# SUPERSONIC INVERSE DESIGN OF WINGS FOR THE FULL CONFIGURATION OF JAPANESE SST

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### Abstract

An inverse design method based on the residualcorrection concept has been developed and applied to aerodynamic design of wings for Japanese SST (supersonic transport). The method is a system of CFD (computational fluid dynamics) softwares. It designs the section geometry at every span station of a wing which realizes the prescribed target pressure distribution. It handles practical models of an aircraft up to a full configuration model. The design system consists of a supersonic inverse problem solver and a Navier-Stokes simulation code. The inverse problem solver has been newly formulated. By means of the design method, two challenging designs are taking place. The first one is the design of a high L/D wing for a wing-fuselage configuration. This has attained a satisfactory design result. The other is the design of a wing for a full configuration SST. For the second challenge, the feasibility has been confirmed that the method can be applied to the design of the wing which is strongly influenced by an engine-nacelle.

### **1** Introduction

Experimental models of a Super-Sonic Transport are under development in Japan. Unlike the traditional way, the aerodynamic shape of the Japanese SST is being primarily designed by numerical tools such as CFD softwares. One of the important tools is the design method which determines the shape of a wing. This SST program has been primarily conducted by the NAL (National Aerospace Laboratory) in Japan and they will perform the first flight test of experimental scaled SST in 2001. The NAL aims to design an NLF (natural laminar flow) wing to reduce a drag. To realize an NLF wing, the pressure distribution on the upper surface of a wing, which is a function of both the camber curvature and the thickness distribution, is of primary importance[1]. However, so far supersonic wing design has been performed considering the load distribution on the wing. Accordingly, most existing methods treat only the camber of the wing, because the thickness does not affect the load. So, the method which treats both the warp curvature and the thickness distribution simultaneously, considering three-dimensional effects was needed.

We have develop a new inverse problem solver for the method for the SST program[2] and have been developing and verifying the method[3, 4] and It is based on Takanashi's integral equation method which was devised for transonic wings[5]. In section 2, the conceptual overview of the design system is described. The formulation process of the supersonic inverse problem is shown in section 3. In section 4 and 5, the new method is applied to highly practical designs involving the NAL's experimental SST models.

### 2 Aerodynamic Design Method

The design procedure for supersonic wings is iterative method. Fig.1 illustrates the procedure. The method determines the wing section's geometry which realizes a specified target pressure distribution at all span stations of a wing. First, a baseline shape is to be guessed. Then the flow field around the wing is analyzed by flow simulation to get the current Cp distribution on the wing surface. Next the inverse problem is solved to obtain the geometrical correction value  $\Delta f$  corresponding to the difference between target and current pressure distributions  $\Delta Cp$ . The new wing is designed by modifying the baseline shape using  $\Delta f$ . Now, the current shape is updated. The next step is to go back to the flowfield analysis. The flow analysis is conducted to see if the current shape realizes target pressure distribution. If the difference between target and current pressure distributions is negligible, the design is completed. Otherwise, the next step is once again to solve the inverse problem and iterate the design loop until the pressure difference becomes negligible. This iterative procedure of reducing the residual is widely used in numerical aerodynamic design.

There are two primary parts; one is a flow analysis part which conducts grid generation and Navier-Stokes flow simulation[6]. It evaluates the residual. The other is a design part where the inverse problem is solved to update the wing geometry. The design part determines the correction which is expected to compensate for the residual. Both parts are completely independent from each other in terms of their algorithms and equations. The accuracy of a design result depends on the analysis part. The flow analysis of the present design method is done by a Navier-Stokes simulation code. So, designed geometry by the method are valid for Navier-Stokes flows. The efficiency of the design method depends on the design part of the inverse problem. The small perturbation of the potential flow approximation is adopted for the formulation of the inverse problem, which will be discussed in the next section.

To conclude this section, the author would

like to mention one important characteristic of the present design method: the design part handles not the flow quantity itself but the  $\Delta$ -form value, which is difference between two states of a flowfield. Accordingly, with the present method, one is approximating the slight change of a Navier-stokes flowfield by the potential flow theories with thin wing approximation.

