

# AN INTEGRATED OPTIMIZATION FOR CONCEPTUAL DESIGNS OF SPACEPLANE

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## Abstract

*This paper presents a new parallel optimization method to solve large-scale design and control optimization problems and its applications to conceptual design of spaceplane. Generally, it takes much computing load and time to solve a large-scale optimization problem. Therefore, the new method divides the problem into several sub-problems which can be optimized in parallel. Firstly, this paper describes the way to decompose the problem and the fundamental algorithm to solve it. The important point of this study is how to deal with the conjunctive constraints among the sub-problems and define an objective function in each sub-problem. Second, a numerical example is solved to show effectiveness of the parallel optimization method. Finally, in this paper, the parallel optimization method is applied to a shape and flight trajectory optimization problem of future space transportation vehicle, i.e. spaceplane. Spaceplane is said to be impossible to develop by present technologies. For this reason, most realistic shapes and flight trajectories of spaceplane are found. The effectiveness of the proposed parallel optimization method is also demonstrated.*

## 1 Introduction

In an aircraft design process, both design and control have to be simultaneously optimized to obtain extremely high performance. Simultaneous design and control optimization, however, is difficult, because the both are in different technical fields and a large number of variables have to be analyzed and optimized. For a conventional aircraft, a configuration and geometrical parameters of the aircraft are

optimized for assumed flight patterns, and, after the aircraft has been designed, flight paths or trajectories are optimized precisely. However, an advanced aircraft, e.g. spaceplane, is required to have severe missions and simultaneous optimization is an important problem. For the large-scale design and control optimization problem, there is an idea that one problem is divided into some small-scale sub-problems, which can be optimized in parallel.

Parallel optimization methods have been studied to solve practical problems for structure and shape designs. The reason is that the number of variables for practical design analysis is huge and cost of computers which can deal with the variables is very high. In addition, aircraft designs are complicated so that many technical fields, for example, fluid, engine, structure, control and so on, are in a jumble. Therefore multidisciplinary design has been fundamentally put into practice, by which these different fields are adjusted and each field is analyzed and optimized independently for itself.

Many studies for large-scale mathematical programming problems begun with Dantzig-Wolfe decomposition algorithm [1] for linear programming reported in 1960. For nonlinear programming problem, two methods called model coordination method and goal coordination method which are reviewed by Kirsch [2] has been well accepted. In addition, a hybrid method [3] combining these two methods has been studied in recent years. However, these decomposition algorithms have not been generally applied. Recently, Sobieski et al. in NASA Langley Research Center has proposed more practical multidisciplinary optimization methods. According to the new method called Collaborative Optimization [4],

optimized variables are shared with variables depending on only each sub-system (field or discipline) and global variables depending on more than two sub-systems. An optimization problem for global variables is defined on upper level than sub-problems optimizing sub-systems, and the global level optimization and sub-system optimization are repeated alternately. In each sub-system optimization, the global variables are optimized to coincide with the values given by the global level optimization. Consequently, after some repetition, the collaborative points of the global variables satisfying all sub-problems are obtained. Sobieski and his fellows strive to apply the proposed method to aircraft designs [4]. While it is more practical than the conventional ones and can refine the nominal design certainly, its convergence characteristic is worse.

Based on these past achievements, first, this study aims to develop the decomposition algorithm which can be used for practical large-scale design problem. The method is called parallel optimization method in this paper. Second, to examine the effectiveness of the proposed numerical method as a parallel method, a simple example is prepared. Finally, as a practical large-scale optimization problem, the body and wing shape and ascent trajectory optimization problems for a future space transportation vehicle, spaceplane, is introduced. Through the problem, conditions to realize spaceplane are demonstrated and validity of the method is confirmed.

## 2 Parallel Optimization Method

In this section, let us one optimized large-scale system composed of three subsystems. The following description covers optimization of the systems with more subsystems. First, an optimized variable is grouped into three parts.

$$\mathbf{x} \equiv (\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_3^T)^T \quad (1)$$

Equality and inequality constraint conditions providing distinctive characteristics of subsystems are also divided.

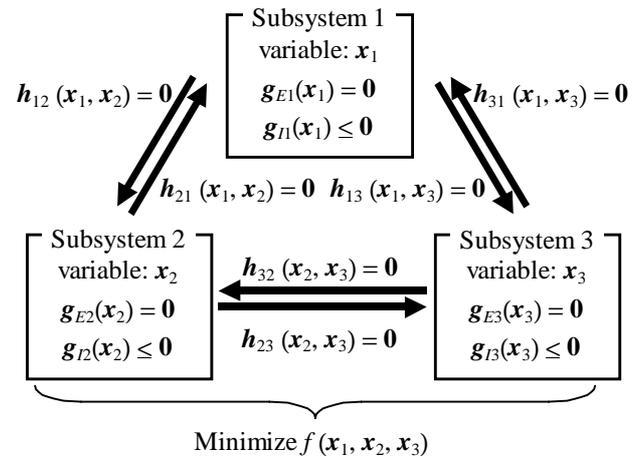


Fig. 1 Three subsystems and conjunctions

$$\begin{aligned} \mathbf{g}_{E_i}(\mathbf{x}_i) &= \mathbf{0} & (2a) \\ \mathbf{g}_{I_i}(\mathbf{x}_i) &\leq \mathbf{0} & (i = 1, 2, 3) \\ & & (2b) \end{aligned}$$

