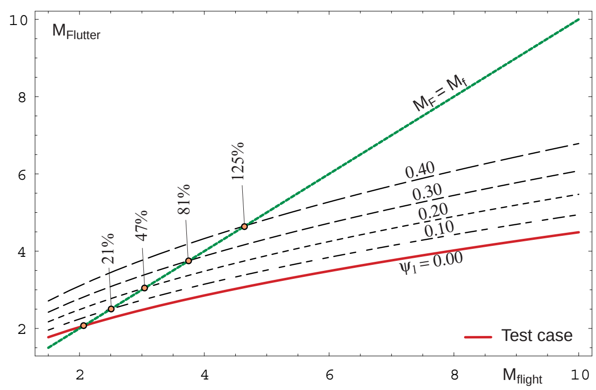


Fig. 4. Mach flutter vs. Mach flight. Influence of the linear control gain ψ_1. The numbers indicate the percentage increase of $M_{Flutter}$ as compared to that of the uncontrolled case ($\psi_1 = 0$).

With the increase of the linear control gain, an increase of the flutter speed is experienced (up to 125% for $\psi_1 = 0.4$).

4. Methodology: the Lyapunov First Quantity (LFQ)

As previously stated, the conditions of catastrophic or benign character of the flutter instability boundary are obtained by the use of the Lyapunov first quantity $L(V_F)$. This quantity will be evaluated next. To this end, the system of governing equations is converted to a system of four differential equations in the form [1,3,8]:

$$\frac{dx_j}{d\tau} = \sum_{m=1}^{4} a_m^{(j)} x_m + P_j(x_1, x_2, x_3, x_4) \tag{9}$$

For the present case the functions $P_j(x_1, x_2, x_3, x_4)$ include both the structural and aerodynamic nonlinearities, as well as the nonlinear control term that can be cast as:

$$P_j(x_1,x_2,x_3,x_4) = \sum_{i=1}^{4} a_{ii}^{(j)} x_i^2 + 2\sum_{\substack{i,l=1\\(i\neq l)}}^{4} a_{li}^{(j)} x_i x_l + \sum_{i=1}^{4} a_{iii}^{(j)} x_i^3 + 3\sum_{\substack{i,l=1\\(i\neq l)}}^{4} a_{iil}^{(j)} x_i^2 x_l + 6\sum_{\substack{i,l,k=1\\(i\neq l\neq k)}}^{4} a_{ilk}^{(j)} x_i x_l x_k \tag{10}$$

Considering the solution of Eqs. (9) under the form $x_j = A_j \exp(\omega t)$, the characteristic equation corresponding to the linearized system counterpart is:

$$\omega^4 + p\omega^3 + q\omega^2 + r\omega + s = 0 \tag{11}$$

As a reminder, for steady motion, the equilibrium is stable in Lyapunov's sense, if the real parts of all the roots of the characteristic equation are negative [15]. It is well known that, the study of stability leads to the Routh-Hurwitz (R-H) conditions, that correspond to stability of the considered state of equilibrium. The R-H conditions reduce to the inequalities:

$$p > 0,\ q > 0,\ r > 0,\ s > 0, \tag{12}$$

and

$$\Re = pqr - sp^2 - r^2 > 0 \tag{13}$$

For the aeroelastic stability problems, the roots of the characteristic equation on the critical flutter boundary, $\Re = 0$, are given by: $\omega_{1,2} = \pm ic$, $\omega_{3,4} = -\varepsilon \pm in$ where $c^2 = r/p$, $\varepsilon = p/2$, $n^2 = sp/r - p^2/4$, $n > 0$. These equations reveal that the required condition for the application of Hopf bifurcation theorem are fulfilled. For sufficient small values of the speed V, all the roots of the characteristic equation are in the left hand side of the complex variable and the zero solution of the system is asymptotically stable.

