IDENTIFICATION OF LONGITUDINAL FLYING CHARACTERISTICS OF AN AEROPLANE AND THE EFFECT OF NONSTATIONARY AERODYNAMICS

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Abstracts

An analysis of identification and of its mathematical-physical variant was performed from gnoseological point of vief. To verify the absence of a significant systematic error a testing quantity has been suggested. To prove the significance of differencies between the considered attributes of a system and of its model a global test of identity or of closeness has been deduced. To facilitate aerodynamic model analysis a classification of nonstationary aerodynamics according to two criteria is given and the distinguishing the angle of attack changes into the "path" and "attitude" onces is introdused. Therefore for the both sorts of the angle of attack changes there could be derived comparable expressions for aerodynamic frequency transfers of the whole aeroplane which are composed from normalized dimensionless transfers of the wing, of the tailplane and of the interaction of the wing on the tailplane. To prove the Strouhal number effect on complex aerodynamic derivatives the "weighted" values of these derivatives have been deduced. The corresponding weight functions are determined by the frequency spectrum of a time history of the elevator deflection and by the frequency transfers for responses of the given aeroplane. An example is attached.

1. Introduction

When analysing results of flight measurements carried out in 1969 with the A 145 light transport aeroplane at constant speed of flight in a calm atmosphere, some differencies were noticed in frequency transfer functions of the aeroplane responses to different deterministic time histories of elevator deflections, that were of triangular, step and sinusoidal shape.(1)

Besides it, differencies in aerodynamic nature of the aeroplane were observed, as values of some aerodynamic derivatives differed according to having been measured at steady or unsteady flights. The greatest differencies reaching 23 percent were stated at the down-wash angle derivative according to the angle of attack at the horizontal tail surfaces.

When estimating the model parameters of the considered aeroplane motion, a model based on the quasi-stationary aerodynamics was used. At that time the hypothesis was expressed that this using of quasi-stationary aerodynamics instead of the more complicated nonstationary one could be the reason of the mentioned differencies. To the analogical conclusion the authors of ref.(2) and (3) have come as well.

By the analysis of flight measurements results the identification method was asserted as a method of recognition the nature of the aeroplane longitudinal motion and of its aerodynamics. The identification method used in this way, however, could just point out the significance of the stated differencies between the aeroplane and its model behaviours but it could not lead to the explanation of their causes. Therefore in later studies attention was focused on the identification errors analysis and to the conditions needed to be fulfilled to transform the identification method to a method of recognition that would be able to confirm or to refuse the suitability of the used model hypothesis. In the case that the errors analysis and their significence confirm that the cause of stated significant differencies might be an unsuitable model, then there must be proved by a further analysis whether this may be caused by the used quasi-stationary aerodynamics. To this purpose the weighted averages of complex aerodynamic derivatives were used. They are the weighted mean values of complex derivatives for circular frequencies which are involved with different weights in the frequency spectrum of the time history of exciting elevator deflection. To estimate them one must know analytical expressions for aerodynamic frequency transfer functions of the whole aeroplane. They were derived on the basis of expressions for the isolated wing and for the horizontal tail surface,(6) and of expressions describing the wing influence on the tailplane at an unsteady flow on the wing. (7) The proper method of estimating the weigheted values of complex derivatives was in comparison with ref. (1) improved.

2. Identification as a Method of Recognition

2.1 Introduction into the Problem

The identification of a cybernetic system may get a recognition method just when the model hypothesis was built up on the basis of an analysis of the investigated system behaviour and of its nature, i.e. when the system is not considered to be a "black box". From the gnoseological point of view, the cybernetic systems identification as a method of recognition the objective reality has the following three stages, see fig.1:

1. The statement of the behaviour of a system defined on the investigated object at the given initial and external conditions.
2. The statement of the behaviour of the system model hypothesis under the same conditions as in 1.
3. The identity verification of signs selec-

ted for behaviours of the system and of its model hypothesis.

If the selected signs may be measured, the system behaviour is staded by measurements of time histories of input and output quantities, see fig. 2. As for the state-

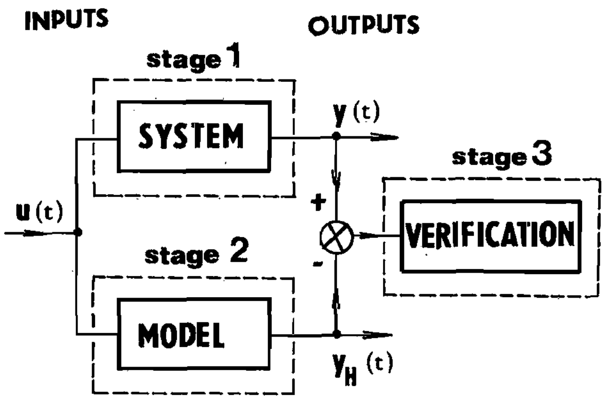


FIGURE 1 – BASIC DIAGRAM OF IDENTIFICATION PROCESS

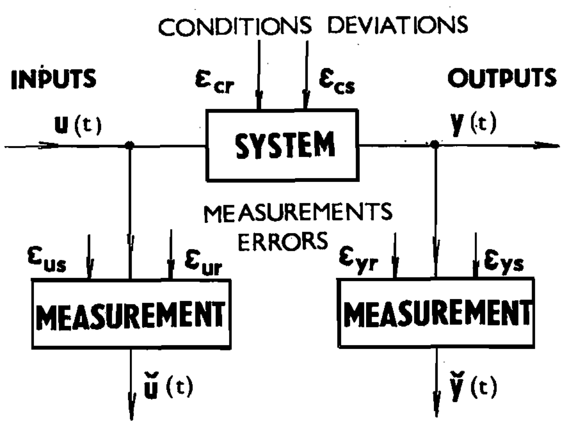


FIGURE 2 – SCHEME DIAGRAM OF ERRORS OF THE STAGE 1

ment of the model hypothesis behaviour, this process is more complicated as at the system itself, see fig. 3.

The identification in order to be a recognition method, the second stage of identification is composed from the following three substages:

a/ The selection of the model hypothesis form on the basis of an analysis of the behaviour and nature of the investigated system.

b/ The estimation of the model hypothesis parameters by a suitable optimisation method from the values of input and output quantities measured and corrected on the system at the given conditions, see e.g. ref. [8] , [9] and application in dynamics of flight see e.g. ref. [10] , [11] , [12] .

c/ The calculation of the output quantities of the model hypothesis for the input quantities values measured (and corrected) on the system and for the estimated values of the model hypothesis parameters.

