THREE-DIMENSIONAL CO-OPERATIVE INSTABILITIES IN WAKE VORTEX PAIRS

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Abstract

In this paper, a numerical study is presented concerning three-dimensional co-operative instabilities in vortex pairs. The vortex pair studied is a generic model for the wake of an aircraft in cruise flight at large distance behind this aircraft. A counter-rotating vortex pair can develop two fundamentally different types of instabilities: short-wavelength or elliptic instabilities and long-wavelength or Crow instabilities. Both types are considered in this study, as is the role of both types in the break-up or decay of an aircraft vortex wake. A brief review is presented of proposed methods to enhance the wake vortex decay. In a number of these methods, the short-wavelength instability plays an important role. Therefore, improving the understanding of the dynamics of this instability is of great relevance to wake vortex studies. In this paper, the dynamics of the short-wavelength instability in a vortex pair is studied in some detail, as is the dependence on Reynolds number.

1 Introduction

A well-known problem in the operation of commercial jet transports is the so-called 'wake vortex hazard'. This problem originates from the strength and persistence of the vortex pair formed in the wake of an aircraft. When a following aircraft encounters this vortex wake, a strong motion is induced. This can be either a rolling motion or an acceleration in the vertical direction, depending on the type of encounter. In the landing and take-off phase of flight the vortex wake is particularly hazardous. Since the 1970s, the wake vortex problem has received a lot of attention from various researchers. Crow[3] presented a linear stability analysis of a counter-rotating vortex pair, predicting sinusoidal instabilities at wavelengths very similar to those observed in condensation trails of jet transport aircraft in cruise flight. Following this publication, this long-wavelength instability (from then on known as Crow instability) has been an important aspect of aircraft wake vortex studies. In the 1970s, more theoretical studies concerning instabilities of vortex flows were performed, for example [10], [11], [12], [4], [9]. The research described in these publications focused on the characteristics of three-dimensional linear instability modes. The temporal evolution of the instability modes was not studied.

The linear stability analyses showed that for the counter-rotating vortex pair, two fundamentally different branches exist: the long-wavelength or Crow instability and the short-wavelength or elliptic instability.

In contrast to the long-wavelength instability, the short-wavelength instability is typically not observed in aircraft trailing vortices, and as a result has not received as much attention in aircraft wake vortex studies as the Crow instability. However, in many of the proposed methods to enhance wake-vortex decay, for example [5], [6], the short-wavelength instability is forced. This short-wavelength instability can be expected to be an effective decay mechanism for wake vortices due to its small-scale structure and large predicted growth rates. Understanding the dynamics of this type of instability is therefore of practical relevance. The numerical results presented in this study highlight some of the complex features of this instability at later stages of the temporal evolution. The numerical results in the present study have been obtained using a three-dimensional high-order accurate method for the Navier-Stokes equations for an incompressible medium, as described in[8]. The method uses a Fourier collocation method in the periodic axial direction of the vortex pair and a compact finite-difference spatial discretization method for the two remaining non-periodic directions.

2 The decay of a wake vortex

The vortex wake of an aircraft decays under the influence of molecular viscosity, (atmospheric) turbulence, stratification and the action of three-dimensional instabilities. This decay or destruction progresses relatively slowly, as will be illustrated in section 4. In the vicinity of airports this problem is most severe and, as a result, various methods to enhance the decay of aircraft wake vortices have been studied by a number of researchers. Examples of proposed methods are:

- configuring the inboard vortices, in a high-lift configuration, such that these vortices remain in a region of high strain. Perturbations on these vortices will then grow very rapidly. These inboard vortices will excite a long-wavelength instability in the wing-tip vortex pair. Examples of such methods were presented in[6] and [1]

- perturbing the wing-tip vortices by internal and external density perturbations. This method was analyzed in [5] as a means of initiating short-wavelength instabilities. The density perturbations were conjectured to be created by temperature variations produced by combustion of fuel in the wing-tip region. This method was envisaged for use during take-off and landings.