The value $V = V_F$ for which the two roots of the characteristic equations are purely imaginary and the remaining two are complex conjugate and remain also in the left hand side of the complex variable, is critical and corresponds to the critical flutter velocity. In order to be able to distinguish the benign portions of the stability boundary from the catastrophic ones, is necessary to solve the stability problem for the system of equations in state-space form in the critical case of a pair of pure imaginary roots. Following the ideas developed by Lyapunov [16] and Bautin [8], the catastrophic and benign portions of the flutter instability boundary can be determined, via determination of the sign of the Lyapunov's coefficient. Following Bautin, the system of equations (9) is reduced to the canonical form. The expression of the Lyapunov first quantity is given under a closed form in [1]. For the present case, the Lyapunov first quantity is expressed in terms of the coefficients $A_{kls}^{(j)}$ as:

$$L(V_F) = \frac{3\pi}{4c}\left(A_{111}^{(1)} + A_{222}^{(2)} + A_{112}^{(2)} + A_{122}^{(1)}\right) \quad (14)$$

where, the terms in the bracket of Eq. (11) are expressed via the coefficients $a_{kls}^{(j)}$ appearing in Eqs. (10). The coefficients of Eq. (11), evaluated on the instability boundary are presented in [14]. The flutter critical boundary is benign (supercritical Hopf-bifurcation) or catastrophic (subcritical Hopf-bifurcation), if the following inequalities, $L(V_F) < 0$ and $L(V_F) > 0$, are fulfilled, respectively. Paralleling the procedure devised in [1] and [3], the benign or catastrophic character of the flutter boundary, may be expressed, respectively, under the concise form as:

$$V_F^{\,2} < V_r^{\,2}, \quad \text{or} \quad V_F^2 > V_r^2 . \quad (15)$$

where:

$$V_r^{\,2} = A_1/A_2 \quad (16)$$

In Eq. (13) A_1 contains the structural nonlinearities and the nonlinear control gain parameter, whereas A_2, includes the aerodynamic nonlinearities.

As a very important consequence, in absence of nonlinear control and for $B < 0$, that is for soft structural nonlinearities, the expression of V_r^2 is negative, and the relation $V_F^2 > V_r^2$ is satisfied for any flight supersonic Mach number [14]. This implies that in this case, even in the presence of the linear control, a subcritical Hopf-Bifurcation is experienced. On the other hand, for $B > 0$, that is for hard structural nonlinearities, in the presence of the linear control, the transition from the benign flutter boundary to the catastrophic one occurs at higher flight Mach numbers.

4.1 Stability Analysis in Presence of Active Control

The stability of the aeroelastic system in the vicinity of the flutter boundary is analyzed next. The effect of structural nonlinearities on the character of the flutter boundary is carried out in terms of the nonlinear parameter B, see Eq. (6).

The graph depicting the Lyapunov first quantity $L(V_F)$ vs M_{flight} for the uncontrolled wing section $(\delta_C = 0; \psi_1 = \psi_2 = 0)$ and for the cases in which aerodynamic and structural hard nonlinearities are retained, are displayed in Figs. 5, 6 and 7.

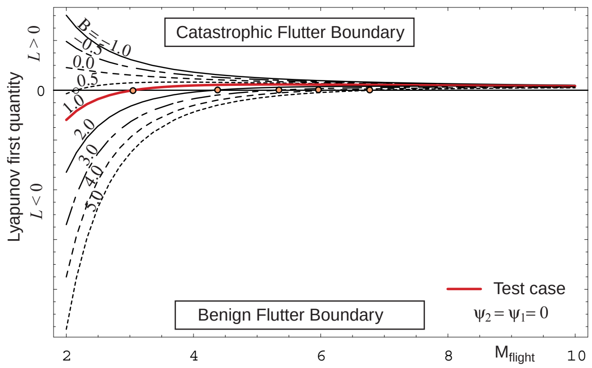


Fig. 5. Lyapunov first quantity (LFQ). Influence of aerodynamic and soft/hard structural nonlinearities on flutter boundary (FB).