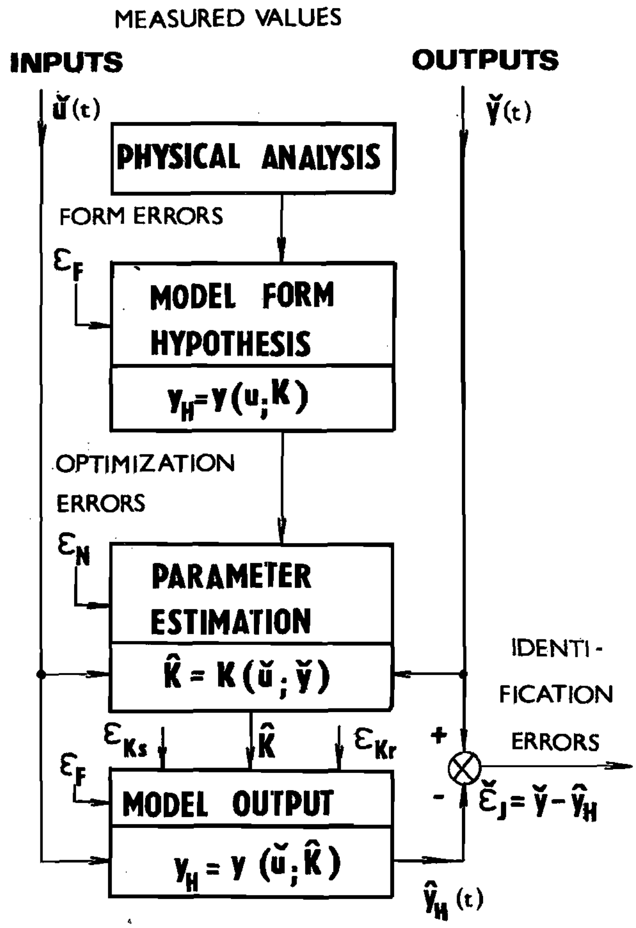


FIGURE 3 – SCHEME DIAGRAM OF ERRORS OF THE STAGE 2 OF THE MATHEMATIC–PHYSICAL VARIANT

As for the third stage of identification, to prove the identity of the behaviours of the system and of its model hypothesis, there must be found suitable objective discriminative criteria in the sense of Leibniz principle, according to which "identical objects are those that cannot be distinguished by means of accessible discriminative criteria". [13] The identification process must be able to be carried out in finite time.

For analysis in dynamics of flight , the "mathematical-physical variant" of identification was used. It is characterised by the fact that the model hypothesis form is derived from a physical analysis of the investigated motion and from the nature of the system defined on the aeroplane and that the estimated values of the model hypothesis parameters have a physical sense.

As values of the selected quantities to be identified are contaminated in the first and second stages by errors of different origins and sorts, the verification depends on their magnitudes and it is , therefore, relative.

2.2 Identification Errors

In the first and second stages of the identification process, some systematic and random errors, ε_s and ε_r, appear. By individual measurements they have concrete values. That are unrecognisable, however. They may be simply algebraically summarised into a total error $\varepsilon_\Sigma = \varepsilon_s + \varepsilon_r$ that is unrecognisable as well.

At random errors, there may be recognised just estimates of statistical characteristics of random errors sets in the case that the identification process was repeated several times by the same method, with the same instrumentation and under the same conditions.

Systematic errors are deterministic quantities. They can be recognised and corrected just in the case that there are known physical laws by which they are governed and values of influence quantities by which they are excited. As the influence quantities values are gained by measurements, the systematic errors estimates have also a nature of random quantities with nonzero mean values. Besides, there remain residuals of systematic errors that cannot be corrected and that for a given series of repeated measurements have originated from the "frozen" values of random errors, e.g. when using the same curves for graduation of measurement instrumentation etc. The residuals of uncorrectible systematic errors are thus determined by conditionally constant random errors in the given series of measurement. If the influence quantities change by random, then a deterministic transformation of random quantities is in question.

There remains thus the problem of the significance of residual incorrected systematic errors of measured quantities that should be solved. The estimate of these residuals in the form of constant parameters involved in optimal parameter estimates of the model hypothesis gives just the constant component of the uncorrectible systematic errors that depends on the model hypothesis form.

The identification errors $\varepsilon_J = y - \hat{y}_H$ may be recognised just as estimates $\check{\varepsilon}_J = \check{y} - \hat{y}_H$.

2.3 A Quantity for Checking the Significance of Systematic Errors Residuals

For the time t_j , where $j = 1,\dots,k$, with repetitions for every j in one series of measurements $\nu = 1,\dots,n_j$, one gets the mean values of one of the measured input quantities $u_{Mj} = \sum_\nu u_{Ej\nu}/n_j$ and of one of the measured output quantities $y_{Mj} = \sum_\nu y_{Ej\nu}/n_j$.

As by flight there is usually not possible to fulfil the same nominal conditions of measurements and insignificantly different time histories of repeated input controlling quantities, there must be incorporated between the first and second stages:

a/ a correction of the measured data for nominal conditions;

b/ at a linear system the averaging of frequency transfer functions just after their Fourier transformation $F_{y.u}(i\omega_j)$ for every $i\omega_j$.

As for given series of repetitions, the optimal values of the model parameters K_r ($r = 1,\dots,n_{Kr}$) are conditionally constant, the estimate of the identification error on the level j (for one output) is

$$\check{\varepsilon}_{Jj} = y_{Mj} - \hat{y}_{Hj} = \varepsilon_{JSj} + \varepsilon_{Jrj} \qquad (1)$$

According to figures 2 and 3 the systematic component estimate of the identification error is a function of partial systematic errors, respectively of their uncorrected residuals,

$$\varepsilon_{JSj} = f_J\,[\varepsilon_{Sj}(u_M),\ \varepsilon_{Sj}(y_M);\ \varepsilon_{Fj},\ \varepsilon_{Kj}] \qquad (2)$$

The identification process to be a recognition method , it must be proved that ˣ⁾

$$\varepsilon_{Sj}(u_M) = \varepsilon_{Sj}(y_M) = \varepsilon_{Kj} = 0$$

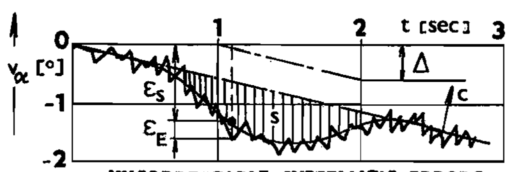
Then the systematic component of the identification error is a function just of model form error, i.e. $\varepsilon_{JSj} = f_J(\varepsilon_{Fj})$.

To prove the significane of $\varepsilon_{Sj}(y_M)$, one can use the checking quantity $v_y(t_j)$ which is defined by the kinematic relation between the necessary and superficient output quantities, i.e.

$$v_y(t_j) = v_y[\check{y}_{lj},\ \check{y}_{vj}] \overset{!}{=} 0 \qquad (3)$$

where $\check{y}_{lj}$ ($l = 1,\dots,n_v$) are the measurement and corrected output quantities necessary to describe the investigated motion of n_v degrees of freedom (i.e. the measured state quantities) and $\check{y}_{vj}$ are the measured and corrected superficient output quantities.

