NUMERICAL STRUCTURAL ANALYSIS OF VISCOUS COMPRESSIBLE FLOW PAST AN AIRFOIL AND A BODY OF REVOLUTION

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Abstract

Application of modern methods of computer analysis enable us to solve problems of viscous compressible flows making use of the Navier-Stokes equations.

The problems to be studied in the present paper are those of a sharp edged airfoil and a body of revolution flown-past by viscous, heat conducting gas, the complete Navier-Stokes equations being used. The case of a boundary layer and an oblique shock wave formed simultaneously in the same region of the flow will be considered, thus enabling us to determine the influence of dissipative properties of the gas on course of those phenomena.

The problem has been solved by the method of decomposition of the set of equations and fractional steps as well as an iterative procedure with respect to time. This set of equations was approximated by an implicit double-layer difference scheme of the Cranck-Nicholson type.

The method of decomposition, which was used, enabled us to devise the method of numerical analysis, based on an implicit difference scheme in the form of an algorithm with alternating directions and separation of physical processes.

1 Introduction

The use of modern methods of computer analysis creates a possibility of finding solutions for viscous flow, making use of the Navier-Stokes equations [1]-[9].

The problems to be investigated in the present paper are those of supersonic viscous flow past a sharp edged airfoil and a body of revolution, which enable us to determine the influence of dissipative properties of the gas on the course of a boundary layer and an oblique shock wave formed simultaneously in the same region.

The method of decomposition, which has been applied for the solution, enabled us to devise the method of numerical analysis making use of an implicit difference scheme in the form of an algorithm with alternating directions and separation of physical processes.

Proceeding in this manner the complex problem has been reduced to a sequence of one-dimensional equations, thus simplifying considerably the computation procedure.

The results of calculation enable us to study the structure of the flow within the region of interaction between the boundary layer and the shock wave as well as in the region where those structures occur separately. They enable us also to determine the influence of Mach and Reynolds numbers on the structure of flow field.

2. Equations of the Problem

Let us consider plane and axisymmetric supersonic flows past a sharp edged airfoil and a body of revolution. It is assumed that the stream of gas is homogeneous and parallel to the symmetry axes of the flown-past bodies.

The gas is treated as a viscous and heat conducting medium, the coefficient of dynamic viscosity μ and that of heat conduction κ being known functions of the temperature T. The gas is assumed to be perfect in the thermodynamic sense, the Prandtl number Pr and the Poisson adiabatic exponent k being constant.

The equations of the problem can be presented in the divergent vector form [1]-[3]

$$\frac{\partial\rho}{\partial t}+div(\rho\mathbf{V})=0 \tag{1}$$

$$\frac{\partial(\rho V)}{\partial t}+div(\rho\mathbf{V}\cdot\mathbf{V}-\mathbf{P})=0 \tag{2}$$

$$\frac{\partial(\rho E)}{\partial t}+div(\rho E\mathbf{V}-\mathbf{V}\cdot\mathbf{P}-\kappa gradT)=0 \tag{3}$$

where Eq. (1) represents mass conservation, Eq. (2) – momentum conservation and Eq. (3) – total energy conservation.

V is velocity vector, ρ – gas density,

$$\mathbf{P}=\sigma_{ij}-p\,\delta_{ij} \tag{4}$$

is total stress tensor, σ_{ij} - viscous stress tensor, p – pressure and δ_{ij} is unit tensor.

$$E=\frac{1}{2}\mathbf{V}^2+e \tag{5}$$

is total energy of gas and

$$e=c_vT \tag{6}$$

is internal energy.

Equations (1)-(3) can be also presented in the divergent vector form as

$$\frac{\partial\mathbf{U}}{\partial t}+div\mathbf{W}=0 \tag{7}$$

Where the vector

$$\mathbf{U}=(\rho,\rho\mathbf{V},\rho E)^T \tag{8}$$

and the vector

$$\mathbf{W}=(\mathbf{W_1},\mathbf{W_2},\mathbf{W_3})^T \tag{9}$$

has the components

$$\mathbf{W}_1=\rho\mathbf{V}$$

$$\mathbf{W}_2=\rho\mathbf{V}\cdot\mathbf{V}-\mathbf{P} \tag{10}$$

$$\mathbf{W}_3=\rho E\mathbf{V}-\mathbf{P}\cdot\mathbf{V}-\kappa gradT$$

Equation (7) together with (8)-(10) is a starting point for the analysis of plane and axisymmmetric viscous and heat conducting supersonic flow past an airfoil and a body of revolution, where the closing equations are:

$$p=R\rho T\ ;\ R=c_p-c_v$$

$$\mu=\left(\frac{T}{T_\infty}\right)^\omega\mu_\infty\ ;\ \omega=\text{const} \tag{11}$$

$$Pr=\frac{\mu c_p}{\kappa}=const$$

3. Formulation of the Plane Problem of Viscous Supersonic Flow past an Airfoil.

Let us consider plane supersonic flow of viscous heat conducting gas past an sharp edged airfoil of length L (Fig.1).