In these numerical simulations both types of nonlinearities have been considered separately and together.
It clearly appears that in presence of aerodynamic nonlinearities only, the Lyapunov first quantity is positive for any flight Mach number. This result reflects the fact that this type of nonlinearity provides a catastrophic character to the flutter boundary, implying that a subcritical H-B occurs.

Similar trends and conclusions are obtained for soft structural nonlinearities. On the other hand, in the presence of hard structural nonlinearities only, the opposite situation is experienced, whereas when both nonlinearities are included, at relatively moderate supersonic flight Mach numbers the flutter is benign, while with the increase of the flight Mach number, the flutter becomes catastrophic.

Moreover, as a consequence of these results, if the aerodynamic nonlinearities are discarded, $(\delta_A = 0)$, the aeroelastic system features a supercritical or subcritical character, for any flight Mach number, depending on whether hard $(B > 0)$ or soft $(B < 0)$ structural nonlinearities are present, respectively.

The influence of the hard structural and aerodynamic nonlinearities $(\delta_S = \delta_A = 1)$ for controlled/uncontrolled system is presented in Figs. 7 and 8.

Both hard structural and aerodynamic nonlinearities ($\delta_A = 1; \delta_S = 1; B = 1$), and aerodynamic nonlinearities only ($\delta_A = 1;\ \delta_S = 0$), are included, respectively.

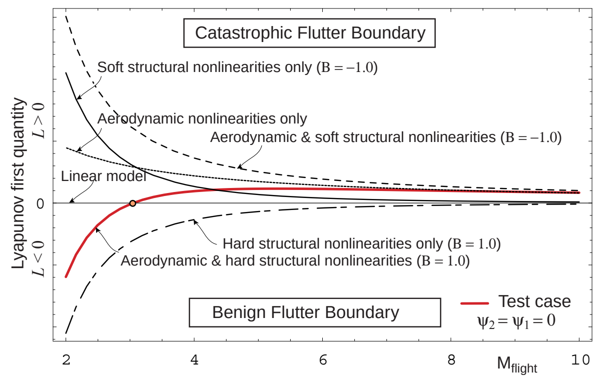


Fig. 6. LFQ. Influence of aerodynamic and soft/hard structural nonlinearities on FB.

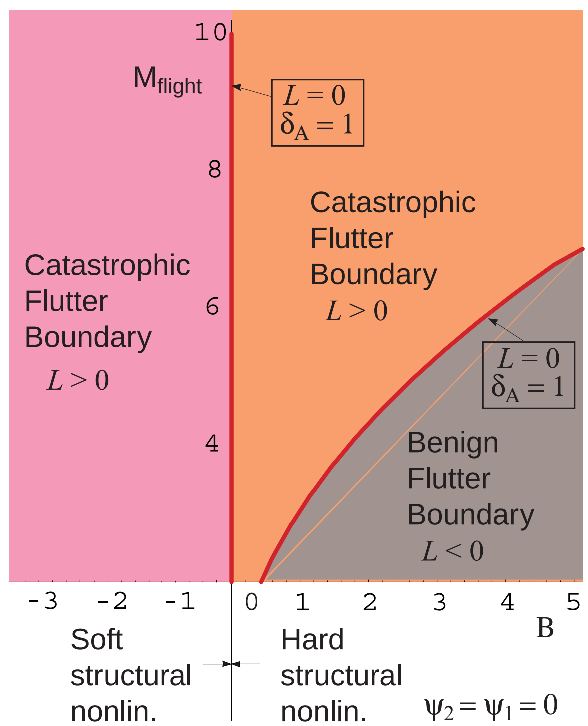


Fig. 7. LFQ. Influence of aerodynamic and soft/hard structural nonlinearities on FB.