If the both mentioned sorts of output quantities are influenced just by random errors, the checking quantity value changes by random in proximity of zero only. If significant uncorrected systematic errors are present, then the mean curve $v_y(t_j)$ has a deterministic shape, see fig. 4 .



s-UNCORRECTABLE SYSTEMATIC ERRORS
c-CORRECTION $\Delta = \Delta\omega_y - k_n \Delta n_F$

FIGURE 4 - THE SCHEME DIAGRAM OF THE CHECKING QUANTITY $v_\alpha(t)$

In this case the time history of $v_y(t_j)$ may be used according to the nature

ˣ⁾ The iterations number being sufficient, $\varepsilon_{Kj} \equiv 0$ is valid. According to the input quantity nature, one can often suppose $\varepsilon_{Sj}(u_M) \equiv 0$

of uncorrected residuals of systematic errors either for a supplementary correction of the measured and corrected output quantities or for the exclusion of the relevant realisation from the further usage in the series of repeated measurements.

As an example, the checking quantity for the aeroplane motion in a calm atmosphere $v_\alpha(t)$ can be stated that was derived in ref.[1] from the kinematic relation $\theta - \gamma - \alpha = 0$, see fig. 4. It serves to check the absence of significant residuals of systematic errors of the measured and corrected quantities ω_y, n_F and α and it reads :

$$v_\alpha(t_j) = \int_0^{t_j} \left(\omega_y - 57{,}3 \ \frac{g}{V_0} \cdot n_F\right) d\tau - (\alpha - \alpha_0) \overset{!}{\equiv} 0 \quad (4)$$

2.4 A Global Test of the Model Hypothesis Closeness

It follows from the identification definition that in the third stage a proof should be done for every value of the independent variable t_j or $i\omega_j$ respectively that the mean value of every output quantity y_{Mj} is identical with the value $\hat{y}_H$ which was estimated by means of the model hypothesis form, see fig. 3. The zero hypothesis then may be defined so that the mean value y_{Mj} being a random variable differs from the conditionally deterministic value $\hat{y}_{Hj}$ just by random. For every level t_j then nonequality is valid [x)]

$$(t_s)_j = (y_{Mj} - \hat{y}_{Hj}) / \ s(y_{Mj}) \leqq t_{\alpha,(n_j-1)} \ , \quad (5)$$

where the random quantity $(t_s)_j$ has the Student distribution with $(n_j - 1)$ degrees of statistical freedom and with its critical value $t_{\alpha,(n_j-1)}$. The quantity α is here the degree of statistical significance (e.g. $\alpha = 0{,}05$) and

$$s(y_{Mj}) = \left[\sum_{\nu=1}^{n_j} \frac{\varepsilon^2_{Ej\nu}}{n_j(n_j-1)} \right]^{1/2} = \sqrt{s^2_{Ej} / n_j} \ . \quad (6)$$

From the zero hypothesis (5) the probability $(1 - \alpha)$ follows that the estimated value $\hat{y}_{Hj}$ lies in the interval defined by the relation

$$P\left\{ y_{Mj} - t_\alpha \cdot s(\hat{y}_{Mj}) < \hat{y}_{Hj} < y_{Mj} + t_\alpha \cdot s(y_{Mj}) \right\} = 1 - \alpha \quad (7)$$

In the case when the estimates $\hat{y}_{Hj}$ distribution about the values y_{Mj} is just by random, the confidence bounderies are passed over also by random with the probability α. When the identification deviation estimate $\hat{\varepsilon}_{3j} = y_{Mj} - \hat{y}_{Hj}$ involves systematic errors residuals, the nonequality $(t_s)_j > t_\alpha$

x) The symbol t_s was used to distinguish from the symbol for time t. In statistical literature t_s is usually denoted by t.

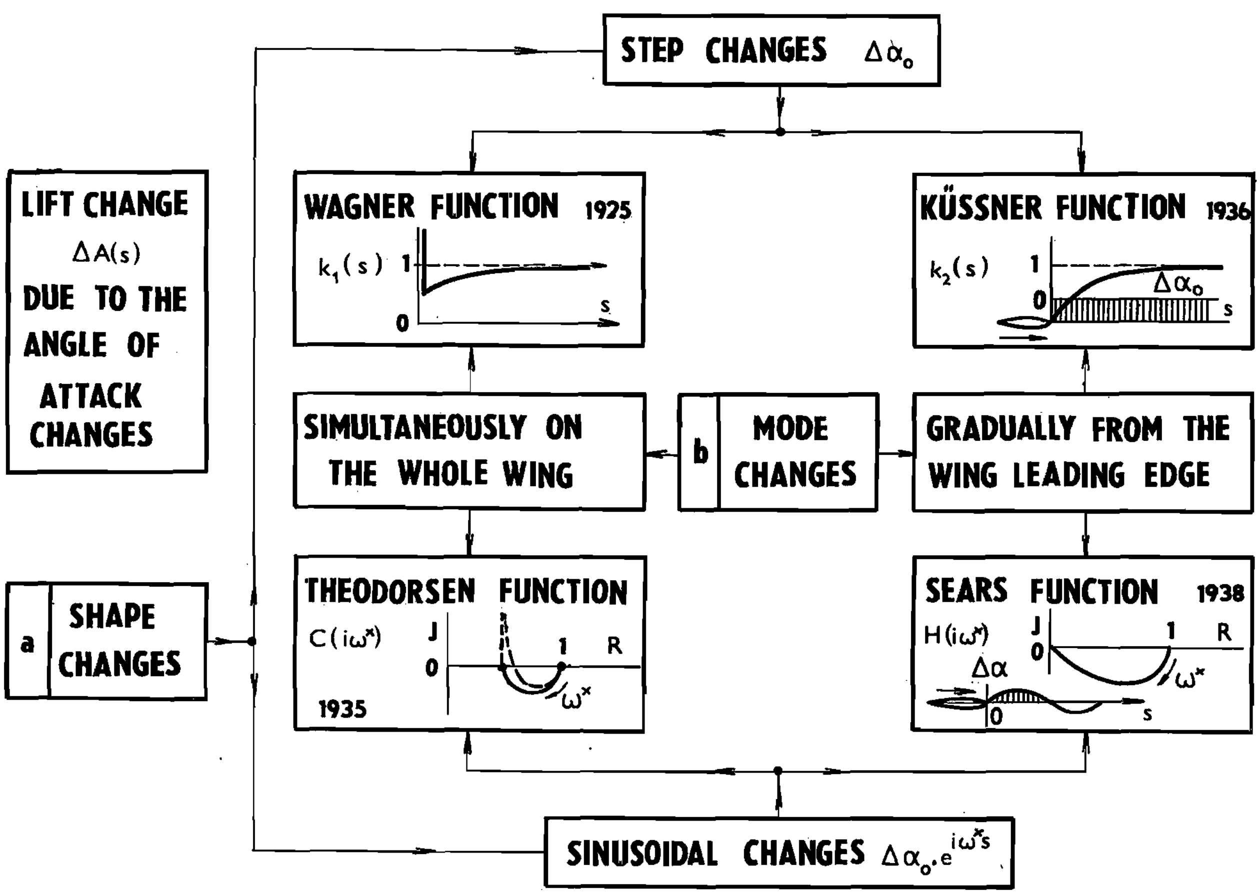


FIGURE 5 – CLASSIFICATION OF THE LIFT CHANGES IN UNSTEADY AERODYNAMICS

appears more often and tends to appear in groups. For this last case a global criterion of closeness was derived in ref.[4] to make easier the decision about the suitability of the model hypothesis. This criterion follows from the t-tests of the zero hypothesis for individual levels j when using the relation $\sum_{j=1}^{k} (t_s)_j^2$.