### **3** Formulation of an Inverse Problem

### 3.1 Basic equations and Green's formula

As stated in the previous section, the inverse problem of the design part should handle the  $\Delta$ -form value, which is difference between two states of a flowfield. The goal of the formulation is to obtain the mathematical function to relate  $\Delta Cp$  to geometrical correction  $\Delta f$ . The flowfield considered here is shown in Fig 2. There are two projection plots of a space. It is a supersonic potential flowfield with freestream Mach number greater than 1.0 ( $M_{\infty} > 1$ ). A wing is located at the z = 0 plane. The *x*-axis is streamwise, the *y*-axis is spanwise. The basic equations are in  $\Delta$ form. Assuming that the small perturbation theory holds and applying a Prandtl-Glauert transformation, the equations are expressed as

$$-\Delta\phi_{xx} + \Delta\phi_{yy} + \Delta\phi_{zz} = 0 \tag{1}$$

$$\frac{\partial}{\partial x}\Delta f_{\pm}(x,\frac{y}{\beta}) = \beta^3 \Delta \phi_z(x,y,\pm 0)$$
(2)

$$\Delta C p_{\pm}(x, \frac{y}{\beta}) = -2\beta^2 \Delta \phi_x(x, y, \pm 0) \qquad (3)$$

where  $\beta = \sqrt{M_{\infty}^2 - 1}$  and, '+' and '+0' indicate the physical quantity on the upper surface of a wing, while '-' and '-0' do that of the lower surface.

Applying Green's theorem for a hyperbolic equation to Eq.(1) and perfoming integral by parts with respect to  $\xi$  to avoid singularity  $\Delta \phi$  is formulated into the following analytical form,

$$\Delta \phi(x, y, z) =$$

$$-\frac{1}{2\pi}\int\int_{\tau_{+}}\frac{1}{r_{c}}\Big[\Delta\phi_{\zeta}(\xi,\eta,+0)-\Delta\phi_{\zeta}(\xi,\eta,-0)\Big]d\xi d\eta$$



Fig. 1 Design method of residual-correction concept.

$$+\frac{1}{2\pi}\int\int_{\tau_{+}}\frac{z(x-\xi)}{[(y-\eta)^{2}+z^{2}]r_{c}}$$
$$(\Delta\phi_{\xi}(\xi,\eta,+0)-\Delta\phi_{\xi}(\xi,\eta,-0))d\xi d\eta \qquad (4)$$

where

1

$$r_c = \sqrt{(x-\xi)^2 - (y-\eta)^2 - z^2}$$
 (5)

The domain of integration is denoted by  $\tau_+$ where  $(x - \xi)^2 - (y - \eta)^2 \ge 0$ . It is shown as a shadowed area on a wing surface in the x - yplane sketch of Fig. 2.

# **3.2 Integral equations for the inverse prob**lem and improper integrals

In order to expose the boundary condition  $\Delta Cp$ and the unknown shape function  $\Delta f$  explicitly, further calculus are conducted on Eq.(4). In fact,  $\Delta Cp$  is associated with  $\Delta \phi_x$  and  $\Delta f$  is associated with  $\Delta \phi_z$ . So, one should differentiate Eq.(4) with respect to x and z respectively, then take the limit as  $z \rightarrow 0$ . There needs to be special treatment of the differentiating process, because the integrand encounters its singularity at the boundary or inside of the region of integration. On the top of this, one should be aware of the order to operate the double integration , in  $\xi$  and  $\eta$  directions, when handling improper integrals. Two extended ideas for integration have to be introduced to evaluate an integral whose integrand is not always definite over the domain. They are found in Refs.[7, 8]. The first idea is the finite part and the other is the principal part of integrals, whose notations used in this particular article are  $f \int$  and  $p \int$  respectively.