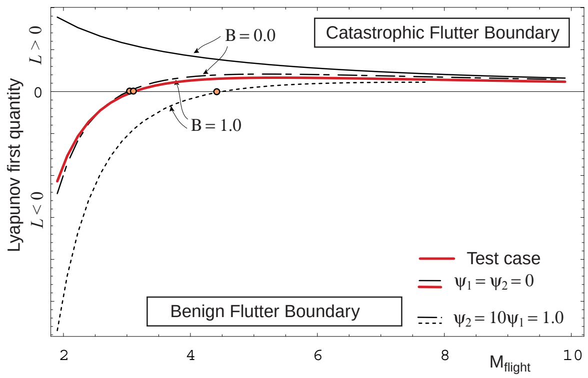
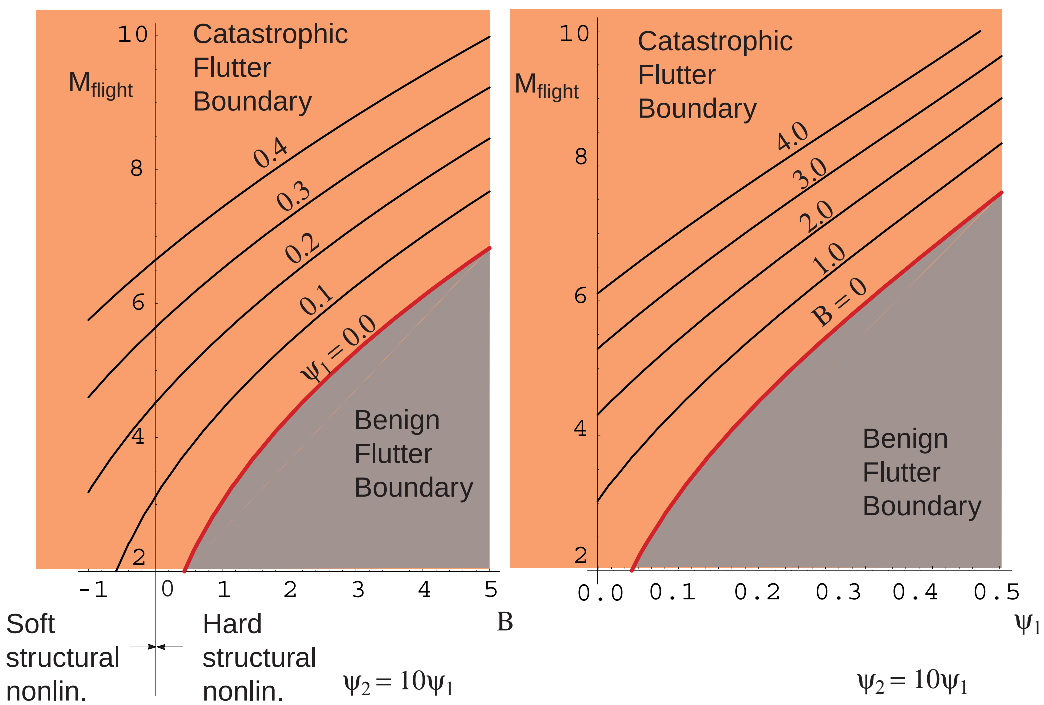


Fig. 8. LFQ. Influence of aerodynamic and structural nonlinearities on FB for the uncontrolled/controlled cases.

The control mechanism acts in both situations toward the stabilization of the system, that is to enhance the flutter behavior. Also in this case, the inherent catastrophic character of the flutter boundary, that corresponds to the case when only aerodynamic nonlinearities are considered, can be converted via the nonlinear control into a benign one.

Figure 9 highlights the effect of the linear/nonlinear control in the presence of structural and aerodynamic nonlinearities. It clearly appears that, the control and the hard structural nonlinearities are beneficial in the sense of expanding the region of benign flutter boundary.