For this reason the estimates of the following variance must be known:

a/ the estimate of the average variance of the measured values round the mean values y_{Mj} on every level j, which is called the "experimental variance", and is given by the relation

$$s_E^2 = s_{Ej}^2/k = \sum_j \sum_\nu \varepsilon_{Ej\nu}^2/k_E \qquad (8)$$

b/ the estimate of the average variance of the mean values y_{Mj} round the conditionally deterministic values $\hat{y}_{Hj}$ on every level j, which is called the "identification variance" and is given by the relation

$$s_J^2 = n_j \sum_j \varepsilon_{Jj}^2/k_J \qquad (9)$$

In the relations (8) and (9) the symbols denote: $k_E = k(n_j - 1)$, $k_J = k - n_{K_r}$ .. degrees of statistical freedom; $\varepsilon_{Ej\nu} = y_{Ej\nu} - y_{Mj}$, $\varepsilon_{Jj} = y_{Mj} - \hat{y}_{Hj}$ ..experimental and identification errors. For the frequency transfer functions the number of levels is 2k instead of k.

In the case when $s_J^2 > s_E^2$, the global closeness test is given by the relation:

$$F = s_J^2/s_E^2 \leqq F_{\alpha;k_J,k_E} \qquad (10)$$

The test quantity F has the F-distribution and the critical value $F_{\alpha;k_J,k_E}$ for the statistical significance degree α and for statistical freedom degrees k_J and k_E.

Other supplementary statistical tests are given in ref. [4].

3. A Model of Nonstationary Aerodynamics of an Aeroplane.

3.1 A Short Introduction into the Problem.

The research of nonstationary aerodynamics problems was started in 1923. A detailed summary of literature on this problem until 1940 is in ref. [5]. A scheme of classification of nonstationary aerodynamics is seen in a block-diagram on fig. 5. Two classification points of vief are considered:

a/ the point of view of the shape of the angle of attack change, i.e. the step or sinusoidal changes as well;

b/ the point of view of the mode of originating the angle of attack on the wing, i.e. the change begins instantaneously on the whole wing or it penetrates gradually from the leading edge.

The dimensionless similarity parameter for these phenomena is the Strouhal number. It is defined in the time domain by $s = Vt/l = t/\tau_A$ and in the frequency domain by $\omega^x = \omega\tau_A$, where $\tau_A = l/V$ is the aerodynamic time unit.

In the fig.5, there are shown for a two-dimensional incompressible flow four basic function:

a/ for a step change of angle of attack the Wagner function $k_1(s)$ from 1925 and the Küssner function $k_2(s)$ from 1936; they are both normalized transition functions, which are called also aerodynamic admittances or indicial lift functions;

b/ for a sinusoidal change the Theodorsen function $C(i\omega^x)$ from 1935 and the Sears function $\varphi(i\omega^x)$ from 1940, which are aerodynamic normalized dimensionless frequency transfer functions.

In the fig. 5 the functions for a gradual angle of attack change are considered with the coordinates origin placed in the leading edge in contradiction to their original form with the origin placed in the middle of the chord. The modified Sears function is also

$$H(i\omega^x) = \varphi(i\omega^x).e^{-i\omega^x/2} \qquad (11)$$

The symbols in fig. 5 and in the following text were arranged according to current practice in dynamics of flight and according to the Standard ISO 1151.

The change of the lift force for a unit span is defined by means of the mentioned functions as follows:

$$\Delta A_j(s) = \frac{\varrho V^2}{2} \cdot l.2\pi.k_j(s).\Delta\alpha \quad [Nm^{-1}] \qquad (12)$$

where j = 1,2 (the Wagner or Küssner functions) and l is a wing chord lenght;

$$\Delta A(s) = \frac{\varrho V^2}{2} \cdot l.2\pi. \left\langle \begin{matrix} C(i\omega^x) \\ H(i\omega^x) \end{matrix} \right\rangle \cdot \Delta\alpha_o.e^{i\omega^x s}, \qquad (13)$$

where $\Delta\alpha_o$ are the angle of attack amplitudes. For a wing with a finite aspect ratio the derivative $C_{A.\alpha}$ instead of 2π is to be used. Between the admittances $A_{A.\alpha}(s)$ and frequency transfer functions $F_{A.\alpha}(i\omega^x)$, which according to fig. 5 correspond one to another, the following relation is valid:

$$\mathcal{F}\{A_{A.\alpha}(s)\} = F_{A.\alpha}(i\omega^x)\frac{1}{i\omega^x}. \qquad (14)$$

For the lift change $\Delta A(t)$ due to an arbitrary angle of attack change $\Delta\alpha(t)$ the convolutory integral will be used in the time domain and the frequency domain the following simple relation is valid:

$$\Delta\bar{A}(i\omega^x) = F_{A.\alpha}(i\omega^x).\Delta\bar{\alpha}(i\omega^x), \qquad (15)$$

which will be used further because it is simple and convenient for identification.

The analytical expressions for $k_2(s)$ and $k_1(s)$ are very complicated functions. They are therefore used to be replaced in analytical solutions in flight dynamics and in identification by one or more exponential functions of the form

$$k_j(s) = 1 - \sum_{i=1}^{n} {}^{\circ}c_{ji}.e^{-b_{ji}^x.s}, \qquad (16)$$

where j = 1,2 (the Wagner or Küssner function). This exponential function after a Fourier transformation has a very simple form:

$$\mathcal{F}\{{}^{\circ}C_{ji}\cdot e^{-b_{ji}^{x}\cdot s}\} = {}^{\circ}C_{ji}\frac{i\omega^{x}}{b_{ji}+i\omega^{x}} = {}^{\circ}C_{ji}\frac{i\omega^{x}T_{ji}^{x}}{1+i\omega^{x}T_{ji}^{x}} \qquad (17)$$

The original aerodynamic admittances and aerodynamic frequency transfers in the approximate exponential form were in the fifties extended also for different wing shapes, for different aspect ratios and for the compressible flow. The original Wagner and Theodorsen functions, which were derived on the basis of velocity circulation on the wing section, were supplemented by terms describing the effect of the air carried into motion by a moving wing, e.g. at a step change of the wing position or at its oscillations. This change of the lift component was called the inertial one ΔA_i in contradiction to the circulatory lift component ΔA_c. A summary of references to these problems is given in ref.[14].

The modern computer techniques have opened new possibilities for improving methods to estimate normalized dimensionless complex coefficients of generalised aerodynamic forces and moments for nonrigid aeroplane of different shapes. The references are summerised in ref.[15]. Nevertheless, for analytical solutions in dynamics of flight and in identification of conventional aeroplanes motions, it is sometimes more convenient to use approximate analytical expressions, as the author has shown in ref. [1] already and whate more recently also in ref.[2] and [3] was shown.