Two fundamental equations for the inverse problem are shown as Eqs.(6) and (9). The first one is obtained by differentiating Eq.(4) with respect to x and taking the limit as  $z \rightarrow 0$ . To lead the differentiated integral equation to the final form blow, one should use the finite part concept for improper integrals as well as a lot of calculus;

$$\Delta w_{s}(x,y) = -\Delta u_{s}(x,y)$$
$$-\frac{1}{\pi} \int \left[ f \int_{\tau_{+}} \frac{(x-\xi)\Delta w_{s}(\xi,\eta)}{[(x-\xi)^{2} - (y-\eta)^{2}]^{3/2}} d\eta \right] d\xi$$
(6)

where

$$\Delta u_s = \Delta \phi_x(x, y, +0) + \Delta \phi_x(x, y, -0)$$

$$= -\frac{1}{2\beta^2} (\Delta C p(x, \frac{y}{\beta}, +0) + \Delta C p(x, \frac{y}{\beta}, -0)) \quad (7)$$

$$\Delta w_s = \Delta \phi_z(x, y, +0) - \Delta \phi_z(x, y, -0)$$

$$= -\frac{1}{\beta^3} (\frac{\partial \Delta f(x, \frac{y}{\beta}, +0)}{\partial x} - \frac{\partial \Delta f(x, \frac{y}{\beta}, -0)}{\partial x}) \quad (8)$$



**Fig. 2** Coordinate system of a supersonic flow-field for formulation..

The other one is obtained by differentiating Eq.(4) with respect to *z* and taking the limit as  $z \rightarrow 0$ . Using the principal part concept, the equation yields

$$\Delta w_a(x,y) = -\Delta u_a(x,y)$$

$$+ \frac{1}{\pi} \int \left[ {}^p \int_{\tau_+} \frac{(x-\xi)\Delta u_a(\xi,\eta)}{(y-\eta)^2 \sqrt{(x-\xi)^2 - (y-\eta)^2}} d\eta \right] d\xi$$
(9)

where

$$\Delta u_a = \Delta \phi_x(x, y, +0) - \Delta \phi_x(x, y, -0)$$
$$= -\frac{1}{2\beta^2} (\Delta C p(x, \frac{y}{\beta}, +0) - \Delta C p(x, \frac{y}{\beta}, -0)) \quad (10)$$

$$\Delta w_a = \Delta \phi_z(x, y, +0) + \Delta \phi_z(x, y, -0)$$
  
=  $-\frac{1}{\beta^3} \left( \frac{\partial \Delta f(x, \frac{y}{\beta}, +0)}{\partial x} + \frac{\partial \Delta f(x, \frac{y}{\beta}, -0)}{\partial x} \right)$ (11)

In Eqs.(6) and (9),  $\Delta w_s(x,y)$  and  $\Delta w_a(x,y)$  are unknowns.  $\Delta w_s$  is associated with the thickness change at (x, y) on a wing.  $\Delta w_a$  is associated with the curvature change of the wing section camber, at (x, y). As for the order of the double integral, the integral with respect to  $\eta$  is to be performed first in both fundamental equations. To guarantee that every section has a closed trailing edge, the solution  $\Delta w_s$  is modified as

$$\Delta w_s^{mod}(x, y) = \Delta w_s(x, y) - \frac{\int_{L.E.}^{T.E.} \Delta w_s(\xi, y) d\xi}{\int_{L.E.}^{T.E.} d\xi}$$
(12)

so as to satisfy the condition:

$$\int_{L.E.}^{T.E.} \Delta w_s^{mod}(\xi, y) d\xi = 0$$
(13)

Finaly, the geometrical correction,  $\Delta f$ , is calculated using  $\Delta w_s^{mod}$  and  $\Delta w_a$ :

$$\Delta f_{\pm}(x, \frac{y}{\beta}) = \frac{1}{\beta^3} \int_{L.E}^{x} \left[ \Delta w_s^{mod}(\xi, y) \pm \Delta w_a(\xi, y) \right] d\xi$$
(14)