A single objective function  $f$  is defined, and all functions are assembled into

$$\text{minimize } f(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \quad (3a)$$

$$\text{subject to } \mathbf{g}_E(\mathbf{x}) \equiv \begin{bmatrix} \mathbf{g}_{E1}(\mathbf{x}_1) \\ \mathbf{g}_{E2}(\mathbf{x}_2) \\ \mathbf{g}_{E3}(\mathbf{x}_3) \end{bmatrix} = \mathbf{0} \quad (3b)$$

$$\mathbf{g}_I(\mathbf{x}) \equiv \begin{bmatrix} \mathbf{g}_{I1}(\mathbf{x}_1) \\ \mathbf{g}_{I2}(\mathbf{x}_2) \\ \mathbf{g}_{I3}(\mathbf{x}_3) \end{bmatrix} \leq \mathbf{0} \quad (3c)$$

$$\mathbf{h}(\mathbf{x}) \equiv \begin{bmatrix} \mathbf{h}_{12}(\mathbf{x}_1, \mathbf{x}_2) \\ \mathbf{h}_{21}(\mathbf{x}_1, \mathbf{x}_2) \\ \mathbf{h}_{13}(\mathbf{x}_1, \mathbf{x}_3) \\ \mathbf{h}_{31}(\mathbf{x}_1, \mathbf{x}_3) \\ \mathbf{h}_{23}(\mathbf{x}_2, \mathbf{x}_3) \\ \mathbf{h}_{32}(\mathbf{x}_2, \mathbf{x}_3) \end{bmatrix} = \mathbf{0} \quad (3d)$$

where  $\mathbf{h}_{ij}$  and  $\mathbf{h}_{ji} = \mathbf{0}$  are respectively called a conjunctive function and a conjunctive condition which connects two subsystems  $i$  and  $j$ . The distinction of two conjunctive functions  $\mathbf{h}_{ij}$  and  $\mathbf{h}_{ji}$  with reverse subscript numbers is clarified later.

Let us arrange an optimization problem in subsystem  $i$ .

$$\text{variable : } \mathbf{x}_i \quad (4a)$$

$$\text{minimize } f(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \quad (4b)$$

$$\text{subject to } \mathbf{g}_{E_i}(\mathbf{x}_i) = \mathbf{0} \quad (4c)$$

$$\mathbf{g}_{I_i}(\mathbf{x}_i) \leq \mathbf{0} \quad (4d)$$

$$\mathbf{h}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{0} \quad (4e)$$

$$\mathbf{h}_{ji}(\mathbf{x}_j, \mathbf{x}_i) = \mathbf{0} \quad (4f)$$

$$(j = 1, 2, 3 \text{ and } i \neq j)$$

It is impossible to solve the sub-problem  $i$  with regard to  $\mathbf{x}_i$  independent of other sub-problems, because the conjunctive function  $\mathbf{h}_{ji}$  is a function of not only  $\mathbf{x}_i$  but  $\mathbf{x}_j$ . Therefore, the crucial point of the study is how to deal with the conjunctive conditions and how to define the sub-problems in order to attain highly independent level and to get superior and steady convergent characteristics. In addition, though the original large-scale optimization problem certainly has only one objective function, the divided optimization sub-problems often have no objective, because the objective function  $f$  doesn't necessarily contain three variables,  $\mathbf{x}_1$ ,  $\mathbf{x}_2$  and  $\mathbf{x}_3$ .

From these points, the following defines a sub-optimization problem of the subsystem  $i$

$$\text{variable : } \mathbf{x}_i \quad (5a)$$

$$\text{minimize } f(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) + \sum_{j=1(i \neq j)}^3 \mathbf{v}_{ij}^T \mathbf{h}_{ij}(\mathbf{x}_i, \mathbf{x}_j) \quad (5b)$$

$$\text{subject to } \mathbf{g}_{E_i}(\mathbf{x}_i) = \mathbf{0} \quad (5c)$$

$$\mathbf{g}_{I_i}(\mathbf{x}_i) \leq \mathbf{0} \quad (5d)$$

$$\mathbf{h}_{ji}(\mathbf{x}_j, \mathbf{x}_i) = \mathbf{0} \quad (5e)$$

$$(j = 1, 2, 3 \text{ and } i \neq j)$$

where  $\mathbf{v}_{ij}$  is a Lagrange multiplier for the conjunctive condition  $\mathbf{h}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{0}$ . Note that the computed variables in the sub-problem  $i$  are  $\mathbf{x}_i$  and  $\mathbf{v}_{ji}$ , and the others are dealt with as constants. One of characteristics of the proposed definition is to use the Lagrange multiplier to add the conjunctive functions to the objective function. The conjunctive conditions are