Extensive research remains necessary before either of these methods can be used in practice. Passive methods have also been studied. These methods have less potential benefit, but could be more easily introduced in practice. A design optimization of a new airliner for minimum vortex-wake generation is an example of a passive method. Since the wake-vortex hazard is particularly severe in the landing phase, this design optimization should emphasize high-lift configurations. The ratio of the vortex core radius and the spacing of vortices in a counter-rotating vortex pair forms the major parameter in the development of cooperative instabilities. Both theoretical and numerical studies indicate that this ratio should be maximized for maximum wake decay. Therefore, a design optimization is needed that leads to a roll-up process that creates a maximum vortex core radius for a given wing span and required lift.

3 Numerical method

The three-dimensional Navier-Stokes equations for an incompressible flow are integrated in time using a fractional-step temporal integration method of second-order accuracy in time. The time-advancement method uses a combination of explicit integration of the non-linear convection terms, employing the second-order accurate Adams-Bashfort method, and implicit integration of the diffusion terms by the Crank-Nicholson method. The spatial discretization method is described in some detail in [8] and is based on a combination of fourth-order accurate compact finite-difference techniques and a Fourier collocation method in the periodic axial direction.

4 Numerical results for Crow instability

Figure 1 shows the temporal evolution of the Crow instability in a vortex pair. The initial vortex pair consists of two Lamb-Oseen vortices with a core radius r_c. The spacing b of the vortex pair in the unperturbed situation is 5 times

