NUMERICAL OPTIMIZATION OF THE WING OF A SUPERSONIC AIRPLANE

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Keywords: *optimization, wing, supersonic airplane*

Abstract

A direct optimization method is developed to design the wing of a supersonic civil airplane. Aerodynamic characteristics of the airplane are calculated within the framework of a model based on Euler's equations with taking into account surface friction drag. The optimization algorithm is based on a modification of Newton's method. Drag is used as the objective function minimized under lift, longitudinal trim and volume constraints.

A quadratic form describing dependence of drag on the design variables is obtained on the basis of a local analysis of the load distribution on the wing. It gives approximations to the true Hessian matrix and gradient vector, and wing form variations that enable the aerodynamic performance to be improved. A fast convergence to the optimum in case of the large number (more than 500) of the variables is provided.

The efficiency of the method is demonstrated on examples of the isolated wing and the wing situated under interference with other elements of the airplane. For the supersonic civil airplane a possibility of lift-to-drag ratio increase on 9.3% is shown. The airplane is designed for cruise flight at Mach number M=2 and performed in tail-first configuration with two air-breathing engines established underneath the wing.

1 Introduction

Numerical optimization design of a flying vehicle remans a difficult task. Investigations are performed for single crucial elements. In case of a supersonic civil airplane such element is the wing that carryes main part of aerodynamic loading. There are two statements of the problem, optimization of the isolated wing and the wing of airplane. In the second case, nonlinear computational fluid dynamics methods are need for correct account of interference between the wing and other elements of the airplane. Thus cost of computations grows considerably. Acceleration of convergence is required to increase research efficience.

Among optimization algorithms, gradient-based and genetic algorithms appeal to many designers. The last are more universal. However, because of a large number of objective function evaluations genetic methods are more time consuming in comparison with methods using function derivatives data [1]. A finite difference computation of sensitivity information requires a large volume of calculations and not always provides acceptable accuracy [2]. An adjoint approach gives the sensitivity information by means of solving adjoint equations without many flowfield calculations and is supposed to be a robust tool [3, 4]. Derivatives of second order can be determined by a quasi-newton method that constructs Hessian matrix through sequential approximation [2]. Computing time increases in proportion to the number of design variables. To keep the computational effort within reasonable bounds one should limit the number of variables. For expansion a variety of shapes generated at optimization process basis functions of aerodynamic shape are used [1, 3].

The analysis presented below solves a variational problem in a simplified statement. The broad review of problems solved within frames of the linear theory and the Newton's theory is presented in the monography [5]. In the case of small perturbation of supersonic flow the linearized theory allows to connect change of pressure at given surface point with form deflection in its vicinity. This connection can be established both theoretically and through numerical calculation. A summation of aerodynamic loading over all elements of the surface gives a quadratic approximation of objective function. On the basis of the information on derivatives of first and second order the Newton's method determines shape variations ensuring a quadratic rate of convergence to the optimum. The variations are utilized in exact solution. This approach has allowed to optimize airplane wing subject to many geometric variables (more than 500). The optimization process looked over thirty versions of the wing.

2 Problem Statement and Wing Description

Most aerodynamic design applications are reduced to constrained minimization of a function of many variables. For an airplane performing a cruise flight at given values of Mach number M_∞, altitude H (or Reynold's number Re), lift L, longitudinal moment Mz, it is convenient to use drag D as objective function. The problem may be stated mathematically as:

$$D\ (g,\ p\ (g)) = min,$$

$$L\ (g,\ p\ (g)) = const,$$

$$Mz\ (g,\ p\ (g)) = const,$$

$$V\ (h) = const,$$

$$T_j\ (h) \le 0,\ j = 1,\ J.$$

The geometric constraints limit the internal volume V and minimum thickness of the wing T_j. The vector of design variables g consists of vector h of n geometric variables and angle of attack α. The surface pressure p is shown among arguments for direct definition functions dependent and independent on the flowfield variables.

The airplane has both supersonic and subsonic cruise. Flight conditions affected on wing plan form that performed with cranked leading-edge and trailing-edge (fig. 1). During optimization the wing is assumed to have the constant shape in the plan view.