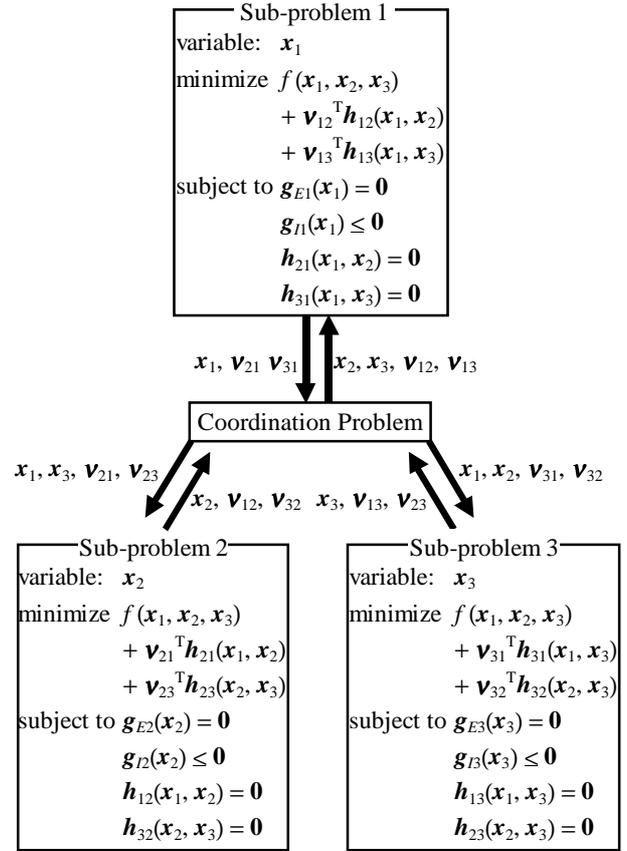


Fig. 2 Three sub-problems and data exchanges

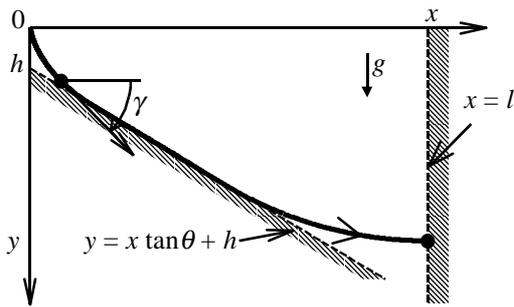
properly allocated to sub-systems, and the sub-problems surely have the objective.

A Fundamental algorithm to solve the sub-problems is summarized as follows:

- (1) Determine proper initial solutions of all the variables  $\mathbf{x}_i$  and Lagrange multipliers  $\mathbf{v}_{ij}$ .
- (2) Solve all sub-problems in parallel with optimization methods by which not only the variables but also the Lagrange multipliers can be computed, e.g. a sequential quadratic programming (SQP) method [5].
- (3) Exchange the obtained variables and multipliers among the sub-problems in a coordination problem as shown in Fig. 2. At this time, to improve convergent characteristic, before the exchanges, these values are modified. Next, return to (2).

### 3 Simple Example

In this section, a numerical example is solved to show effectiveness of the parallel optimization method.



**Fig. 3 Brachistochrone problem**

A brachistochrone problem with an inequality constraint is considered. A particle moves in a constant gravity field. The motion equations are

$$\dot{x} = \sqrt{2gy} \cos \gamma \quad (6a)$$

$$\dot{y} = \sqrt{2gy} \sin \gamma \quad (6b)$$

where  $x$  is horizontal distance,  $y$  is vertical distance (positive downward),  $g$  is the acceleration due to gravity, and  $\gamma$  is path angle to the horizontal (see Fig. 3). The particle starts moving from

$$x(0) = y(0) = 0 \quad (7)$$

and reach

$$x(t_f) = l \quad (8)$$

at the final time  $t_f$ , with the path constraint

$$y \leq x \tan \theta + h \quad (9)$$

where  $l$ ,  $\theta$  and  $h$  are constant, and find  $\gamma(t)$  to minimize the final time. Thus, the objective function is formulated as

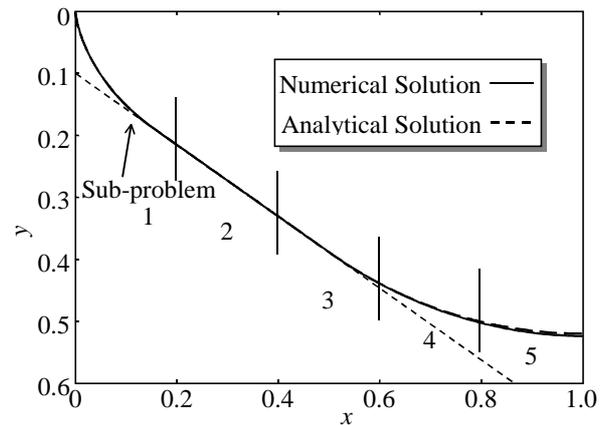
$$J = t_f \quad (10)$$

The following values are given to the parameters.