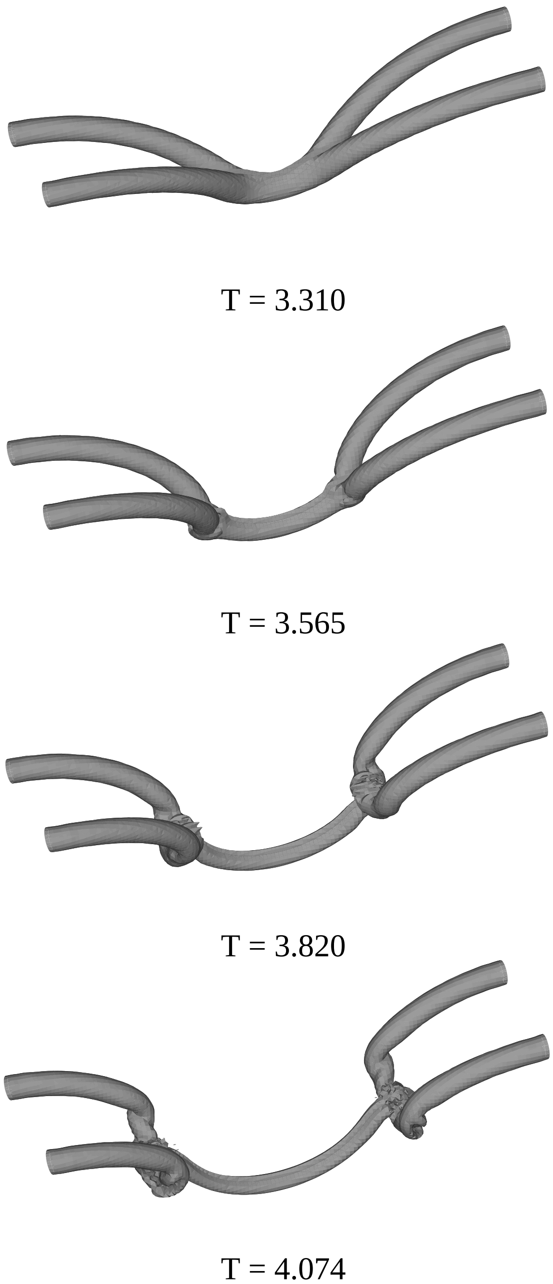


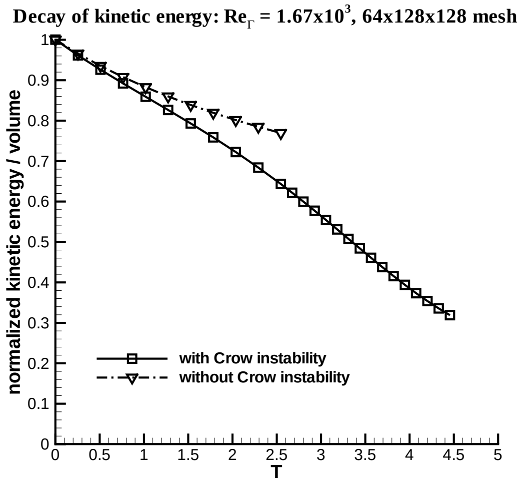
Fig. 1 Temporal evolution of Crow instability: $Re_\Gamma = 1.67 \cdot 10^4$, 128^3 mesh, $r_c/b = 0.2$, $\lambda/b = 8.0$. Shown are iso-surfaces $|\underline{\omega}| = 500\ s^{-1}$.

this core radius, i.e. $r_c/b = 0.2$. Linear stability predicts a wavelength λ for the most amplified Crow instability mode of 8 times this spacing. The computational domain is chosen such that the dimension is 8.0 b in each of the three coordinate directions. In this paper, T denotes non-dimensional time $T = t \cdot \Gamma/2\pi b^2$, i.e. time is scaled with the time needed for the unperturbed vortex pair to sink a distance b.

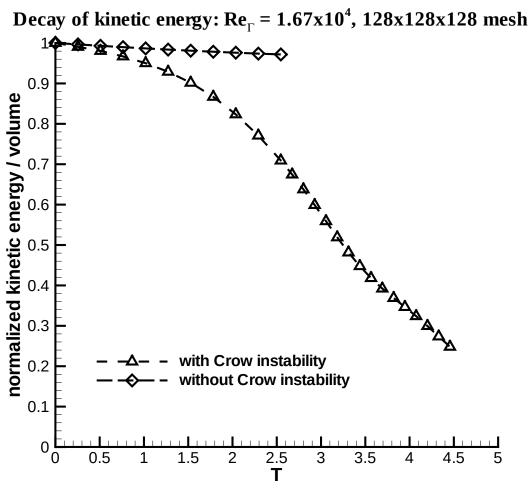
The figure shows iso-surfaces of vorticity magnitude $|\underline{\omega}| = 500\ s^{-1}$. This level was chosen since it highlights many of the characteristic features of the evolving instability.

The features shown in figure 1, such as the reconnection of the vortices and the formation of vortex-ring structures, are typical for the Crow instability.

To obtain quantitative information on the effect of the long-wavelength instability on the decay of the vortex pair, the kinetic energy of the flow contained in the computational domain is computed. A frame of reference is used that moves downward with the vortex pair. Then, a Fast Cosine Transform in the axial direction of the flow is applied to the computed kinetic energy contained within the computational domain. The resulting coefficients give information about the kinetic energy contained in the finite domain at specific wavelengths. The mode $k = 0$ represents the total kinetic energy of the computational domain. The evolution of this quantity is shown in figure 2 for two Reynolds numbers. Shown is this energy normalized by the initial value. The contribution of the Crow instability can be seen by comparison with the result for the vortex pair without evolving Crow instability. As can be expected, the decay of the total kinetic energy is stronger for the lower Reynolds number simulation (which has a kinematic viscosity that is 10 times higher than that of the higher Reynolds number simulation). For both Reynolds numbers, the effect of the Crow instability is very significant. However, the kinetic energy decay is weak for the practical situation of the wake of a large jet transport. For example at T = 3, about half of the kinetic energy is left for a Reynolds number that is a factor 100 lower than for a wake of a large jet transport. Translating T to a distance behind the aircraft, shows that for a Boeing 747 in cruise flight, T = 3 corresponds to a distance of more than 16 kilometers. Although the kinetic energy studied here is not a direct measure for the threat posed by a wake vortex pair, it does show that the process of decay or destruction of a vortex wake by