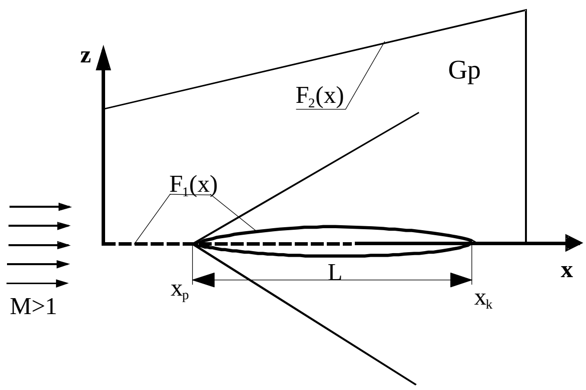

Fig. 1

The problem can be expressed in the rectangular coordinates x,z in the form of the conservation law

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial z}=0 \tag{12}$$

where **U** is the vector of conserved quantities

$$\mathbf{U}=\left(\rho,\rho v_x,\rho v_z,\rho E\right)^T \tag{13}$$

and **F**, **G** are flux vectors

$$\mathbf{F}=\left(F_1,F_2,F_3,F_4\right)^T \tag{14}$$

$$\mathbf{G}=\left(G_1,G_2,G_3,G_4\right)^T \tag{15}$$

where

$$\begin{gathered}F_1=\rho v_x\\ F_2=\rho v_x^2+p-\sigma_{xx}\\ F_3=\rho v_x v_z-\sigma_{xz}\\ F_4=v_x\left(\rho E+p-\sigma_{xx}\right)-v_z\sigma_{xz}-\kappa\frac{\partial T}{\partial x}\end{gathered} \tag{16}$$

and

$$\begin{gathered}G_1=\rho v_z\\ G_2=\rho v_x v_z-\sigma_{xz}\\ G_3=\rho v_z^2+p-\sigma_{zz}\\ G_4=v_z\left(\rho E+p-\sigma_{zz}\right)-v_x\sigma_{xz}-\kappa\frac{\partial T}{\partial z}\end{gathered} \tag{17}$$

where

$$E=\frac{1}{2}\left(v_x^2+v_z^2\right)+e \tag{18}$$

components of viscous stress tensor are

$$\begin{gathered}\sigma_{xx}=2\mu\frac{\partial v_x}{\partial x}-\frac{2}{3}\mu div\mathbf{V}\\ \sigma_{xz}=\mu\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)\\ \sigma_{zz}=2\mu\frac{\partial v_z}{\partial z}-\frac{2}{3}\mu div\mathbf{V}\end{gathered} \tag{19}$$

The boundary conditions of the problem can be assumed to be at the surface of the body

$$v_x=v_z=0;\quad \frac{\partial T}{\partial n}=0 \tag{20}$$

where n indicates the direction normal to the surface of the body and

$$\begin{gathered}v_x=v_\infty;\quad v_z=0;\\ T=T_\infty;\quad \rho=\rho_\infty\end{gathered} \tag{21}$$

in the region undisturbed by the flown past body.

Equation (12) together with (13)-(19) and boundary conditions (20)-(21) constitute a complete formulation of the plane problem of viscous supersonic flow past an airfoil.

4. Formulation of the Axisymmetric Problem of Viscous Supersonic Flow past a Body of Revolution

Let us consider axisymmetric supersonic flow of viscous, heat conducting gas past a sharp edged body of revolution (Fig. 2).

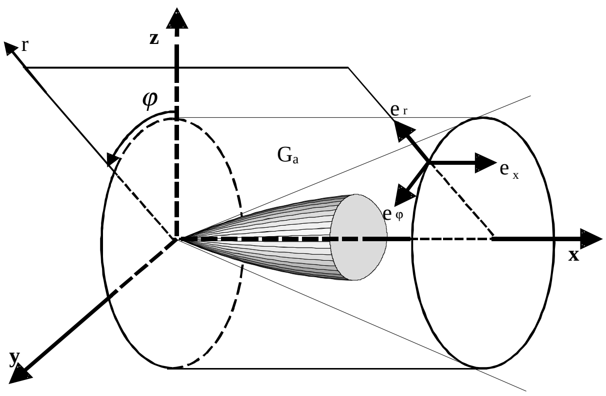

Fig. 2

The problem can be formulated in the polar coordinates system (r,φ,x) under assumption that the flow parameters do not depend on coordinate φ. The velocity vector

$$\mathbf{V}=\left(v_x,v_r\right)^T \tag{22}$$

and the equation of flow

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial r}=\mathbf{H} \tag{23}$$

where the vector of conserved quantities

$$\mathbf{U}=\left(r\rho,r\rho v_x,r\rho v_r,r\rho E\right)^T \tag{24}$$

and flux vectors

$$\begin{aligned}\mathbf{F} &= (F_1, F_2, F_3, F_4)^T \\ \mathbf{G} &= (G_1, G_2, G_3, G_4)^T \\ \mathbf{H} &= (H_1, H_2, H_3, H_4)^T\end{aligned} \tag{25}$$