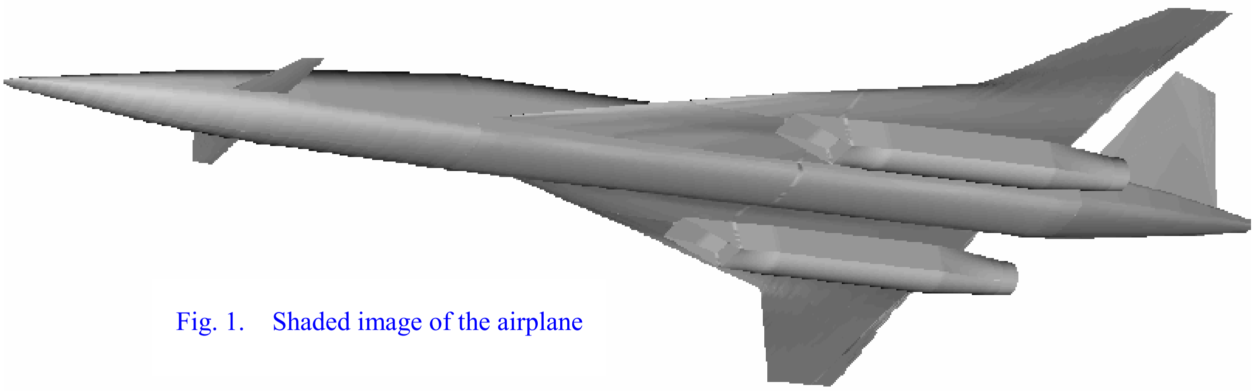

Fig. 1. Shaded image of the airplane

The success of any optimization is crucially dependent on geometrical description of shape. For an ill-conditioned problem with system of two variables linear convergence (at the rate of a geometric progression) proves to be insufficient. In theory, the most simple methods (such as coordinate and gradient descent methods) determines an optimum after infinite number of cycles. In practice, the optimization process is terminated at a considerable distance from the optimum point because of numerical errors. At the same time, information on objective function topography allows to find the optimum after 2-3 cycles of coordinate-wise descent [6]. At first system of variables are selected arbitrarily. After the analysis the variables are replaced.

The most simple representation of a wing is a set of elements making its surface up. Moving these elements one can model the diversity of wing geometry. The partition into elements is introduced to wing projection onto the base plane (fig. 2). The base plane is defined

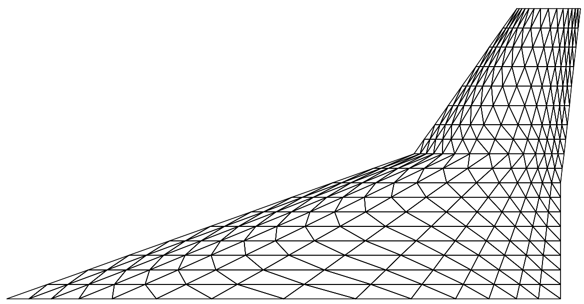

Fig. 2. Wing partition into elements

as plane, intersecting the vertical symmetry plane at right angle along the inboard chord of the wing. 19 longitudinal sections are allocated, each of which is partitioned into 14 segments. The nodal points are condensed to leading and trailing edges and define the apexes of triangular elements forming the upper and lower wing surfaces. Geometrical parameters h_i, $i=1,n$, are stated as displacements of nodal points in the normal direction of the base plane. As a result, each half wing is partitioned into N=1008 elements. The number of parameters is equal n=517. The wing edges are assumed to be sharp. The wing which profiles are formed by circle arcs are taken as a starting one. The relative thickness of wing profiles is 3%.

3 Direct Optimization Method

The direct design method combines direct problem solving within the framework of Euler's model and variational problem solving by Newton's method.

The variational problem is solved in the simplified formulation allowing analytical approximation of objective function and aerodynamic-geometric constraints. For the first time such approach was applied to optimization of an isolated wing [6, 7]. The approximation of objective function was received on the basis of direct dependence of surface pressure on local angle of attack. At research of airplane wing the technique is advanced by means of taking into account computed flow field data.