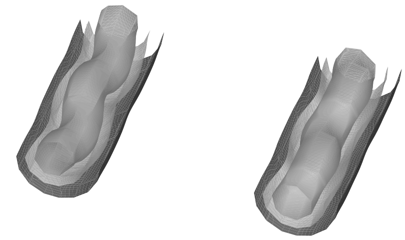


$Re_\Gamma = 1.67 \cdot 10^3$

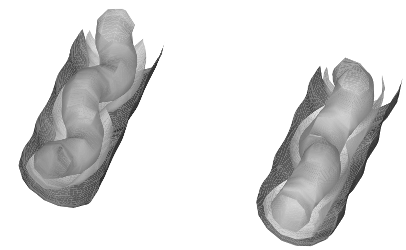


$Re_\Gamma = 1.67 \cdot 10^4$

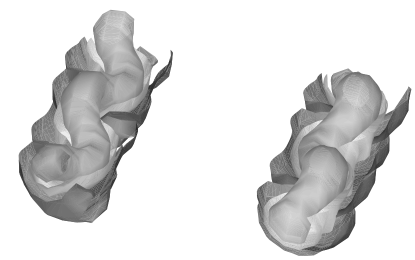
Fig. 2 Kinetic energy within computational domain: evolution of coefficient $k = 0$ of Fast Cosine Transform in axial direction. With and without developing Crow instability.



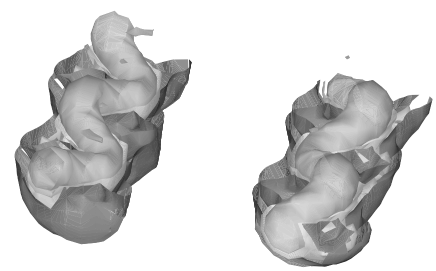
T = 0.000



T = 0.637



T = 1.273



T = 1.909

Fig. 3 Temporal evolution: $\tilde{k} = 2.261$, $r_c/b = 0.2$, $64 \times 128 \times 128$ mesh, $Re_\Gamma = 1.67 \cdot 10^4$. $|\underline{\omega}| = 600\ s^{-1}$, $900\ s^{-1}$ and $1200\ s^{-1}$.

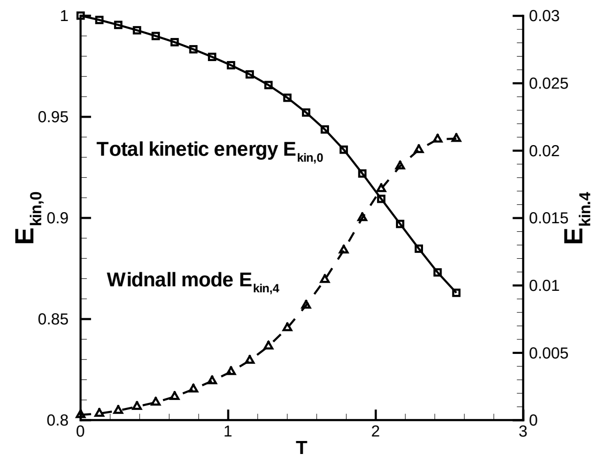
the Crow instability is slow. This motivates the research into devices that enhance this deacy.