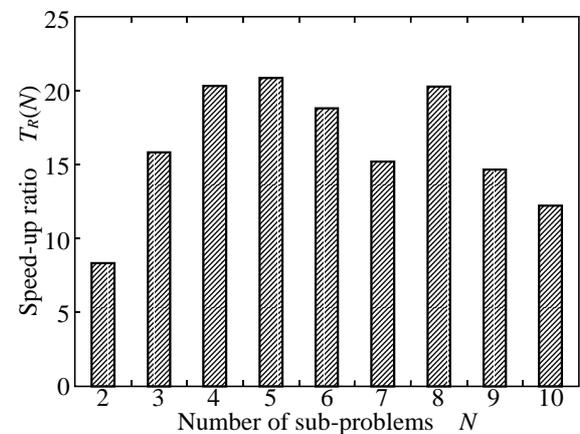
$$g = l = 1 \quad (11a)$$

$$\theta = 30 [\text{deg}], h = 0.1 \quad (11b)$$

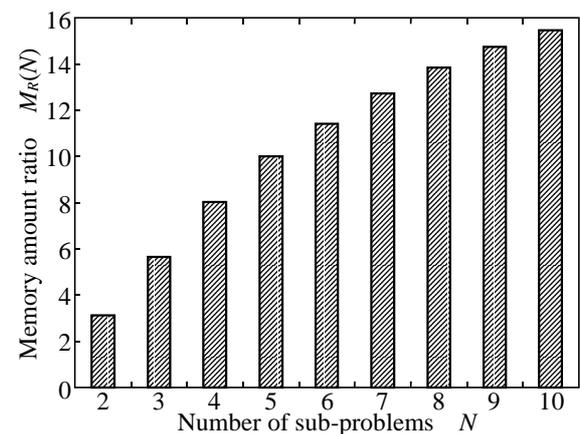
The optimization problem in this section is what is called an optimal control problem that deals with dynamic state and control variables  $x$ ,  $y$ ,  $\gamma$  depending on time, and different from the optimization problem with static variables in the previous section. In this paper, according to a



**Fig. 4 Optimal solutions**



**Fig. 5 CPU time improvement**



**Fig. 6 Memory improvement**

BDH method [6], time, variables and constraints are discretized to 200 elements, and the optimal control problem is transformed into nonlinear programming with static variables. The BDH method is one of numerical methods for the optimal control problem. The advantage is that it is easy to deal with various constraints, such

as differential equations and inequality constraints, and that the variables quickly converge to solutions. The weak point in this method is that a huge number of variables should be solved. For example, separation of two state variables and one control variable to 200 elements produces more than 600 optimized variables. Therefore time from the initial 0 to the terminal  $t_f$  is divided into some equal intervals gathering the elements to form sub-problems, so that the characteristics of the proposed parallel optimization method are examined.

Figure 4 shows that a numerical solution obtained from five sub-problems and an analytical one. The both are in good agreement.

Figure 5 indicates improvement of CPU time by dividing a large problem into some smaller sub-problems. A vertical axis is a speed-up ratio of the parallel optimization method to a conventional method solving all variables simultaneously. From this graph, it is certified that the proposed parallel optimization method has excellent performance. In addition, more division doesn't make speed-up, and it takes the least CPU time to solve the problem with five sub-problems.

Figure 6 shows a ratio of the required memory amount of the proposed parallel optimization method to a conventional method. This figure indicates that, the more sub-problems are, the less computer program memory is required.

In conclusion, the parallel optimization method proposed in this paper works well for the simple example.

#### 4 Spaceplane Design Study

In this section, an integrated optimization problem for shape and ascent trajectory of a spaceplane is solved to examine the realization of future space transportation vehicles and show effectiveness of the proposed parallel optimization method.

Figure 7 gives three technical fields in this problem, body design field, aerodynamic analysis field and trajectory planning field, and an objective is to maximize payload weight.

Besides these, the problem in the trajectory planning field is an optimal control problem and its size is larger than the others. This makes the problem separated into smaller two or four sub-problems. Therefore the large-scale optimization problem is divided into four or six sub-problems assigned to the technical fields.

These fields are not independent but bound to exchange their results. Every field contains some variables of body shape, wing shape, performance and aerodynamic coefficients, whose variables must be same values in an optimal solution. This defines conjunctive conditions that the same variables in different fields are equal. The following describe outlines on these fields.

#### 4.1 Problem Definition

##### 4.1.1 Body Design

The spaceplane shape model adopted in this paper is illustrated in Fig. 8. Takeoff weight is specified, from 100 to 500 ton, and a simple body is composed of an elliptical cylinder body,

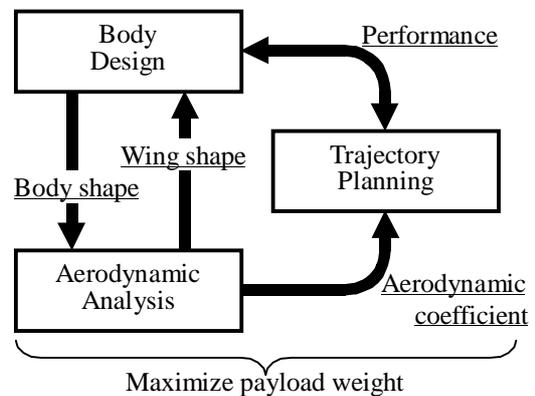


Fig. 7 Three technical fields

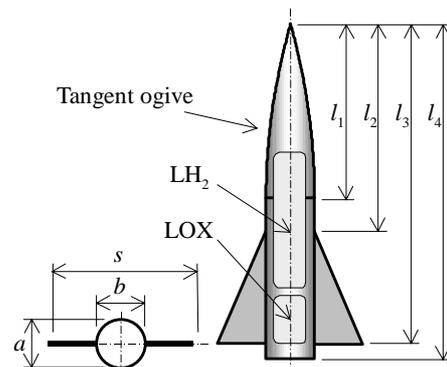


Fig. 8 Spaceplane model

a tangent ogive nose and a delta wing. The design variables in this field are  $l_1, l_2 (\geq l_1), l_3 (\geq l_2), l_4 (\geq l_3), a (\geq 6 \text{ m}), b (\geq 6 \text{ m})$  and  $s (\geq b)$ , and performance variables, viz. maximum dynamic pressure  $q_{\max} (\leq 100 \text{ kPa})$ , and maximum load factor  $n_{LF\max} (\leq 4 \text{ G})$ . Note that the tank volume of fuel compounded from liquid hydrogen (LH<sub>2</sub>) and liquid oxygen (LOX) must be less than 70 % of the total body volume.