Figs. 9. a) B vs M_{flight}. b) ψ_1 vs M_{flight}. Uncontrolled/controlled cases.

The results emerging from these figures reveal also that soft structural nonlinearities ($B < 0$) result in a catastrophic flutter boundary, and that via the active nonlinear control the unstable LCO can become stable. In addition, in [14] was reported that when soft structural and aerodynamic nonlinearities are present, the linear active control cannot change the character of the flutter boundary.

Using Eqs. (12) and (13), the character of the flutter boundary is examined and has been plotted in Figs. 10, 11 and 12 for $\delta_S = 1; B = 1$ and $\delta_A = 1$. These graphs display the benign $(L < 0)$ and catastrophic $(L > 0)$ characters of the flutter boundary for the actively controlled wing section, where $\psi_1 = 0; 0.1; 0.2; 0.3; 0.4$; $\psi_2 = 0; 10\psi_1$.

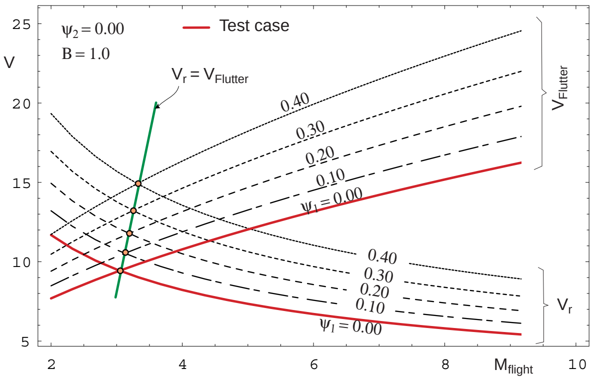


Fig. 10. Influence of the linear control on the character of the flutter boundary ($V_r > V_{Flutter}$ ⇒ Benign FB; $V_r < V_{Flutter}$ ⇒ Catastrophic FB.

The intersection of the curves $V_{Flutter}$ and V_r identifies the transition between catastrophic and benign flutter boundary. It appears that a nonlinear control is more effective toward a stabilization of the aeroelastic system than the linear one. The Liapunov first quantity corresponding to the cases identified in Figs. 10, 11, and 12 are depicted in Figs. 13, 14 and 15, respectively. These plots help understanding of the behavior of the aeroelastic system in the presence of linear and nonlinear active controls. From the present numerical simulations, the aeroelastic system appears to be characterized by a catastrophic flutter boundary in the region where $V_r > V_F$ (unstable LCO), and by a benign flutter boundary in the region where $V_r < V_F$ (stable LCO). It appears that even, in the conditions of a hard structural nonlinearity, when, at relatively moderate supersonic flight Mach numbers, the flutter is benign, with the increase of the flight Mach number the flutter becomes catastrophic. This implies that for large flight Mach numbers the effects of the aerodynamic nonlinearities become prevalent. It is also shown that neglect of aerodynamic nonlinearities, yields inadvertent results related to the character of the flutter boundary, especially at high flight Mach numbers.

Figure 16 highlights the fact that the increase of hard physical nonlinearity, measured in terms of B, results in the increase of the benign portions of the stability boundary and, at the same time, in a shift of the benign character of the flutter boundary toward larger flight Mach numbers.

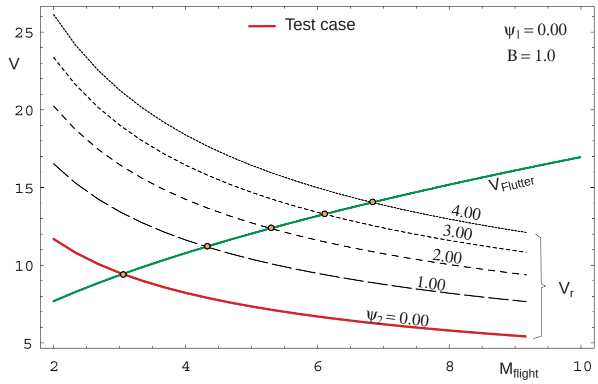


Fig. 11. Influence of the nonlinear control on the character of the flutter boundary.