On the basis of linearized theory it is supposed that a change of a nodal point position influences on gas-dynamic properties on contiguous elements solely. Each nodal point (except for points lying on the wing edges) is surrounded with six elements (fig. 2).

The simplest assessment of pressure variation on an element may be obtained from the wavy-wall theory for small disturbance. The spatial movement of the element requires turn of velocity vector on angle Θ so that it remains parallel to the element plane. The turn of the flow results in pressure variation. For weak waves a relation of pressure with a turn angle is established by a ratio:

$$p_n = p\left(1 - \frac{\gamma \cdot M^2}{\sqrt{M^2 - 1}}\Theta\right),$$

where p, M - pressure and Mach number before turn, p_n - pressure after turn, γ - ratio of specific heats. A similar approach was used in an inverse design method to find wing ensuring given surface pressure distribution [8]. At that, variations at which the angle between velocity vector and surface normal remains right are ignored.

On the other hand, numerical calculation is the most exact tool. A direct variation of a nodal point position gives assessment of pressure variation on all contiguous elements.

In the present research a relation between pressure and geometric variables are established numerically within a simplified formulation, without taking wing thickness and angle of attack into account. This calculation is carried out once before the optimization process and its cost is on the order of one computational analysis of flow around the airplane. In result for each element we have the linear ratio between pressure and the displacements of nodal points (for simplification, indexes 1, 2, 3 are used):

$$p_n = p(1 + k_1h_1 + k_2h_2+ k_3h_3) .$$

Others gas-dynamic properties are established in the following way. Density is determined in the assumption of entropy invariability. The value of velocity (and then Mach number) is calculated from energy equation. Using the approximation for relation $p(h)$ it is easy to get quadratic approximations of objective function and aerodynamic constraints.

The separate remark is required for inequality constraints. Each nodal point on the lower wing surface has correspondent nodal point on the upper surface. On wing edges these points coincide. Distance between the points determines thickness of the wing. Conditions $T_j(h)\leq 0$ are entered for each pair of such points and limit minimum value of thickness. The functions are linear. In space of geometrical variables they set planes that limit area of permissible values. The inequality constraint is ignored at optimization process as long as it is executed strictly. If the optimization path intersected the boundary plane the correspondent inequality constraint would be transformed to equality constraint.

Each equality constraint is equivalent to relation between design variables and results in reduction of number of independent variables on unit. It is tentatively assumed that the last variable is excluded from the analysis. First of all one should exclude the angle of attack. Thus, the problem is reduced to an unconstrained extremum value problem for a function of many variables. The number of independent parameters decreases from n to n'.

The resulting quadratic form of drag approximation may be recorded as:

$$D = D(\bar{h}_0) + \langle D'(\bar{h}_0), \bar{h} - \bar{h}_0 \rangle +$$
$$+ 0.5\langle D''(h_0)(h - h_0), h - h_0 \rangle$$

where h_0 - vector of variables for given wing shape, $D(h_0)$ - value of drag, $D'(h_0)$ - gradient, $D''(h_0)$ – Hessian matrix. A scalar product of vectors is represened as

$$\langle a, b \rangle = \sum_{i=1}^{i=n'} a_i b_i \ .$$

The developed approach allows to determine derivatives of first and second order for aerodynamic functions on the base of data on first order derivatives for gas-dynamic properties. The information on objective function behaviour contained in Hessian matrix provides more fast convergence to the optimum in comparison with a gradient algorithm.

Eigen values of Hessian matrix are varied over a wide range that indicates ill conditionality of the problem. Maximum and minimum eigen values differ in value on 4-5 orders. Hence, lines (or surfaces) of constant level of the objective function are extremely stretched. For great eigen values correspondent eigen vectors define wing shape variations which affect separate surface segments. On the contrary, eigen vectors for small eigen values determine long-wave variations affecting all wing. Because the theoretical analysis was developed under assumption that shape deformations are small it is necessary to divide treatment of short-wave and long-wave variations. For the first, search of the optimum is carried out on the base of data on derivatives of first and second order, for the second - with account for derivatives of first order. Thus, each cycle of optimization process includes descent on two directions. In practice, about 95% of eigen vectors of Hessian matrix determine variations which can be referred to short-wave variations.