4.1.2 Aerodynamic Analysis

The aerodynamic characteristics of the model are analytically computed by CRSFLW program [7]. This program is based on the concept that the normal-force distribution over a body is made up of a potential term given by slender-body theory and a viscous crossflow term modified by Newtonian theory, and calculates normal force, axial force, and pitching moment. Nonlinear effects due to vortex shedding from the nose of the fuselage or the leading edge of the lifting surface are not included in this method. This is much simpler than present representative CFD methods, but can be easily applied over a wide range of angles of attack, Mach numbers, and Reynolds numbers to estimate aerodynamic characteristics. Five sampling points are selected from low speed to hypersonic speed, where aerodynamic coefficients to compute lift coefficient and drag coefficient are calculated.

Constraint functions in this field are the equations relating the body and wing shape in Fig. 8 to the aerodynamic parameters.

4.1.3 Trajectory Planning

The spaceplane takes off, rises and is accelerated by air-turboramjet (ATR) engine (to Mach 6), scramjet (SCR) engine (switched from ATR and useable to Mach 12) and rocket (ROC) engine (useable with ATR and SCR at the same time). Then, after the engine is cut-off above 90 km, it zooms up to 400 km with no thrust in an elliptical orbit. Finally, it is put into a 400 km circular orbit at the apogee in the elliptical orbit.

Fundamental nomenclatures are represented in Fig. 9. State variables are altitude  $h (= r - R_0)$ , velocity  $v$ , flight-path angle  $\gamma$  and

weight  $m$ . A control variable is defined as the angle of attack  $\alpha$ . Motion equations of the spaceplane are expressed as

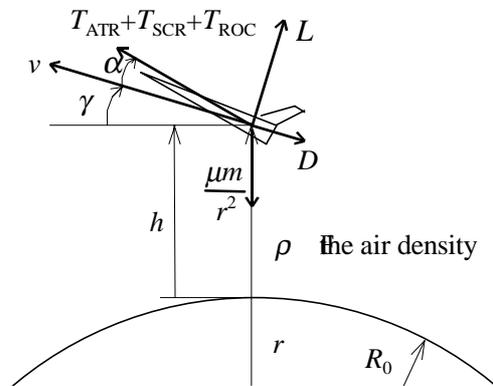


Fig. 9 Symbols

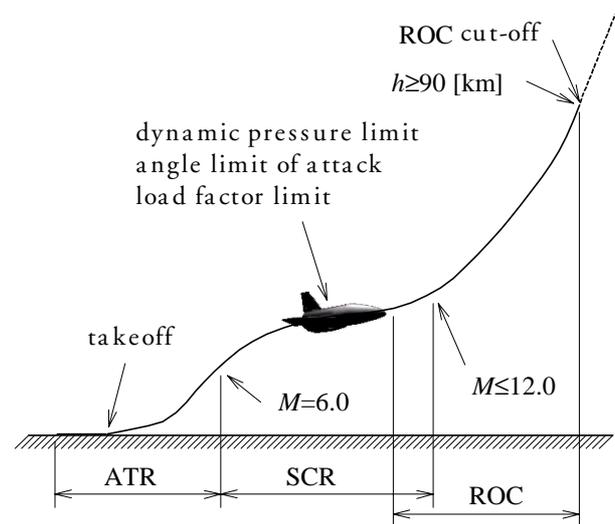


Fig. 10 Ascent trajectory to RE cut-off

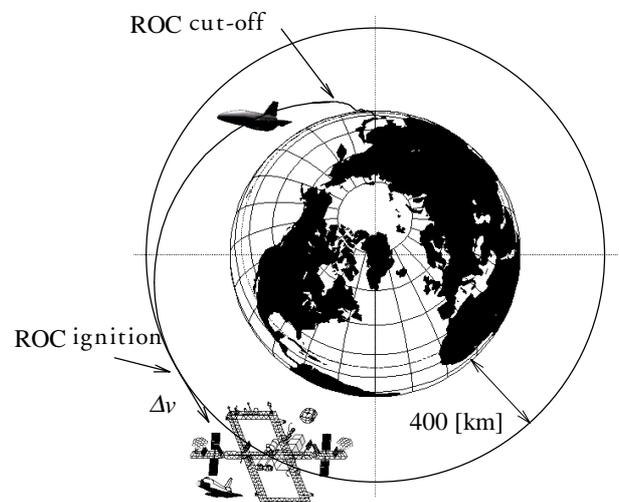


Fig. 11 Zooming up to a 400 km orbit

$$\frac{dh}{dt} = v \sin \gamma \quad (12a)$$

$$\frac{dv}{dt} = \frac{(T_{ATR} + T_{SCR} + T_{ROC}) \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2} \quad (12b)$$

$$\frac{d\gamma}{dt} = \frac{(T_{ATR} + T_{SCR} + T_{ROC}) \sin \alpha + L}{mv}$$

$$+ \left( \frac{v}{r} - \frac{\mu}{vr^2} \right) \cos \gamma \quad (12c)$$

$$\frac{dm}{dt} = - \left( \frac{T_{ATR}}{I_{SPATR}} + \frac{T_{SCR}}{I_{SPSCR}} + \frac{T_{ROC}}{I_{SPROC}} \right) \frac{1}{g_0} \quad (12d)$$

where  $\mu$  is the gravity constant,  $g_0$  is the gravity acceleration at the ground level ( $h = 0$ ,  $r = R_0$ ), and  $D$  and  $L$  are lift and drag respectively, which are computed by the aerodynamic coefficients.  $T_{ATR}$ ,  $T_{SCR}$  and  $T_{ROC}$  are the thrust of air-turboramjet (ATR) engine, scramjet (SCR) engine and rocket (ROC) engine,  $I_{SPATR}$ ,  $I_{SPSCR}$  and  $I_{SPROC}$  are specific impulse of each engine.