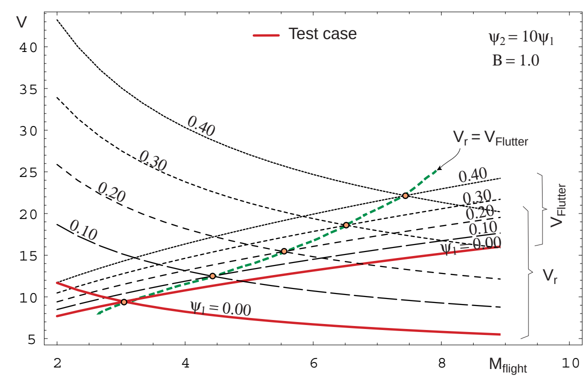


Fig. 12. Influence of linear/nonlinear controls on the character of the flutter boundary.

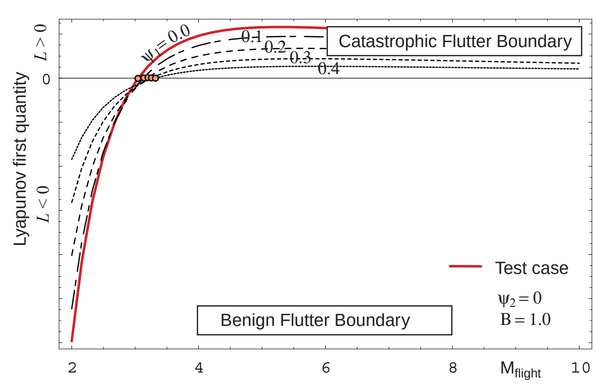


Fig. 13. LFQ corresponding to the case of Fig. 10.

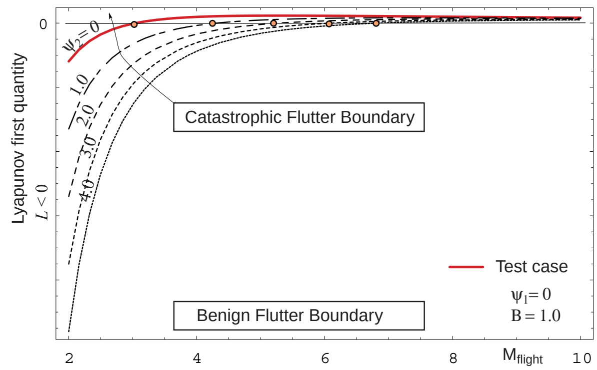


Fig. 14. LFQ corresponding to the case of Fig. 11.

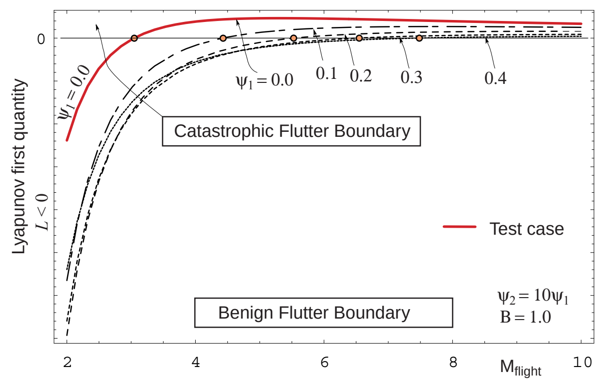


Fig. 15. LFQ corresponding to the case of Fig. 12.