In further chapters, from the four mentioned aerodynamic normalized dimensionless functions the aerodynamic frequency transfer functions will be used, the basis of which are the Theodorsen and Sears functions respectively.

3.2 The Realisation of the Angle of Attack Changes.

The angle of attack is defined as an angle between two semi-straight-lines, one of them being the direction of the relative velocity vector of the air and the second one the direction of a reference axis fixed on the wing or on the aeroplane. An angle of attack change may be thus effected by two modes:

a/ by a rotation of just one of these directions;

b/ by a simultaneous rotation of both these directions, see fig. 6.

As each of these directions is given by a physically different object one can expect that each of the mentioned modes of the angle of attack changes will display aerodynamically in a different way.

The change of the angle of attack due to the change of orientation (attitude) of the reference axis of the aeroplane was called the "attitude" change $\Delta\alpha_\Theta = \Delta\Theta$.

A rotation of the velocity vector may be realised in different ways. In aeroelasticity it is usually done by shifting a wing section perpendicularly to the velocity vector direction, e.g. when bending the wing, in flight dynamics it may be done by shifting the whole aeroplane. Because this shifting means a change of the airpath inclination angle $\Delta\gamma = -\frac{\Delta\dot{z}_a}{V} = -\Delta\alpha_\gamma$, this change of angle of attack was called "path" change $\Delta\alpha_\gamma = -\Delta\gamma$.

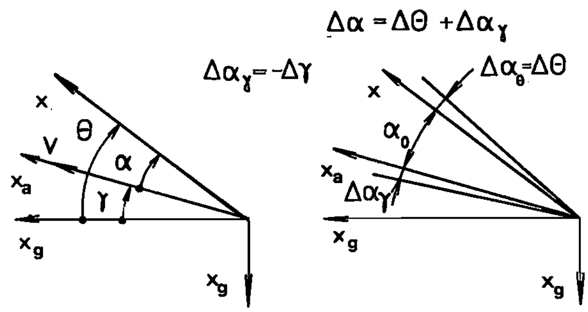


FIGURE 6 – SCHEME DIAGRAM OF ANGLES

Therefore to describe the angle of attack change $\Delta\alpha = \Delta\Theta - \Delta\gamma = \Delta\alpha_\Theta + \Delta\alpha_\gamma$, two generalised coordinates $\Delta\Theta$ and $\Delta\alpha_\gamma$ are needed.

In aerodynamics, in aeroelasticity and sometimes also in flight dynamics, the two generalised coordinates $\Delta\Theta$ and Δz_a are used, i.e. a rotation of the wing (aeroplane) reference axis and a translation of the origin of this axis. A disadvantage of the coordinates $\Delta\Theta$ and Δz_a is in the fact that they are dimensionally and physically not homogeneous and analytical expressions for aerodynamic forces and moments excited by them are not comparable.

In flight dynamics the two generalised coordinates $\Delta\Theta$ and $\Delta\alpha = \Delta\Theta + \Delta\alpha_\gamma$ are used as well or sometimes with suspect to the possibility of measurement the quantities $\Delta\dot{\Theta} = \omega_y$ and $a_{z_a} = \Delta\ddot{z}_a \doteq V.\Delta\dot{\alpha}_\gamma$ are also used.

In the following aerodynamic considerations the use of physically homogeneous pair of the angle of attack changes $\Delta\Theta$ and $\Delta\alpha_\gamma$ is preferred. Expressions for aerodynamic responses may be easily transformed to other generalised coordinates as well.

3.3 A Simple Model of the Whole Aeroplane.

At a nonstationary longitudinal motion of an aeroplane in calm atmosphere the changes of the lift and pitching moment coefficients are expressed by approximate relations which after the Fourier integral transformation have the form:

$$\Delta\overline{C}_A = F_{C_A\cdot\alpha_\gamma}(i\omega^x)\,\Delta\overline{\alpha}_\gamma + F_{C_A\cdot\Theta}(i\omega^x)\Delta\overline{\Theta} + F_{C_A\cdot\eta}(i\omega^x)\Delta\eta \qquad (18)$$

$$\Delta C_m = F_{C_m\cdot\alpha_\gamma}(i\omega^x)\Delta\overline{\alpha}_\gamma + F_{C_m\cdot\Theta}(i\omega^x)\Delta\overline{\Theta} + F_{C_m\cdot\eta}(i\omega^x)\Delta\eta \qquad (19)$$

The two first frequency transfers in each relation consist of aerodynamic frequency transfer functions of the wing and of the tailplane into which the interaction of the wing must be involved, see ref. [7]. The effect of the fuselage is not considered here. The third ones are determined just by aerodynamic transfer functions of the tailplane. To express all these transfer functions the formulae from appendices of the book by Scanlan and Rosenbaum [6] have been used. As a reference point for both the wing and the tailplane a common point in the aeroplane centre of gravity was considered. For the relative distance "a" in ref. [6] of the reference point behind the middle of the lifting surface chord length the following values have been therefore substituted:

a/ for the wing $\tilde{a}_F = a_F/l = \tilde{x}_S - 0{,}5$;

b/ for the tailplane $\tilde{\tilde{a}}_H = a_H/l_H = \tilde{a}_H/\tilde{l}_H = -(\tilde{r}_H/\tilde{l}_H + 0{,}25)$

A survey of characteristic lengths on an aeroplane is given in fig. 7.

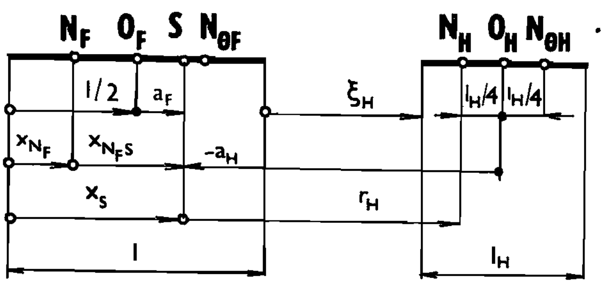


FIGURE 7 – SCHEME DIAGRAM OF LENGTHS

After having transformed the expressions from ref. [6] into contemporary symbols of flight dynamics according to the standard ISO 1151 and after having them algebraically arranged and synthetized for the whole aeroplane the expressions for aerodynamic frequency transfer functions dependent on angle of attack changes are obtained that are convenient to use in practice.