Initial conditions at time  $t = 0$  are specified as

$$h(0) = 0 \text{ [km]} \quad (13a)$$

$$\gamma(0) = 0 \text{ [deg]} \quad (13b)$$

$$m(0) = W_{\text{takeoff}} \quad (13c)$$

$$L \cos \alpha + (T - D) \sin \alpha \geq m(0) g_0 \quad (13d)$$

$$v(0) \leq 150 \text{ [m/sec]} \quad (13e)$$

where  $W_{\text{takeoff}}$  is a takeoff weight specified as an arbitrary constant. Terminal conditions at the engine cut-off time  $t = t_f$  is expressed as

$$h(t_f) \geq 90 \text{ [km]} \quad (14a)$$

$$\gamma(t_f) \geq 0 \text{ [deg]} \quad (14b)$$

and there is a equality condition that the apogee altitude computed by the terminal states,  $h(t_f)$ ,  $\gamma(t_f)$  and  $v(t_f)$ , needs to be 400 km.

In addition, the following path constraints are defined.

$$h \geq 0 \text{ [km]} \quad (15a)$$

$$q \leq q_{\text{max}} \quad (15b)$$

$$\alpha \leq 20 \text{ [deg]} \quad (15c)$$

$$n_{LF} \leq n_{LF \text{ max}} \quad (15d)$$

where  $q$  is dynamic pressure and  $n_{LF}$  is load factor.

It should be noted that there are times that the motion equations change discontinuously since the operating engines are switched according to the flight conditions. Therefore the trajectory planning field is subdivided into four stages, that is, ATR, SCR SCR+ROC and ROC stage, which provide four sub-problems.

If an objective function is given, the optimization problem in this field is an optimal control problem as solved in the previous section. Dynamic variables and constraints are discretized according to BDH method to formulate nonlinear programming problems.

Considering the circumstances mentioned above, decided variables are four state and one control variables, initial and terminal time in every stages, the engine performances  $S_{ATR}$ ,  $S_{SCR}$ ,  $T_{ROC}$ , the flight performances  $n_{LF \text{ max}}$ ,  $q_{\text{max}}$ ,  $T_{\text{max}}$ , the aerodynamic coefficients and the fuel amount  $W_{\text{propellant}}$  consumed until arriving at the circular orbit. As this field is subdivided, the variables not required in each sub-problem are left aside.

#### 4.1.4 Objective Function

An object in all fields is to maximize a payload weight. In order to compute this weight, a structural weight  $W_{\text{structure}}$  can be estimated from the body and wing size, according to WAATS [8] program. Considering a propellant weight  $W_{\text{propellant}}$  and an auxiliary weight  $W_{\text{auxiliary}}$  including control and power systems and crews, the payload weight  $W_{\text{payload}}$  is defined as

$$W_{\text{payload}} = W_{\text{takeoff}} - (W_{\text{structure}} + W_{\text{propellant}} + W_{\text{auxiliary}}) \quad (16)$$

Terms in an above equation are supposed to be related to the variables in the body design field. Therefore, the only body design field has an above objective function, and there are no objects in the other fields. The other objective

functions are composed of conjunctive constraint functions between the fields.

Let us give an example of conjunctive constraint functions between the body design and aerodynamic analysis field. In the two fields, there are synonymous variables,  $l_1$ ,  $l_4$ ,  $a$  and  $b$ , concerning the body shape. Equations that the variables between these fields are equal are defined as constraint conditions in the aerodynamic analysis and an objective function in the body design field. On the other side, equalities on the wing shape,  $l_2$ ,  $l_3$ , and  $s$  in the body design and aerodynamic analysis fields are defined as constraint conditions in the body design field and an objective function in the aerodynamic analysis. Notice that the fields in which the conjunctive conditions are defined as the constraint or the objective are different. The conjunctive functions in the other fields are also allocated properly in same manner. Proper definitions provide objective functions for every field.

### 4.2 Optimal Solutions

Supposing a takeoff weight to be 300 ton ( $W_{\text{takeoff}} = 300 [\text{ton}]$ ), an obtained optimal solution is shown in Figs. 12 to 16 and Table 1. The maximized payload weight is negative,  $-13.75$  ton, and the spaceplane cannot reach the orbit even without the payload. It is general that the weight estimation by present technological level indicates the negative payload weight. It means that weight reduction more than 5 % is required to realize it.

Figure 12 and table 1 show the optimized wing area, and the intake area of ATR are very small. It can be considered that the wing area and ATR are respectively the limit size in order to take off and fly the vehicle against the aerodynamic drag. In addition, the intake area of SCR is  $0 \text{ m}^2$ , which means that SCR is unnecessary in this calculation. The reason is that, without SCR, the volume of  $\text{LH}_2$  is reduced and that the SCR engine is heavy.