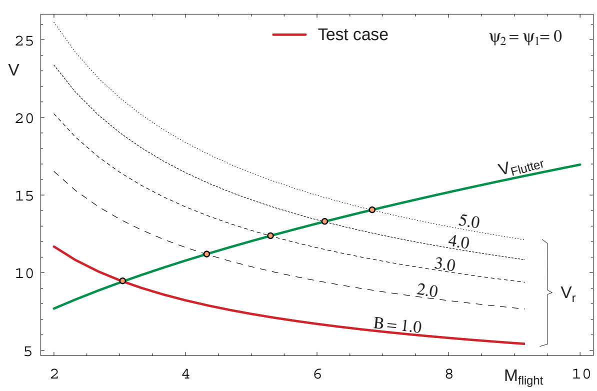


Fig. 16. Influence of hard structural and aerodynamic nonlinearities on the character of the flutter boundary. Uncontrolled case.

This result, as it was shown in Figs. 7 and 9, reflects the fact that this type of nonlinearity provides a benign character to the flutter boundary. Figure 17 highlights the effects of the frequency ratio ϖ on the LFQ and implicitly on the nature of the flutter boundary.

This plot, in conjunction with Fig. 2, shows that with the increase of ϖ, the transition from the benign to catastrophic flutter occurs at lower flight Mach numbers. At the same time, with the increase of a ϖ, lower flutter speed is experienced.

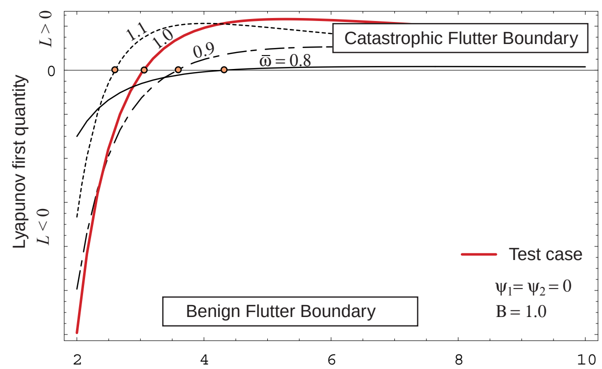


Fig. 17. LFQ. Influence of the frequency ratio ϖ on the character of the flutter boundary.

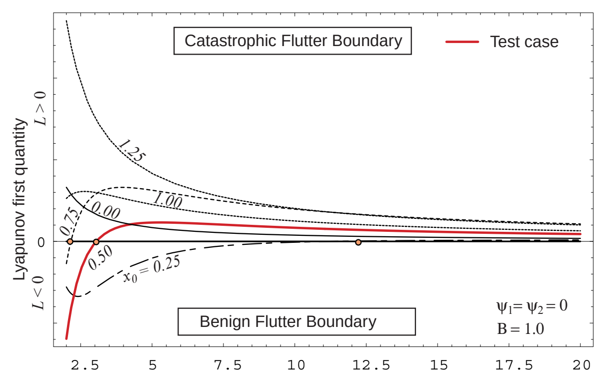


Fig. 18. LFQ. Influence of location of the elastic axis on the character of the flutter boundary.

The effects of the position of the elastic axis, x_0, measured from the leading edge, on the LFQ is presented in Fig. 18.

From this preliminary investigation, it appears that the location of elastic axis has significant implications on determination of the character of the flutter boundary. In particular, transitions from catastrophic to benign and from benign to catastrophic occur for various locations of the elastic axis.

As concerns the issue of generating the active control moment, this one was not addressed. It is the authors' belief that this can be produced via a device operating similarly to a spring, whose linear and non-linear characteristics can be controlled. However, additional analysis is required to confirm this assertion.

It should be noticed, that in spite of the great practical importance, the literature dealing with the aeroelasticity of aircraft structures in the presence of both structural and aerodynamic nonlinearities is quite void of such results.