5 Numerical results for Widnall instability

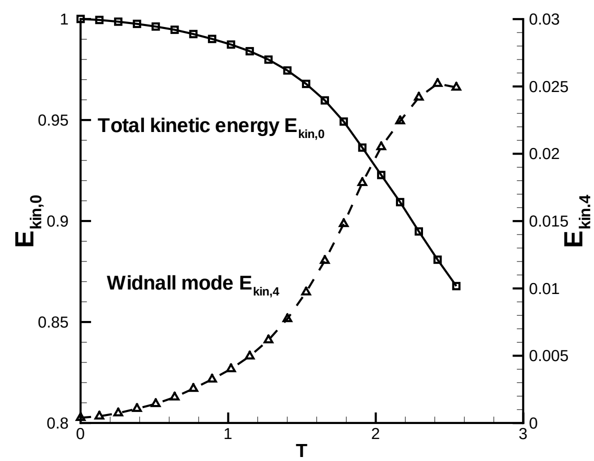
Figure 3 shows the temporal evolution of the elliptic instability in a vortex pair. The figure shows iso-surfaces of vorticity magnitude $|\underline{\omega}|$. The iso-surfaces 600 s^{-1} and 900 s^{-1} show the outer part of the vortices and are only shown for the lower half of the computational domain. The iso-surfaces 1200 s^{-1} show the structure of the inner part of the vortices. The instability mode is characterized by a strong distortion of the vortices, stronger in the inner part of the vortices than in the outer region of the vortices. This pattern is qualitatively very different from the structure of the Crow instability mode. The time-accurate simulations in this section are initialized with a pair of Lamb-Oseen vortices with two elliptic instability modes superposed on them. The spatial structure of the initial instability is computed from a normal mode analysis of an isolated Lamb-Oseen vortex, using a procedure similar to that of [7]. This analysis gives a set of non-dimensional axial wave numbers $\tilde{k}$ and a normal mode corresponding to these wave numbers for a chosen azimuthal wave number. Here, the smallest non-dimensional axial wave number, i.e. $\tilde{k} = 2.261$, is used. This wave number corresponds to a axial wavelength of 2.512 r_c, i.e. roughly two and a half times the vortex core radius. The structure is then computed by superposition of the normal modes for azimuthal wave numbers -1 and $+1$. Figure 3 shows the computed initial flow at T = 0.000. Noticeable is the relatively large initial amplitude of the instability mode that can be used with this precise initialization. The plots of figure 3 show the evolving flow at a series of instances in time. The rapid growth of the instability is apparent, as is the formation of complex flow structures in the vortices at later stages. The complexity of the flow makes it difficult to obtain quantitative data from simulations. As with the discussion of the Crow instability of section 4, the kinetic energy contained in the computational domain is used as a means to extract quantitative information about the flow evolution.

5.1 Kinetic energy decay

To obtain quantitative information on the evolution of the short-wavelength instability, the kinetic energy of the flow contained in the computational domain is computed. A frame of reference is used that moves downward with the vortex pair. Then, a Fast Cosine Transform in the axial direction of the flow is applied to the computed kinetic energy. The resulting coefficients give information about the kinetic energy contained in the finite domain at specific wavelengths. Results of this analysis are shown in figure 4. Results are shown from simulations at



$Re_\Gamma = 1.67 \cdot 10^4$



$Re_\Gamma = 1.67 \cdot 10^5$

Fig. 4 Kinetic energy within computational domain: evolution of coefficients of Fast Cosine Transform in axial direction $k = 0$ (total) and $k = 4$ (wave number of instability).

two different Reynolds numbers. As can be expected, the decay of the total kinetic energy is stronger for the lower Reynolds number simulation (which has a kinematic viscosity that is 10 times higher than that for the higher Reynolds number simulation). Both plots show an initial growth of the energy at the wave number of the elliptic instability, indicating a rapid growth of the instability in the vortex pair. However, this increase in kinetic energy stops at later stages of the evolution, indicating a saturation of the growth of the elliptic instability mode. The phenomenon was previously discussed by [7] and has serious implications for the relevance of this instability in aircraft wakes.

5.2 Elliptic instability in aircraft wakes

The numerical results presented in this section show the saturation of the amplitude growth. Further numerical study, including test cases with a different ratio r_c/b, shows that the saturation amplitude is a function of r_c^2/b^2. For a typical aircraft (cruise) configuration, this ratio is very small ($r_c/b \leq 0.05$), so the elliptic instability can be expected to saturate at a very small amplitude. In a high-lift configuration, with multiple vortex pairs, this elliptic instability can become more important if one of the flap vortices is placed at a location with a strong straining field induced by the main wing-tip vortices. This will enhance the destruction of the flap vortex. However, this offers only a partial solution to the problem, since the effect on the decay of the main wing-tip vortex will be far smaller. Thus, forcing the elliptic instability could become a useful mechanism in the enhanced destruction of the trailing vortex wake. As discussed earlier, the presence of of multiple vortices (see [2], [6]) and active forcing (for example [5]) are typical features of devices to enhance wake decay.

6 Conclusion

The dynamics of both long-wavelength and short-wavelength instabilities in a counter-rotating vortex pair is studied numerically. The emphasis in on the evolution of the kinetic energy contained in the computational domain. The significant effect of the Crow instability on this decay is shown. Furthermore, the saturation of the amplitude growth of the short-wavelength instability is shown using this kinetic energy analysis, followed by a discussion of these features for the dynamics of aircraft wake vortices.

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