Next, the takeoff weight which has been fixed to 300 ton is changed. Figure 17 shows the maximized payload weight for the various takeoff weights from 100 to 500 ton. The

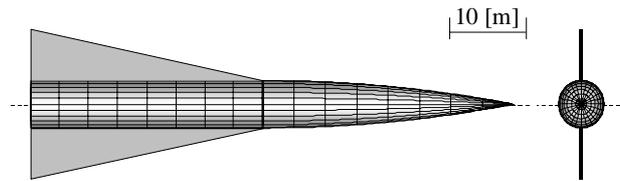


Fig. 12 Optimal shape

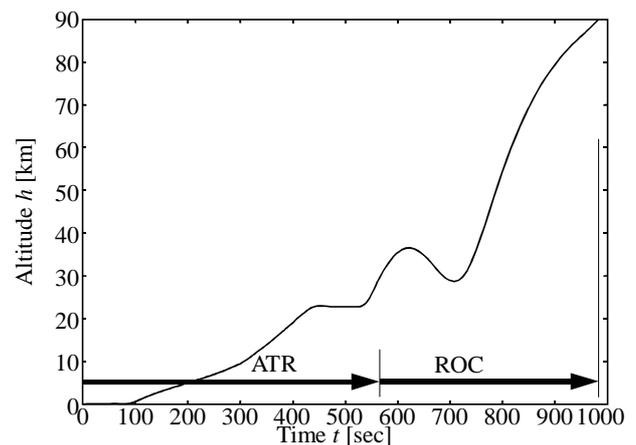


Fig. 13 Time history of altitude

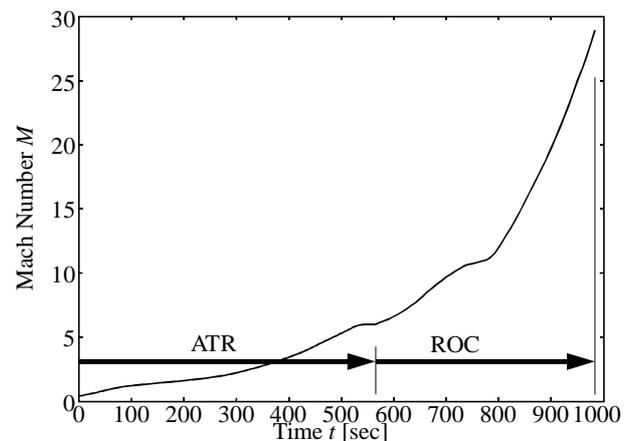


Fig. 14 Time history of Mach number

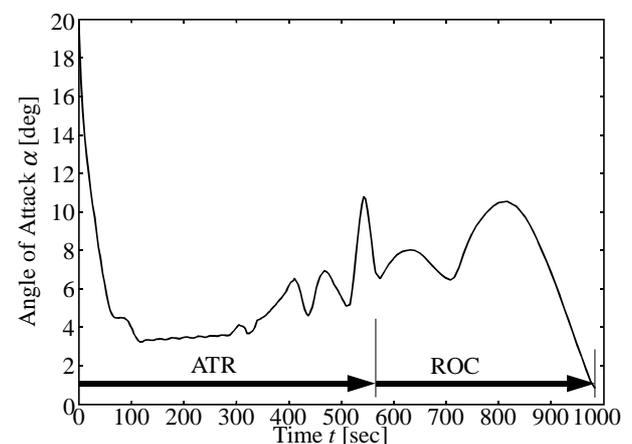


Fig. 15 Time history of angle of attack

payload weight slightly increases with the takeoff weight by only about 4 ton. This indicates that the increase of the take-off weight

Table 1 Optimal spaceplane with takeoff weight 300 ton

Characteristics		Solution
Body length	$l_4$ [m]	63.48
Body height	$a$ [m]	6.00
Body width	$b$ [m]	6.36
Wing span	$s$ [m]	10.02
Intake area of ATR	$S_{ATR}$ [m <sup>2</sup> ]	12.59
Intake area of SCR	$S_{SCR}$ [m <sup>2</sup> ]	0.00
Thrust of ROC	$T_{ROC}$ [ton]	226.6
Max. thrust	$T_{max}$ [ton]	226.6
Max. dynamic pressure	$q_{max}$ [kPa]	100.0
Max. load factor	$n_{max}$ [G]	3.82
Payload weight	$W_{payload}$ [ton]	-13.75