where

$$\begin{aligned}F_1 &= r\rho v_x \\ F_2 &= r\rho v_x^2 + rp - r\sigma_{xx} \\ F_3 &= r\rho v_x v_r - r\sigma_{xr} \\ F_4 &= r v_x(\rho E + p - \sigma_{xx}) - r v_r \sigma_{xr} - r\kappa \frac{\partial T}{\partial x}\end{aligned} \tag{26}$$

$$\begin{aligned}G_1 &= r\rho v_r \\ G_2 &= r\rho v_x v_r - r\sigma_{rx} \\ G_3 &= r\rho v_r^2 + rp - r\sigma_{rr} \\ G_4 &= r v_r(\rho E + p - \sigma_{rr}) - r v_x \sigma_{xr} - r\kappa \frac{\partial T}{\partial r}\end{aligned} \tag{27}$$

$$\begin{aligned}H_1 &= H_2 = 0 \\ H_3 &= p - \sigma_{\varphi\varphi} \\ H_4 &= 0\end{aligned} \tag{28}$$

where

$$E = \frac{1}{2}(v_x^2 + v_r^2) + e \tag{29}$$

components of viscous stress tensor are

$$\begin{aligned}\sigma_{xx} &= 2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu div\mathbf{V} \\ \sigma_{xr} &= \mu\left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x}\right) \\ \sigma_{rr} &= 2\mu \frac{\partial v_r}{\partial r} - \frac{2}{3}\mu div\mathbf{V} \\ \sigma_{\varphi\varphi} &= 2\mu \frac{v_r}{r} - \frac{2}{3}\mu div\mathbf{V}\end{aligned} \tag{30}$$

The boundary conditions of the problem can be assumed to be

$$\frac{\partial v_x}{\partial r} = \frac{\partial T}{\partial r} = v_r = 0 \tag{31}$$

at the symmetry axis of the flow, for r=0,

$$v_x = v_r = 0; \ \frac{\partial T}{\partial n} = 0 \tag{32}$$

on the surface of the body and

$$\begin{aligned}v_x &= v_\infty; \ \ v_r = 0 \\ T &= T_\infty; \ \ \rho = \rho_\infty\end{aligned} \tag{33}$$

in the region undisturbed by the flown past body.

At the rear boundary of the computation region, we assume approximate boundary conditions which are necessary to close the boundary-value problem. It is taken, for instance, that the gradients in the flow direction can be set to zero

$$\frac{\partial v_x}{\partial s} = \frac{\partial v_r}{\partial s} = \frac{\partial T}{\partial s} = 0 \tag{34}$$

5. Method of Solution

The problem of supersonic viscous flow is treated as an initial-boundary-value problem and the steady-state field of flow will be determined by an iteration process as a limit of unsteady fields.

The initial conditions can be, for t=0, in the case of the flow past an airfoil

$$\begin{aligned}\rho(x,z,t) &= \rho_0(x,z) \\ v_x(x,z,t) &= v_{x0}(x,z) \\ v_z(x,z,t) &= v_{z0}(x,z) \\ T(x,z,t) &= T_0(x,z)\end{aligned} \tag{35}$$

and in the case of the axisymmetrical flow past a body of revolution

$$\begin{aligned}\rho(x,r,t) &= \rho_0(x,r) \\ v_x(x,r,t) &= v_{x0}(x,r) \\ v_r(x,r,t) &= v_{r0}(x,r) \\ T(x,r,t) &= T_0(x,r)\end{aligned} \tag{36}$$

Where the functions on right-hand side of the initial conditions must satisfy the conditions on the surface of the flown past bodies.

They may represent the parameters of undisturbed field of flow or a field of flow determined in the course of the iteration process.

The equations of the problem will be considered in a dimensionless form. In this connection the following dimensionless quantities are introduced for the plane flow

$$\bar{x}=\frac{x}{L};\quad \bar{z}=\frac{z}{L};\quad \bar{t}=\frac{t\cdot v_{x\infty}}{L};$$
$$\bar{v}_x=\frac{v_x}{v_{x\infty}};\ \bar{v}_z=\frac{v_z}{v_{x\infty}} \qquad (37)$$

and for the axisymmetrical problem

$$\bar{x}=\frac{x}{L};\ \bar{r}=\frac{r}{L};\ \bar{t}=\frac{t\cdot v_{x\infty}}{L};$$
$$\bar{v}_x=\frac{v_x}{v_{x\infty}};\ \bar{v}_r=\frac{v_r}{v_{x\infty}} \qquad (38)$$

The remaining dimensionless quantities which occur in both problems are

$$\bar{\rho}=\frac{\rho}{\rho_\infty};\ \bar{T}=\frac{T\cdot c_p}{v_{x\infty}^2};\quad \bar{p}=\frac{p}{\rho_\infty\cdot v_{x\infty}^2};$$
$$\bar{\mu}=\frac{\mu}{\mu_\infty};\quad \bar{\kappa}=\frac{\kappa}{\kappa_\infty} \qquad (39)$$

Where the quantities with the index ∞ refer to the undisturbed flow.