#### 4 Wing Design for a Wing-Fuselage Model

The design of the first NAL's experimental SST model has been completed successfully by using the present method[9]. The model is a wingfuselage combination without propulsion system. The wing of the model is aerodynamically designed at its cruising condition which is of  $M_{\infty} =$ 2.0 and 2.0 degree-angle of attack. Such SST planform and the overview of the flowfield about it is illustrated in Fig. 3. The design concept is to obtain a high L/D wing by reducing drag force. Then, we prescribe a target pressure whose elliptical load distribution minimizes the induced drag and whose upper surface distribution keeps the laminar boundary layer significantly longer than traditional wings. Wings of this type are called as NLF wings. Thus, the goal of the design method is to determine a wing section geometry which realizes the prescribed target pressure distribution. For solving the inverse problem, the half span of the wing is divided into 82 (spanwise)  $\times$ 50 (chordwise) panels.



Fig. 3 Wing-fuselage configuration of SST.

The wing design starts from the baseline shape. The baseline is the result of planform and warp optimizations in terms of the L/D ratio, while the shape of the fuselage is determined using the area rule. The thickness distribution of the NACA66003 airfoil is adopted for each span station of the baseline wing. Despite optimizations, the performance of the wing of the wing-fuselage model is not as efficient as expected. This is because these optimizations were done for a wing alone. In other words, they did not take the wingfuselage interaction into consideration. Therefore, improvement of the aerodynamic shape of the SST wing by a method which can account for this interaction is necessary.

The design results at two span stations are shown in Figs.4 and 5. Those results were obtained after twelve iterations of the design loop.

Fig.4 presents the wing section geometry and the realized pressure distribution along the chord at 30%-semispan station. The dashed line and '+-line' indicate, respectively, the geometry and pressure distribution of the baseline wing section, while the solid line and 'Diamond- line' indicate those of the designed wing section. The target pressure is indicated by chain lines. Fig.5 shows those at the 70%-semispan station. The resulting wing realizes almost identical pressure distribution to the target. The improvement from the baseline wing by the design method is excellent. One of the most characteristic features of a NLF



Fig. 4 Design process at 30% semi-span.



Fig. 5 Design process at 70% semi-span.

wing is the sudden expansion of the upper surface Cp distribution at the leading edge. Another one is, on the upper surface, a flat roof type of Cpdistribution along the chord. These features have been realized by the designed wing geometry in Figs.4 and 5.

# 5 Wing Design for a Full-Configuration Model

The next challenge is going on. That is to design a wing for a full-configuration model using the same target pressure as the first one. The new model is plotted in Fig. 6. This is the preliminary baseline shape for the NAL's experimental SST model with propulsion system and its flowfield at the speed of  $M_{\infty}$ = 2.0 with the angle of attack of 2.0 degrees. The baseline airplane consists of the resulting wing- fuselage combination model of the first design, diverter, cylindrical shape engine nacelle, as well as vertical and horizontal tail wings. The pressure contours on the SST and a cross-flow sectional plane can be observed there. Comparing with the flowfield about the first model in Fig. 3, one should recognize complicated flow physics occur by the strong interaction among the wing, fuselage and engine elements on the lower surface of the wing.



Fig. 6 Full configuration model of SST.

The design processes at the 30%, 50% and 70% semi-span stations are presented in Figs. 7-9, respectively. The first (left-hand side) plot of

each figure is the Cp vs. X/C graph of "diamond" lines associated with the baseline geometry. The other plot (right-hand side) is the Cp vs. X/Cgraph of "square" lines associated with the updated geometry modified by the design method. In both plots, the target Cp distribution, which is the same as the first design, is also seen as + symbols. The design has not been completed yet. The updated one is the result after three iteration of the residual-correction loop.

Although the wing is the resulting one of the design for the first model, the baseline pressure distribution is coming off the target, especially on the lower surface of the wing. It is because of the strong effect by an engine nacelle. This implies that to adopt the same target pressure distribution on the lower surface is not always appropriate for the second design. At the 30% and 50% semispan stations, the effect looks strong and immediate. On the other hand, at the 70% semi-span station, the same target pressure distribution could be available because the influence effect becomes small. At the present, inside the 65% semi-span station, the target pressure distribution is chordwisely modified. >From the viewpoint of spanwise pressure distribution, however, it is desirable to construct the whole target pressure distribution three-dimensionally. Unfortunately, such strategy has not been established. Therefore, the goal of the first design project of the full configuration SST is to recover the NLF target pressure distribution on the upper surface of the wing and to reduce the pressure curve variation of ups and downs on the lower surface.