The flow of ideal gas about the airplane is computed by a marching method. A gas-dynamic properties jump on a head shock wave is allocated strictly. Inside shock waves and other flow discontinuities are treated without tracking their spatial location. The Euler equations are integrated by using MacCormack explicit finite-difference scheme implemented on multizone grids [9]. The maximum number of mesh-points in cross section reaches 50 000. The surface friction drag is determined by a semiempirical calculation method for a turbulent boundary layer [10].

4 Results

The airplane is performed in tail-first configuration with two air-breathing engines established underneath the wing. The optimization of the wing is carried out for conditions appropriate to the cruise flight - Mach number M_∞=2, altitude H=20 kilometres. The values of lift coefficient C_L=0.145 and longitudinal moment coefficient m_z=0 are chosen from conditions of realization of the maximal lift-to-drag ratio and longitudinal trim. Planform wing area and mean aerodynamic chord are adopted as reference values. The results are shown as comparison of geometrical and aerodynamic dependences for the starting variant of the wing (dash-and-dot curves) and optimum variants received by optimization of the isolated wing (dashed-line curves) and the wing, established on the airplane (solid curves).

Optimization process consisted of three cycles including descent on two directions. The relative reduction of drag achieved as a result of the first, second and third cycles, was 7.8%, 8.4% and 8.5%, respectively. The design process required nearly 50 hours of central processing unit of Pentium-400. Thirty versions of the wing were looked over.

As the optimal wing geometry is approached, the wing volume is redistributed toward the fuselage. The relative thickness c decreases over the wingspan z from 3.6% near the inboard section to 0.2% near the tip edge (fig. 3). For the outboard wing the thickness reaches the extreme permissible from geometric constraints values. The warp φ of the optimal wing changes from small positive values for inboard wing sections up to negative values (about 2-3 degrees) for central sections.

The wing modifications result in more

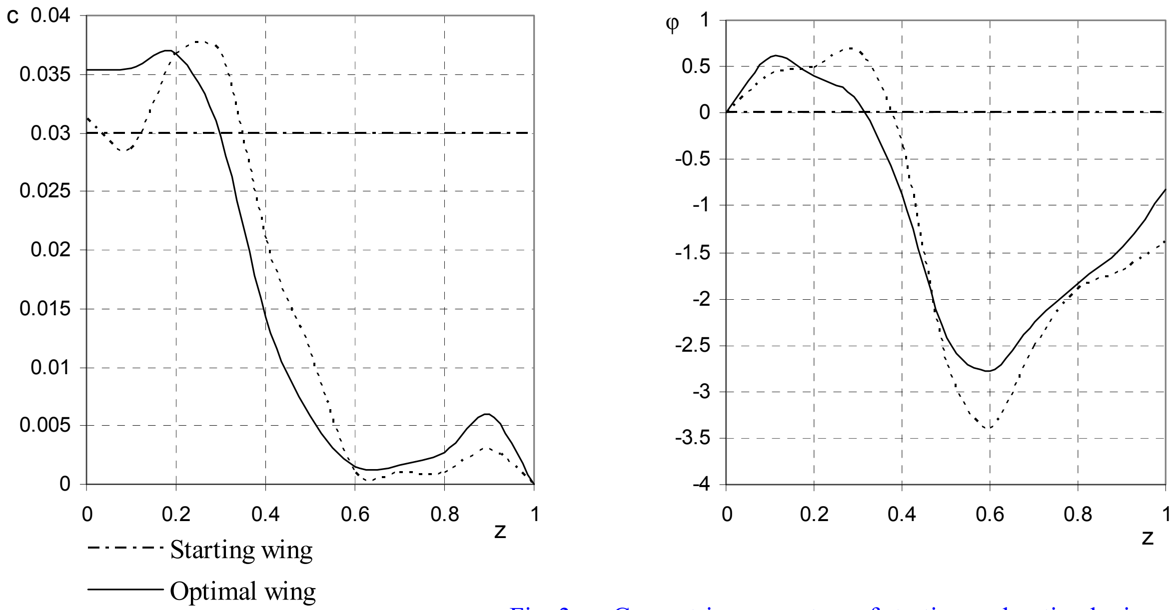

Fig. 3. Geometric parameters of starting and optimal wings

uniform spanwise pressure distribution, load diminishing near the leading edges (fig. 4). Reduction of vortex-type flow regions on the upper surface of the wing is observed. Shock waves pressures on the outboard wing lower surface are significantly reduced.