After these transformations we obtain also constant values

$$\mathrm{Re}=\frac{\rho_\infty v_{x\infty} L}{\mu_\infty};\ \ \mathrm{Pr}=\frac{c_p\mu_\infty}{\kappa_\infty}=0.71 \qquad (40)$$

and the equation of state

$$p=\frac{k-1}{k}\rho T;\quad k=\frac{c_p}{c_v} \qquad (41)$$

The computation region G_p for the plane flow and G_a for the axisymmetrical flow are assumed on the physical plane (Fig. 1) and in space (Fig. 2). They will be transformed on the auxiliary plane of variables q_1, q_2 into the square of unit side length.

This transformation enables us also to condense difference meshes in the physical plane or space within the region of the sharp edges and the boundary layer. The mesh remains uniform in the computation plane q_1,q_2. Numerical solution to the set of equations of the problem can be found by the method of decomposition of the equations and fractional steps as well as an iterative procedure with respect to time. This set of equations was approximated by an implicit double-layer difference scheme of the Cranck-Nicholson type [8], [9].

6. Results of Analysis

Making use of the equations and method of solution presented above numerical analysis of viscous supersonic flow past an airfoil and body of revolution has been performed. Some results of analysis will be presented in next figures.

In Figure 3 we can see the density of gas ρ variations in flows past a sharp edged airfoil, of g=10% thickness, for Reynolds number Re=20000, Pr=0.71 and small supersonic Mach numbers M=1.15; 1.25; 1.3; 1.5.

In the front part of the airfoil we see the developing shock wave over a thin boundary layer and in the rear part of the airfoil expansion waves can be seen.

For M=1.15 (Fig.3a) the headwave is rather a weak shock wave which is easily extinguishing shock, and for greater values of Mach number M =1.25; 1.3; 1.5; (Figs. 3b,c, d) the front shock wave becomes an oblique shock wave which expands over the airfoil.

For greater Mach numbers all waves are thinner and more leaning over the airfoil.

In Fig. 4 the courses of gas density ρ for various positions along the airfoil are presented and in Fig. 5 similar courses for the velocity $w = \sqrt{v_x^2 + v_z^2}$ are shown.

In these figures we can see the run of gas density ρ and velocity w in the boundary layer, in the region between the boundary layer and the shock wave and across the shock wave.

For x-1.2 L; 1.4 L there can be also seen the rear shock beyond the airfoil.

In the Fig. 6 the courses of velocity w in the boundary layer and in the wake beyond the airfoil one presented for various positions along the airfoil and in Fig. 7 the similar courses of the temperature ratio T/T_∞ are shown.

In the next Figs. 8 and 9 the runs of the temperature ratio T/T_∞ and the pressure p/p_∞ in the whole region of flow for various position along the airfoil are determined.

We can see the courses between the boundary layer and the shock wave as well as across the shock wave.

The viscous supersonic flow past a body of revolution will be considered by way of example of a circular cone (Fig. 2) with apex angle $=40^0$, flown along its axis with Mach number M=3, Re=20000. Essential results of numerical analysis of supersonic viscous flow past the circular cone are presented in Figs.10 to 13.

In Fig. 10 we can see the courses of the gas density ρ for various cross-sections of the cone. In this figure we can find the runs of ρ in the boundary layer, in the region between the boundary layer and the shock wave and across the shock wave.

In Fig. 12 the courses of velocity $w = \sqrt{v_x^2 + v_r^2}$ are presented for the same cone cross-sections, and in Fig. 11 the runs of the mach number M., while in Fig. 13 the ratio of pressures p/p_∞ can be found.

7. Conclusions

The method of solution and the results of numerical analysis of viscous supersonic flow past an airfoil and a body of revolution are presented in this paper.

Making use of the complete Navier-Stokes equations, the solution is obtained by the method of decomposition of the set of equations, fractional steps and an iterative procedure with respect to time.

The set of equations is approximated by implicit double-layer difference scheme of Cranck-Nicholson type.

Results of numerical analysis are presented in a series of figures. In these figures we can see the courses of gas density, velocity, pressure and temperature variations in the region of boundary layer, between the boundary layer and shock wave and across the shock wave. These results enable us to determine the influence of dissipative properties of gas on the course of phenomena under investigation.

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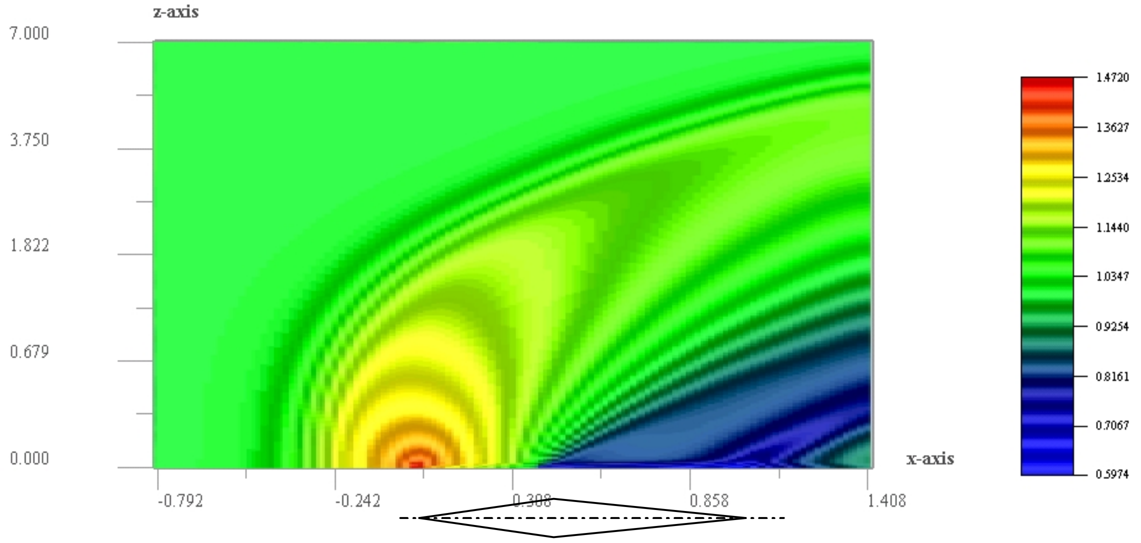