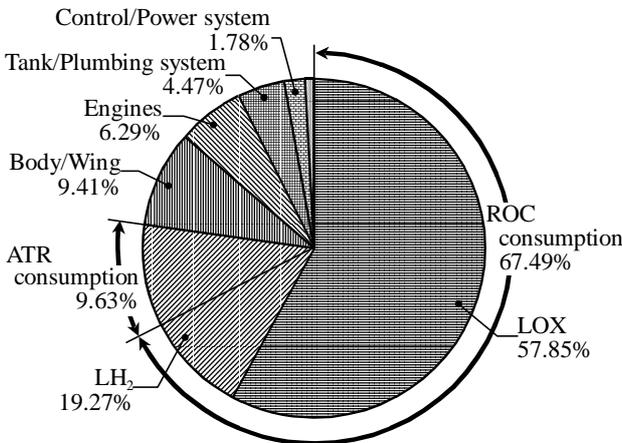


Fig. 16 Weight distribution

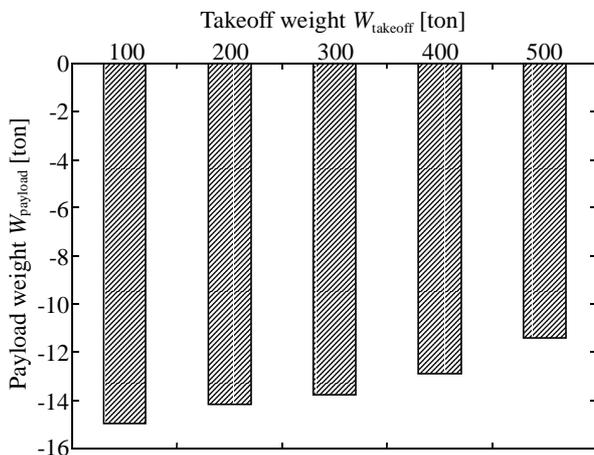


Fig. 17 Takeoff weight and payload weight

cannot improve the performance index greatly, and the weight reduction is the essential key technology to realize future space transportation vehicles.

### 4.3 Validity of Parallel Optimization

Let us compare the parallel optimization method with a conventional method solving all variables simultaneously. In this section, the conventional method is called All-At-Once method. The number of variables optimized in this problem is more than one thousand, and constraint

Table 2 Required memory per process

Optimization method	Memory amount [kB]
All-At-Once	96184
Parallel Optimization	56392

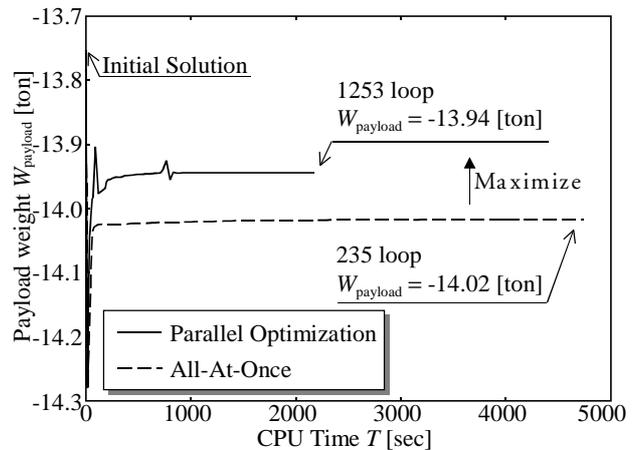


Fig. 18 Comparison of the payload weight

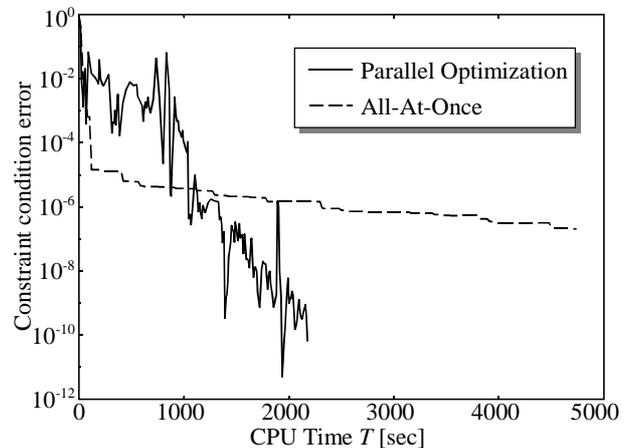


Fig. 19 Comparison of the constraint condition error

functions are similar in quantity. Here, the optimal solution in a previous subsection is set up as an initial solution, ROC cut-off altitude is changed from 90 to 100 km, and an optimization is computed again.

Figures 18 and 19 show the variations in objective function value and constraint condition error with computation time respectively, and Table 2 gives required memory amount per computer. Iteration number of the parallel optimization method is much more than that of the All-At-Once method, and computation time per loop in the parallel optimization method is much shorter. Therefore, in this problem, the variable in the parallel optimization method converges into an optimal solution faster than that in All-At-Once. The computation time and iteration number are, however, influenced by parameters in optimization computer programs. Besides this, considering the required memory amount per computer, which depends on the number of the variables, the memory in parallel optimization method is smaller.

According to Fig. 18, characteristic of the parallel optimization method is that the maximized objective function value calculated from the variable obtained in the parallel optimization method is larger than that in All-At-Once method. In addition to this, Fig. 19 indicates the constraint condition error of the variables in the parallel optimization method is smaller. There is a possibility that the parallel optimization method improves the variable which the All-At-Once method cannot make better any longer.

## 5 Conclusions

First, this article proposed the new parallel optimization method for a large-scale system design with a huge number of variables and constraint conditions. This method divides the problem into some small optimization sub-problems based on subsystems constituting the system, which are solved in parallel. Second, The method was applied to a simple example and compared with a conventional method solving all variables simultaneously. This result

indicates that the parallel optimization method can reduce computation load. Third, a shape and ascent trajectory optimization problem for spaceplane was studied by this method. Consequently, obtained numerical solutions indicate useful design guides to develop a spaceplane. Through these studies, it was confirmed that the proposed parallel optimization method was effective for the complex large-scale optimization problems.

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