Figure 5 compares the aerodynamic characteristics. The wing optimization has allowed to increase the maximal lift-to-drag ratio *L/D* of the airplane at 9.3% and to ensure longitudinal trim. For the wing optimized alone the relative increment of lift-to-drag ratio is less on 2%. In the latter case, the longitudinal moment constraint is not implemented.

5 Conclusions

A direct optimization technology has been developed for designing of the wing of a supersonic civil airplane. Drag is used as the objective function minimized under lift, longitudinal trim and volume constraints. The variational problem is solved in a simplified statement allowing analytical formulation of the objective function and aerodynamic-geometric constraints. It gives approximations to the true Hessian matrix and gradient vector, and wing form variations that enable the aerodynamic performance to be improved. A fast convergence to the optimum in case of the large number (more than 500) of the variables is provided.

The efficiency of the method is demonstrated for the airplane performed in tail-first configuration with two air-breathing engines established underneath the wing. For cruise flight at Mach number M=2 a possibility of lift-to-drag ratio increase on 9.3% is shown. The optimization process required nearly 50 hours of central processing unit of Pentium-400. Thirty versions of the wing were looked over.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research, project No. 02-01-00757.

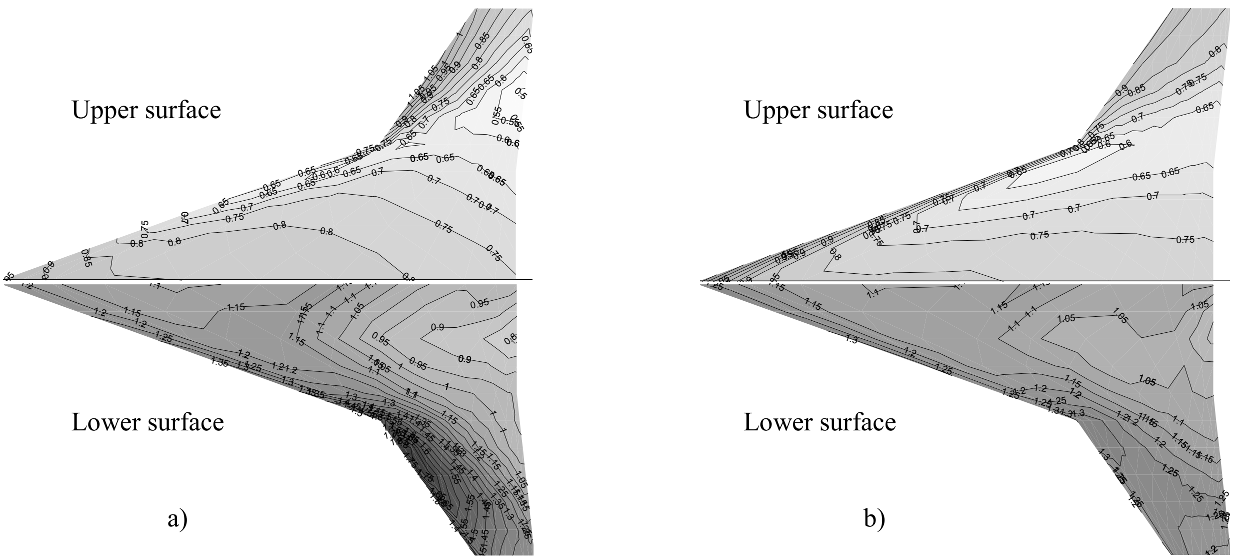

Fig. 4. Surface pressure contours (P/P_∞), M_∞=2 and C_L=0.145:

a) starting wing and b) optimal wing

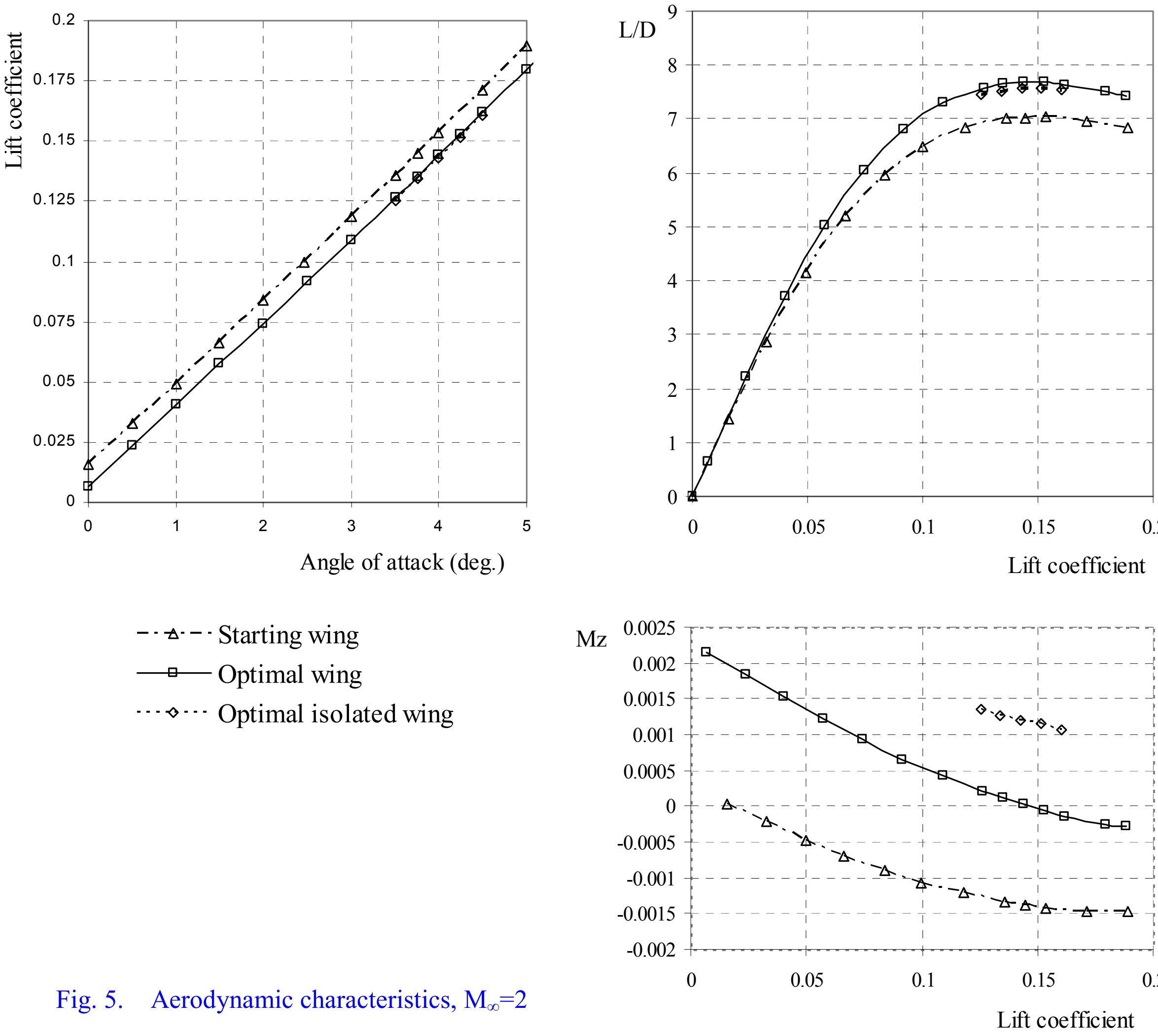

Fig. 5. Aerodynamic characteristics, M_∞=2

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