Fig 3.a M=1.15 (Re=20 000; g=10%; x_g=0.4)

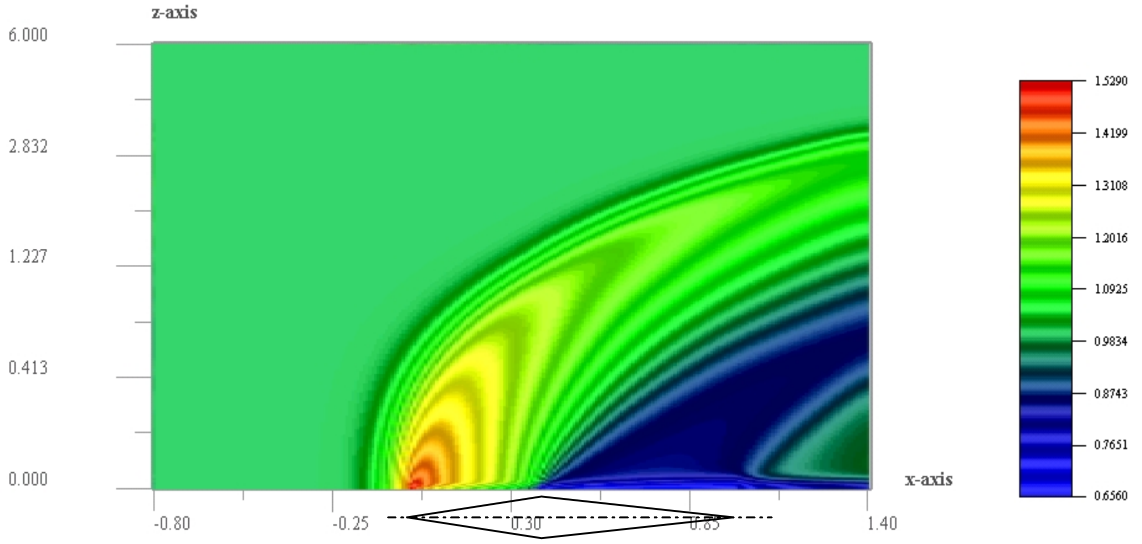

Fig 3.b M=1.25 (Re=20 000; g=10%; x_g=0.4)

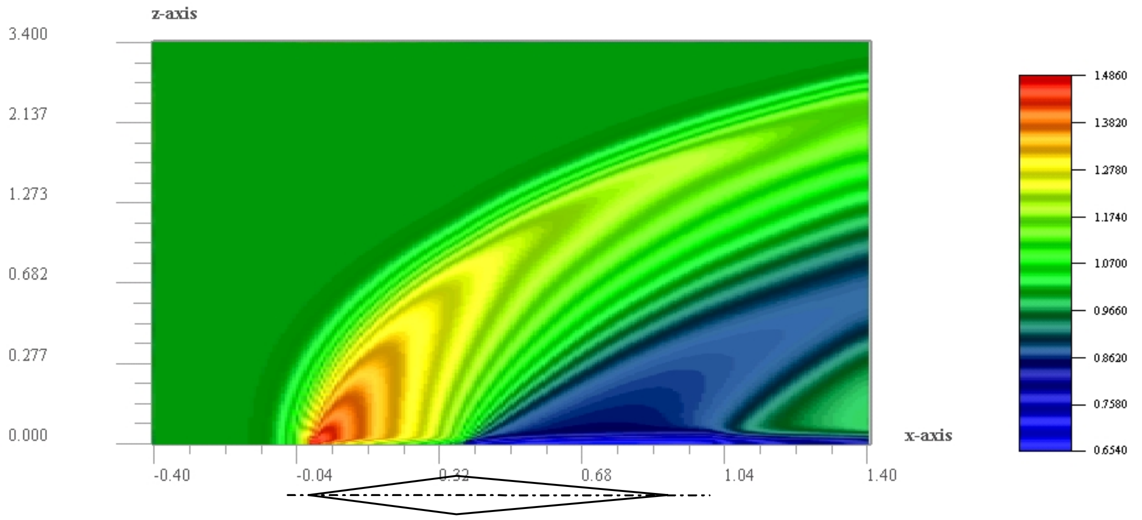

Fig 3.c M=1.25 (Re=20 000; g=10%; x_g=0.4)