Comparing the two plots of each figure of Figs. 7-9, the updated Cp distributions are much closer to the target ones than the baseline. The extent of the agreement of updated and target pressures is better on both of the upper and lower surface. Improvement by a strategic use of the present method is obvious.

## **6** Conclusions

A inverse design method for wings of SST has been developed and applied to the design of experimental models of Japanese SST. It is a CFD

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Fig. 8 Design process at 50% semi-span.



Fig. 9 Design process at 70% semi-span.

system and uses only computers. The design method consists of the inverse problem solver and the Navier-Stokes flow analysis code. That inverse problem solver has also been developed for the method. It has good availability and portability.

The equations to solve the supersonic inverse problem are derived from the supersonic small disturbance equation and thin wing theory. Since the formulation for the inverse problem is done in  $\Delta$ -form and the design method adopted the residual-correction concept, a designed wing by the method is valid for a wing-fuselage model of a SST in Navier-Stokes flow-fields when the Navier-Stokes analysis is conducted about a wing-fuselage configuration at each iteration step.

Two challenging designs are taking place by means of the method. The first one is the design of a high L/D wing for a wing-fuselage configuration. This has attained a satisfactory design result. The other is the design of a wing for a full configuration SST. For the second challenge, the feasibility has been confirmed that the method can be applied to the design of the wing which is strongly influenced by complicated structure of the airplane components, e.g. diverters and engine nacelles. Considerable effort and study are needed to establish the general strategy for the usage of the method for the full configuration model.

### References

- Yoshida, K., Iwamiya, T., Ueda, Y., and Ishikawa, H.: Boundary Layer Transition Analysis of the Scaled Supersonic Experimental Airplane, 2<sup>nd</sup> *International CFD Workshop on SST Design*, Tokyo JAPAN, 2000.
- [2] Jeong, S., Matsushima, K., Iwamiya, T., Obayashi, S., and Nakahashi, K.: inverse Design Method for Wings of Supersonic Transport. *AIAA paper 98-0602*, 1998.
- [3] Matsushima, K., Iwamiya, T. Jeong, S., and Obayashi, S.: Aerodynamic Shape Design of a Wing for NAL's SST Using an Invese Problem, *NAL-SP*, Vol. 41,pp 191-196, 1998.

- [4] Matsushima, K., Jeong, S., Takaki, R., Obayashi, S., Nakahashi, K. and Iwamiya, T.: Supersonic Inverse Wing Design Method and Its Application to Japanese SST, *Proceedings 16th ICNMFD*, *Lecture notes in Physics*, Vol. 55, pp 79-84, Arcachon FRANCE, 1998.
- [5] Takanashi, S.: Iterative Three-dimensional Transonic Wing Design Using Integral Equations, J. Aircraft, Vol. 22, No. 8, pp 655-660, 1985.
- [6] Takaki, R., Iwamiya T., and Aoki, A.,: CFD Analysis Applied to the Supersonic Research Airplane, 1<sup>st</sup> International CFD Workshop on SST Design, Tokyo, JAPAN, 1998.
- [7] Lomax, H., Heaslet, M. A. and Fuller, F. B.: Integrals and Integral equations in Linearized Wing Theory, *NACA Rep.* 1054, 1951.
- [8] Heaslet, M. A. and Lomax, H.: Supersonic and Transonic Small Perturbation Theory. Vol. 6 of *High-Speed Aerodynamics and Jet Propulsion*, Princeton University Press, Vol. 6, pp 122-344, 1954.
- [9] Matsushima, K., Iwamiya, T, and Zhang, W.: Inverse Wing Design for the Scaled Supersonic Experimental Airplane Ensuring Design Constraint, 2<sup>nd</sup> International CFD Workshop on SST Design, Tokyo JAPAN, 2000.