For the lift coefficient change $\Delta\bar{C}_A$ the aerodynamic frequency transfers for the whole aeroplane are of the form:

$$F_{C_A\cdot\alpha_\gamma}(i\omega^x) = F_{C_A\cdot\alpha_\gamma\cdot c}(i\omega^x) + F_{C_A\cdot\alpha_\gamma i}(i\omega^x) \qquad (20)$$

$$F_{C_A\cdot\Theta}(i\omega^x) = F_{C_A\cdot\alpha_\gamma}(i\omega^x) + i\omega^x\cdot F_{C_A\cdot\dot{\Theta}^x}(i\omega^x) \qquad (21)$$

where

$$F_{C_A\cdot\dot{\Theta}^x}(i\omega^x) = F_{C_A\cdot\dot{\Theta}^x_c}(i\omega^x) + F_{C_A\cdot\dot{\Theta}^x_i}(i\omega^x) \qquad (22)$$

The individual aerodynamic frequency transfers of the lift coefficient in (20), (21) and (22) are given in the form:

$$F_{C_A\cdot\alpha_\gamma c}(i\omega^x) = A_{F_1}\cdot C_F(i\omega^x) + A_{H_1}\left[C_H(i\omega^x) + - h_H(i\omega^x)\,\frac{d\alpha_a}{d\alpha}\cdot C_{\alpha_a}(i\omega^x)\right] \qquad (23)$$

$$F_{C_A\cdot\alpha_\gamma i}(i\omega^x) = (A_{F2} + A_{H2})\cdot i\omega^x = K_{A.\dot{\alpha}}\cdot i\omega^x \qquad (24)$$

$$F_{C_A\cdot\dot{\Theta}^x_c}(i\omega^x) = A_{F3}\cdot C_F(i\omega^x) + A_{H3}\cdot C_H(i\omega^x) \qquad (25)$$

$$F_{C_A\cdot\dot{\Theta}^x_i}(i\omega^x) = (-A_{F4} + A_{H4})\cdot i\omega^x = K_{A\dot{\Theta}}\cdot i\omega^x \qquad (26)$$

For the pitching moment coefficient change $\Delta\bar{C}_m$ the aerodynamic frequency transfers for the whole aeroplane are:

$$F_{C_m\cdot\alpha_\gamma}(i\omega^x) = F_{C_m\cdot\alpha_\gamma c}(i\omega^x) + F_{C_m\alpha_\gamma i}(i\omega^x) \qquad (27)$$

$$F_{C_m\cdot\Theta}(i\omega^x) = F_{C_m\cdot\alpha_\gamma}(i\omega^x) + i\omega^x\cdot F_{C_m\cdot\dot{\Theta}^x}(i\omega^x) \qquad (28)$$

where

$$F_{C_m\cdot\dot{\Theta}^x}(i\omega^x) = F_{C_m\cdot\dot{\Theta}^x_c}(i\omega^x) + F_{C_m\cdot\dot{\Theta}^x_i}(i\omega^x) \qquad (29)$$

The individual aerodynamic frequency transfers of the moment coefficient in (27), (28) and (29) are:

$$F_{C_m\cdot\alpha_\gamma c}(i\omega^x) = m_{F1}\cdot C_F(i\omega^x) + m_{H1}\cdot\left[C_H(i\omega^x) + - h_H(i\omega^x)\,\frac{d\alpha_a}{d\alpha}\cdot C_{\alpha_a}(i\omega^x)\right] \qquad (30)$$

$$F_{C_m\cdot\alpha_\gamma i}(i\omega^x) = (m_{F2} - m_{H2})\, i\omega^x = K_{m.\dot{\alpha}}\cdot i\omega^x \qquad (31)$$

$$F_{C_m\cdot\dot{\Theta}^x_c}(i\omega^x) = m_{F3}\cdot C_F(i\omega^x) + m_{H3}\cdot C_H(i\omega^x) \qquad (32)$$

$$F_{C_m\cdot\dot{\Theta}^x_i}(i\omega^x) = -(m_{F4} + m_{H4}) - (m_{F5} + m_{H5})\cdot i\omega^x = K_{m\dot{\Theta}^x} + K_{m\ddot{\Theta}}\cdot i\omega^x \qquad (33)$$

Expressions to calculate the constants A_{F1}, ..., m_{H5} from (23) up to (26) and (30) to (33) are given in table 1, where one can find also examples of their values for the A 145 light transport aeroplane. Main characteristics of this aeroplane are given in table 3 taken from ref. [1]. Approximate expressions for normalized dimensionless frequency transfers $C_F(i\omega^x)$, $C_H(i\omega^x)$, $h_H(i\omega^x)$ and $C_{\alpha_a}(i\omega^x)$ are presented in appendix to this paper and values of their contants for the A 145 aeroplane are to be found in table 4.

There follows from (20), (21), (27) and (28) that the frequency transfers $F_{C_{A.\Theta}}$ and $F_{C_{m.\Theta}}$ contain the transfers $F_{C_{A.\alpha_\gamma}}$ and $F_{C_{m.\alpha_\gamma}}$. One can therefore express in (18) and (19) the sum of the first two terms on the right-hand side as follows:

$$F_{C_y\alpha_\gamma}\cdot\Delta\bar{\alpha}_\gamma + (F_{C_y\alpha_\gamma} + F_{C_y.\Theta})\cdot\Delta\Theta = F_{C_y.\alpha_\gamma^x}(\Delta\bar{\alpha}_\gamma + \Delta\Theta) + F_{C_y.\dot{\Theta}^x}\cdot(i\omega^x\Delta\Theta) \qquad (34)$$

where y = A, m. It follows therefore that

WING		VALUES	TAILPLANE		VALUES
A_{F1}	a	+4,735	A_{H1}	$+a_1 k_H \tilde{S}_H$	+0,581 043
A_{F2}	$a K_{\lambda F}$	+1,263 331	A_{H2}	$+a_1 k_H \tilde{S}_H K_{\lambda H} \tilde{\tilde{l}}_H$	+0,147 983
A_{F3}	$a \tilde{x}_{SN_\Theta} = a\,(0{,}75-\tilde{x}_S)$	+2,381 705	A_{H3}	$+a_1 k_H \tilde{S}_H (\tilde{\tilde{r}}_H + 0{,}50)\tilde{\tilde{l}}_H$	+2,232 788
A_{F4}	$a K_{\lambda F}\,(\tilde{x}_S - 0{,}50)$	−0,319 623	A_{H4}	$+a_1 k_H \tilde{S}_H (\tilde{\tilde{r}}_H + 0{,}25) K_{\lambda H} \tilde{\tilde{l}}_H^2$	+0,541 815
m_{F1}	$\tilde{x}_{N_F S} \cdot a$	+0,572 935	m_{H1}	$-\tilde{r}_H a_1 k_H \tilde{S}_H$	−1,939 427
m_{F2}	$(\tilde{x}_S - 0{,}50)\, a K_{\lambda F}$	−0,319 623	m_{H2}	$+(\tilde{\tilde{r}}_H + 0{,}25) . a_1 k_H \tilde{S}_H \tilde{l}_H K_{\lambda H} \tilde{\tilde{l}}_H$	+0,519 690
m_{F3}	$\tilde{x}_{N_F S} . a . (0{,}75 - \tilde{x}_S)$	+0,288 186	m_{H3}	$-\tilde{r}_H a_1 k_H \tilde{S}_H (\tilde{\tilde{r}}_H + 0{,}50)\tilde{\tilde{l}}_H$	−7,452 683
m_{F4}	$0{,}25\, a K_{\lambda F}$	+0,315 833	m_{H4}	$+0{,}25\, a_1 k_H \tilde{S}_H K_{\lambda H} \tilde{\tilde{l}}_H$	+0,036 996
m_{F5}	$[(\tilde{x}_S - 0{,}50)^2 . K_{\lambda F} + 1/128] . a$	+0,117 857	m_{H5}	$+a_1 k_H \tilde{S}_H \tilde{l}_H [(\tilde{\tilde{r}}_H + 0{,}25)^2 . K_{\lambda H} + 1/128] . \tilde{\tilde{l}}_H$	+2,624 703
$a = \partial C_{AF}/\partial \alpha_F$		$a_1 = \partial C_{AH}/\partial \alpha_H$	$\tilde{\tilde{r}}_H = r_H / l_H = \tilde{r}_H / \tilde{l}_H$		$\tilde{\tilde{l}}_H = \tilde{l}_H / \sqrt{k_H}$
$K_{A\dot{\alpha}} = +(A_{F2} + A_{H2}) = +1{,}411\,314$;			$K_{A\dot{\Theta}} = +(-A_{F4} + A_{H4}) = +0{,}861\,438$;		
$K_{m\dot{\alpha}} = +(m_{F2} - m_{H2}) = -0{,}839\,313$;		$K_{m\dot{\Theta}} = -(m_{F4} + m_{H4}) = -0{,}352\,829$;	$K_{m\ddot{\Theta}} = -(m_{F5} + m_{H5}) = -2{,}742\,560$		