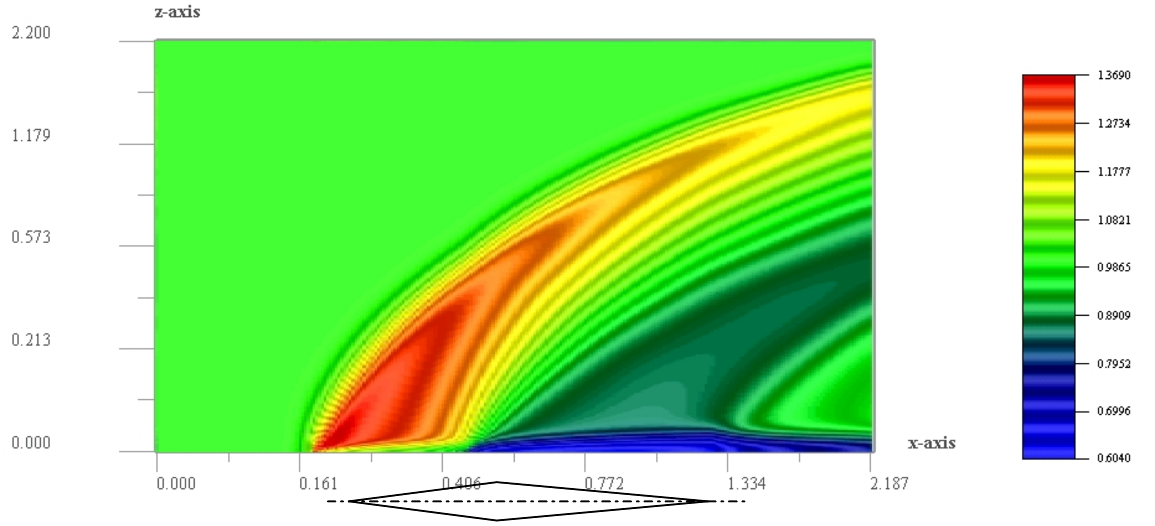

Fig 3.d M=1.5 (Re=20 000;g=10%;x_g=04)