4 Conclusions

A comprehensive study related to the influence of structural and aerodynamic nonlinearities on the character of the flutter boundary of aircraft wing sections in a supersonic flight speed regime is presented. Via the use of the Lyapunov first quantity a general picture of the variation of catastrophic and benign parts of the flutter boundary, as a function of the considered nonlinearities and of the parameters characterizing the aeroelastic model can be obtained. In addition, the potentialities of the linear and nonlinear active control enabling to enhance the flutter instability behavior and convert the catastrophic flutter boundary into a benign one are highlighted in the paper. The expected outcomes of this study are: a) to greatly enhance the scope and reliability of the aeroelastic analysis and design criteria of advanced supersonic flight vehicles and, b) to provide a theoretical basis for the analysis of more complex nonlinear aeroelastic systems.

Acknowledgement

The support of this research by the NASA Langley Research Center through Grants NAG-1-2281 and NAG-1-01007 is acknowledged.

References

[1] Librescu L. Aeroelastic stability of orthotropic heterogeneous thin panels in the vicinity of the flutter critical boundary. *Journal de Mécanique*, Part I, Vol. 4, No. 1, pp. 51-76, 1965; Part II, Vol. 6, No. 1, pp. 133-152, 1967.

[2] FAA FAR Part 25-- *Airworthiness Standards*, Sec. 25.629 Aeroelastic stability requirements.

[3] Librescu L. *Elastostatics and kinetics of anisotropic and heterogeneous shell-type structures, aeroelastic stability of anisotropic multilayered thin panels*. 1st edition, Noordhoff Internat. Publ., Leyden, The Netherlands, pp. 106-158, 1975.

[4] Strganac TW, Ko J, Thompson DE and Kurdila AJ. Identification and control of limit cycle oscillations in aeroelastic systems. *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, pp. 1127-1133, 2000.

[5] Librescu L and Na SS. Bending vibration control of cantilevers via boundary moment and combined feedback control laws. *Journal of Vibration and Controls*, Vol. 4, No. 6, pp. 733-746, 1998.

[6] Hopf E. Bifurcation of a periodic solution from a stationary solution of a system of differential equations, *Ber. Math. Phys. Klasse*, Sächsischen Acad. Wiss. Leipzig (Germany), Vol. XCIV, pp. 3-32, 1942.

[7] Marsden JE and McCracken M. *The Hopf bifurcation and its applications*. 1st edition, Applied Mathematical Sciences, Vol. 19, Springer-Verlag, New York, pp. 1-26, 1976.

[8] Bautin NN. *The behaviour of dynamical systems near the boundaries of the domain of stability*. 1st edition, Nauka, Moskva, (In Russian), pp. 70-77, 1984.

[9] Lee BHK, Price SJ and Wong YS. Nonlinear aeroelastic analysis of airfoils: bifurcation and chaos. *Progress in Aerospace Sciences*, Vol. 35, pp. 205-334, 1999.

[10] Ashley H and Zartarian G. Piston theory – A new aerodynamic tool for the aeroelastician, *Journal of Aeronautical Science*, Vol. 23, pp. 1109-1118, 1956.

[11] Carafoli E. *Wing theory in supersonic flow*. 1st edition, International Series on Monographs in Aeronautics and Astronautics, Vol. 7, Pergamon Press, Oxford, pp. 463-524, 1969.

[12] Thuruthimattam BJ, Friedmann PP, McNamara JJ and Powell KG. Aeroelasticity of a generic hypersonic vehicle. *43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference,* Denver, CO, Paper 2002-1209, 2002.

[13] Bisplinghoff RL and Ashley H. *Principles of Aeroelasticity*. Dover Publications, Inc., 1996.

[14] Marzocca P, Librescu L and Silva WA. Flutter, post-flutter and control of a supersonic 2-D lifting surface. *Journal of Guidance, Control, and Dynamics,* in press, 2002.

[15] Malkin IG. *Theory of stability of motion*. Translation Series, AEC-tr-3352, Physics and Mathematics, pp. 178-184, 1963.

[16] Lyapunov AM. *General theory of the stability of motion*. Izd., Moskva, 1950, English translation by Academic Press, New York, 1966.