TABLE 1 – PARAMETERS OF AERODYNAMIC FREQUENCY TRANSERS. EXAMPLE FOR THE A 145 AEROPLANE

instead of the generalized coordinates $\Delta\bar{\alpha}_{\gamma}$, $\Delta\bar{\Theta}$ there may be used the generalized coordinates $\bar{\Delta\alpha} = \Delta\bar{\alpha}_{\gamma} + \Delta\bar{\Theta}$ and $i\omega^x \Delta\bar{\Theta} = \Delta\bar{\Theta}^{\cdot} = \bar{\omega}_z^x$.

It is evident from (23) to (26) and (30) to (33) that from a comparison of the corresponding coefficients A_{F1} and A_{H1}, ... , m_{F5} and m_{H5} one can estimate the participation of the wing and that of the tailplane in the considered frequency transfer.

From the given expressions the formulae for the quasi-stationary aerodynamic model may be easily deduced when taking $C_F(i\omega^x) = C_H(i\omega^x) = h_H(i\omega^x) = 1$ and when using for $C_{\alpha_a}(i\omega^x)$ the usually considered quasi-stationary model

$$C_{\alpha_a}(i\omega^x) = 1 - i\omega^x . \tau_W^x \tag{35}$$

where $\tau_W^x = \frac{\xi_W}{V_H} \cdot \frac{1}{\tau_A}$ is the transport lag of the trailing edge vortex on the path from the wing to the tailplane. The definitions of the length ξ_W (often $\xi_W = r_H$ is used) and of the velocity V_H are considered with different opinions, see ref. (2), (3), (7). The transport lag is also difficult to be estimated from measured values of the downwash, see ref. (1).

From the expressions one can also see that to the aerodynamic transfers for $\omega^x = 0$ the complex aerodynamic derivatives correspond in quasi-stationary aerodynamics, i.e. there is valid

$$F_{C_{y.x}}(0) = U_{C_{y.x}}(0) + i\omega^x V'_{C_{y.x}}(0) \tag{36}$$

and therefore

$$\bar{C}_{y.x} = C_{y.x} + i\omega^x C_{y.\dot{x}^x}$$

where $y = A$, m and $x = \alpha_{\gamma}, \Theta, \eta$. Thus the following relations are valid:

$$V_{C_{y.x}}(0) = \omega^x . V'_{C_{y.x}}(0) \text{ and } U_{C_{y.x}}(0) = C_{y.x} \text{ , } V'_{C_{y.x}}(0) = C_{y.\dot{x}^x}$$

To have an idea of aerodynamic frequency transfers some examples for the A 145 aeroplane are given in fig. 8, 9, 10. The values of complex derivatives are there drawn too. One can see here that imaginary components may be remarcably different at the circulation terms and at the resultants comprising inertial terms. The real components remain unchanged except at the transfer $F_{C_{m}.\dot{\Theta}^x}$. It is important too that the circulation terms converge to zero with the growing frequency $\omega^x \to \infty$ or to a finite real value whilst the inertial terms grow into infinity.

The correctness of aerodynamic frequency transfers depends not only on the correctness of normalized dimensionless transfers C_F, C_H, h_H, which are determined by lifting surfaces and by the Mach number, but also on the correctness of the function $C_{\alpha_a}(i\omega^x)$ which involves the evolution of the circulation on the wing, the development of the velocity field around the wing and the transport lag of the trailling edge vortices. The expression of the function C_{α_a} was taken from ref. (1) where it was proposed on a basis of the identification analysis. It is given with the constants values for the A 145 aeroplane in the appendix, see table 4.

4. Weighted Values of Complex aerodynamic Derivatives for a Longitudinal Motion of an Aeroplane at a Constant Velocity.

4.1 Basic Relations.

A longitudinal unsteady motion of a rigid aeroplane at a constant velocity that was excited by an elevator deflection at a steady flight in the calm atmosphere is described by a system of two differential equations of the second order. Their matrix form convenient from the cybernetic point of view with matrices physically homogeneous after the Fourier transformation is given in the appendix, equations (47). In the case that starting from a certain value of the Strouhal number ω^x the effect of nonstationary aerodynamics begins to be evident one must substitute in the matrices of motion equations for aerodynamic responses forces and for input forces the aerodyna-