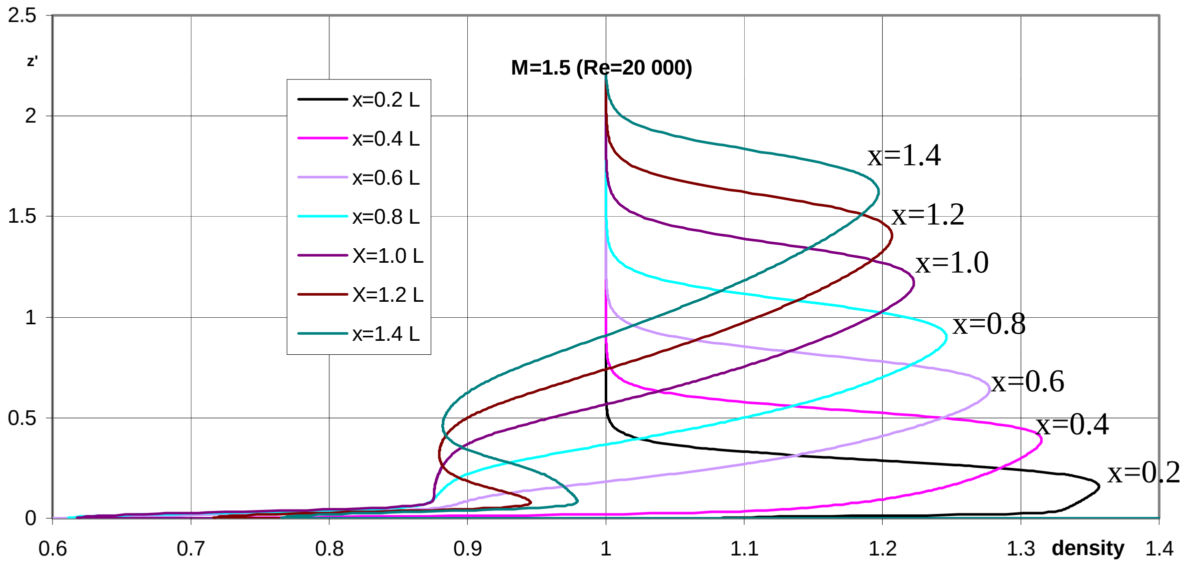

Fig. 4

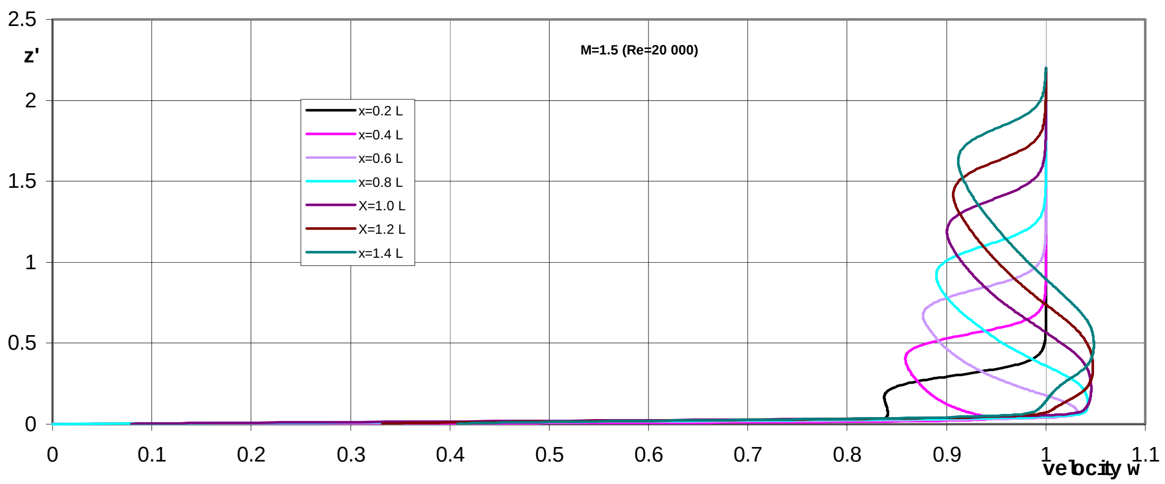

Fig. 5

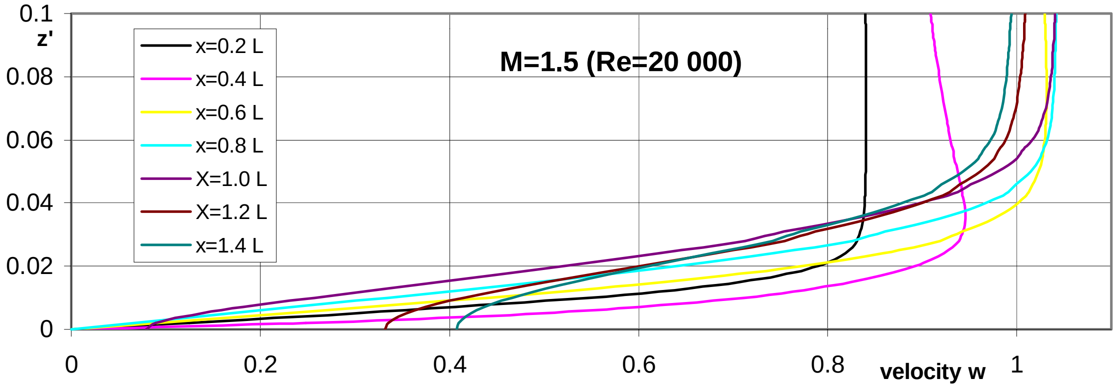

Fig. 6

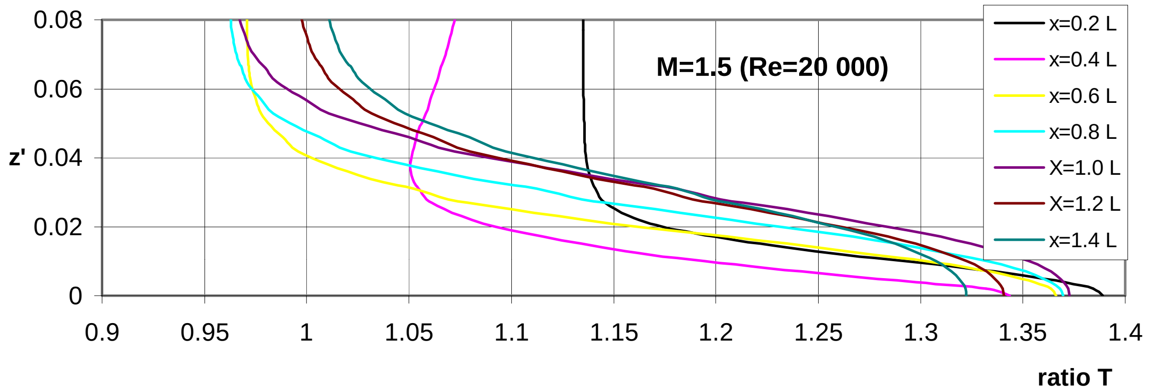

Fig. 7

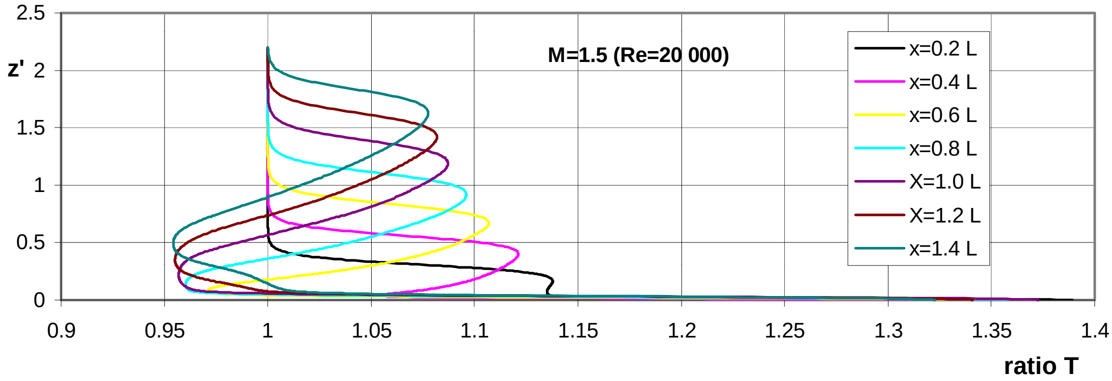

Fig. 8

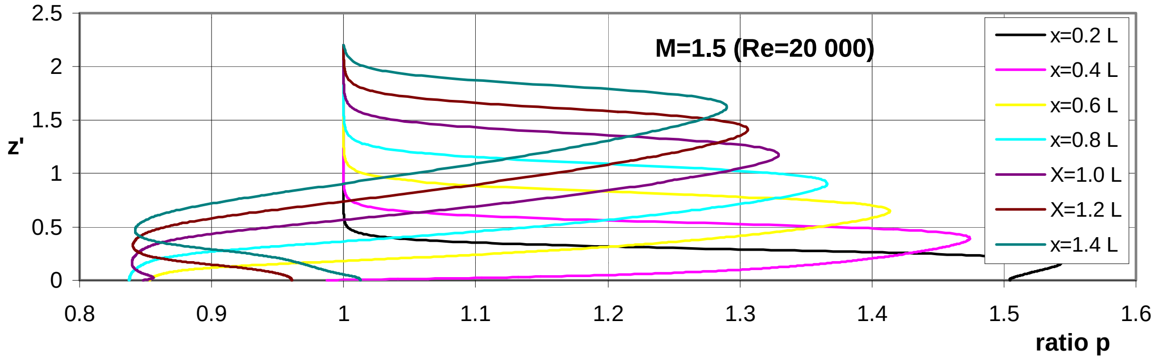

Fig.9

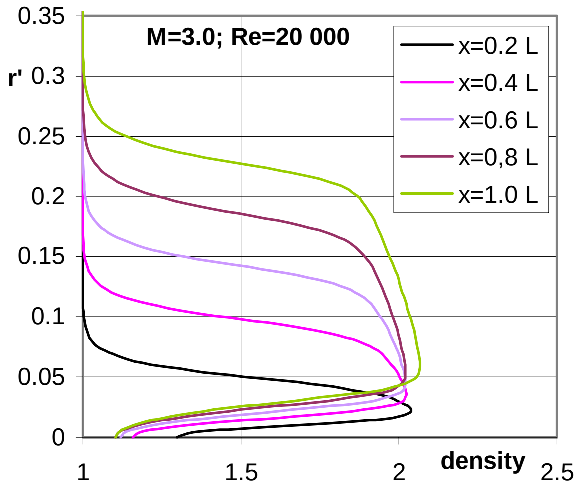

Fig. 10

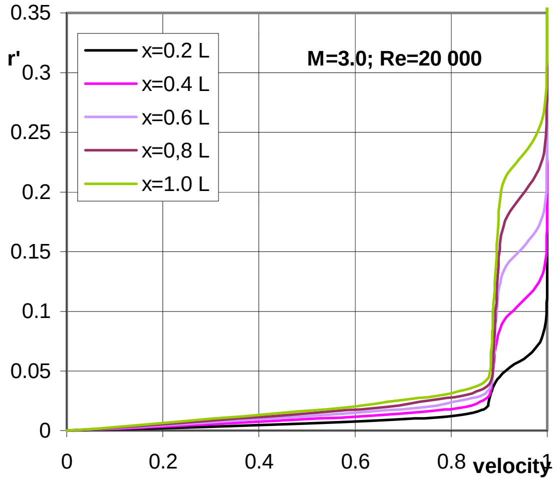

Fig. 12

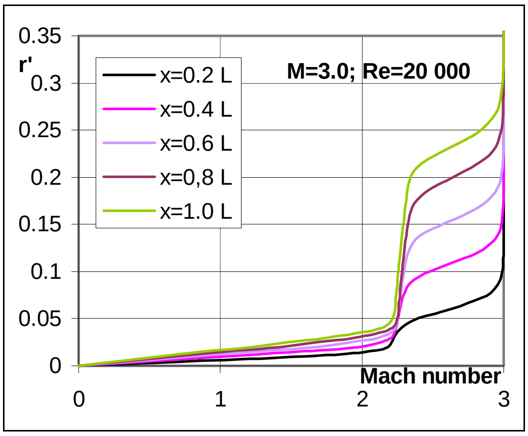

Fig.11

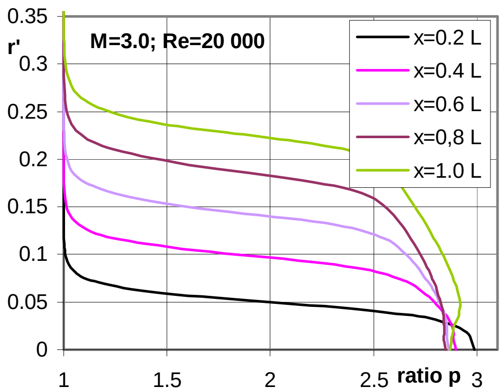

Fig. 13