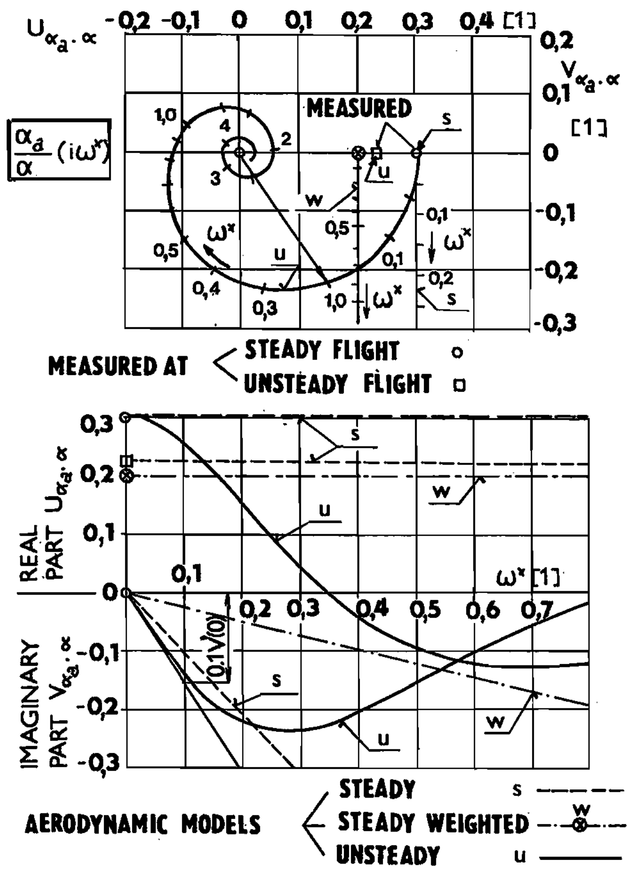
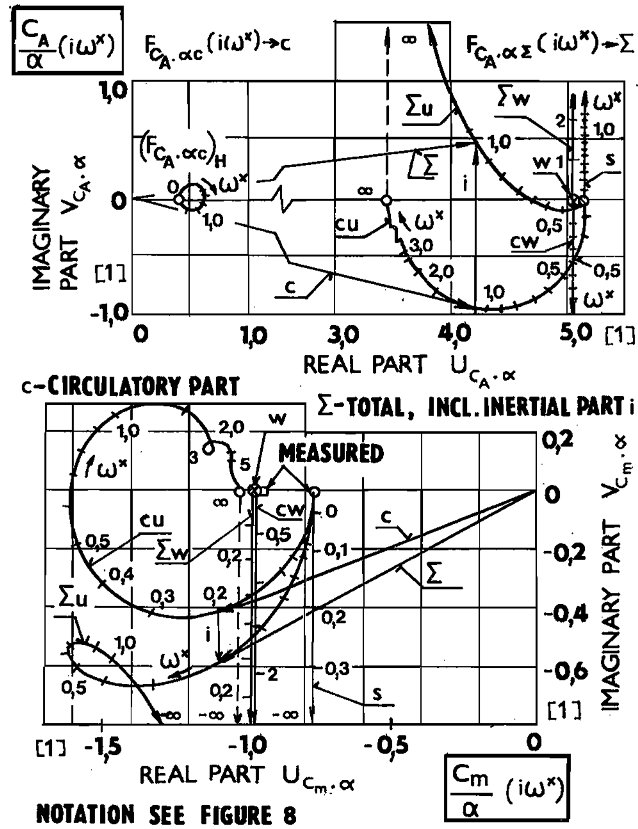


FIGURE 8 - FREQUENCY TRANSFER $F_{\alpha_a.\alpha}(i\omega^x)$ OF THE A 145 AEROPLANE



NOTATION SEE FIGURE 8

FIGURE 9 - FREQUENCY TRANSFERS $F_{C_A.\alpha}(i\omega^x)$, $F_{C_m.\alpha}(i\omega^x)$ OF THE A 145 AEROPLANE

mic frequency transfers for the aerodynamic complex derivatives.

An elevator motion may be replaced by Fourier series by means of a sum of harmonic components with a spectrum $\bar{\Delta}\eta(i\omega_j^x)$.

Every harmonic component of the deflection excites a harmonic response of the aeroplane of the form: $F_{x.\eta}(i\omega_j^x).\bar{\Delta}\eta(i\omega_j^x)$,

where $x = \alpha$, $\dot{\Theta}^x$. The effect of aerodynamic frequency transfers $F_{y.x}(i\omega_j^x)$, where $y = \alpha_a$, C_A, C_m and $x = \alpha$, $\dot{\Theta}^x$, η, may be replaced in the motion equations in a certain interval of the Strouhal number by weighted mean values of the form

$$(\bar{C}_{y.x})_W = (u_{y.x})_W + i\omega^x (v_{y.x})_W \tag{37}$$

The values of these complex derivatives may be defined as follows

$$\sum_j [(u_{y.x})_W + i\omega^x (v_{y.x})_W].\bar{\Delta}x(i\omega_j^x) = \sum_j F_{y.x}(i\omega_j^x).\bar{\Delta}x(i\omega_j^x) \tag{38}$$

where $\bar{\Delta}x(i\omega_j^x) = F_{x.\eta}(i\omega_j^x).\bar{\Delta}\eta(i\omega_j^x)$

are weighted functions which are determined by the frequency transfers of the aeroplane responses

$$\frac{\Delta x}{\Delta \eta}(i\omega_j^x) = U_{x.\eta}(\omega_j^x) + i\,V_{x.\eta}(\omega_j^x)$$

and by the spectrum of the elevator deflection $\Delta\bar{\eta}(i\omega_j^x)$.

From (38) after rearranging one gets a system of two algebraic equations with unknowns $(u_{y.x})_W$ and $(v_{y.x})_W$ into form:

$$\begin{bmatrix} a_1, b_1 \\ a_2, b_2 \end{bmatrix} \cdot \begin{bmatrix} u_W \\ v_W \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad (39)$$

in which the coefficients are:

$$a_1 = \sum_j a_1'(\omega_j^x), \qquad a_2 = \sum_j a_2'(\omega_j^x), \qquad (40a,b)$$

$$b_1 = -\sum_j \omega_j^x a_2'(\omega_j^x), \quad b_2 = +\sum \omega_j^x a_1'(\omega_j^x), \qquad (41a,b)$$

$$c_1 = \sum_j c_1'(\omega_j^x), \qquad c_2 = \sum_j c_2'(\omega_j^x) \qquad (42a,b)$$

where the real and imaginary parts of the weighted function $\Delta\bar{x}(i\omega_j^x)$ are:

$$a_1' = \Delta\bar{x}_R(\omega_j^x) = U_{x.\eta}\cdot\Delta\bar{\eta}_R - V_{x.\eta}\cdot\Delta\bar{\eta}_J \qquad (43a)$$

$$a_2' = \Delta\bar{x}_J(\omega_j^x) = U_{x.\eta}\cdot\Delta\bar{\eta}_J + V_{x.\eta}\cdot\Delta\bar{\eta}_R \qquad (43b)$$

and

$$c_1' = U_{y.x}\cdot\Delta\bar{x}_R - V_{y.x}\cdot\Delta\bar{x}_J \qquad (43c)$$

$$c_2' = U_{y.x}\cdot\Delta\bar{x}_J + V_{y.x}\cdot\Delta\bar{x}_R \qquad (43d)$$

The coefficients of (39) contain response transfers of the aeroplane $F_{x.\eta}(i\omega^x)$ that are given in the appendix and aerodynamic frequency transfers $F_{y.x}(i\omega^x)$ given in chapter 3.2 . The spectra of elevator deflections may be very various. The extreme cases are the unit impulse $\Delta\bar{\eta}(i\omega^x) = 1$ and the step change $\Delta\bar{\eta}(i\omega^x) = 1/i\omega^x$. In practise a triangular impulse is often found, see fig. 11.

From the eq. (40) up to (43) one can see the weighted derivatives values for a given aeroplane to be dependent on the elevator deflections spectrum. The values of these derivatives with an increasing interval of the Strouhal number $\omega_W^x \in \langle 0;1,5\rangle$ are convergent or at least semiconvergent within the extent of circular frequencies which may be considered for the short-period motion of a rigid aeroplane as a whole.

4.2 An Example for the A 145 Aeroplane

The mass and geometrical characteristics of the A 145 light two-engined transport aeroplane have been taken from ref. (1) and are given in the appendix, table 3 . Coefficients values of other used aerodynamic functions are presented in table 4 . The calculations were started with aerodynamic derivatives estimated from measure-

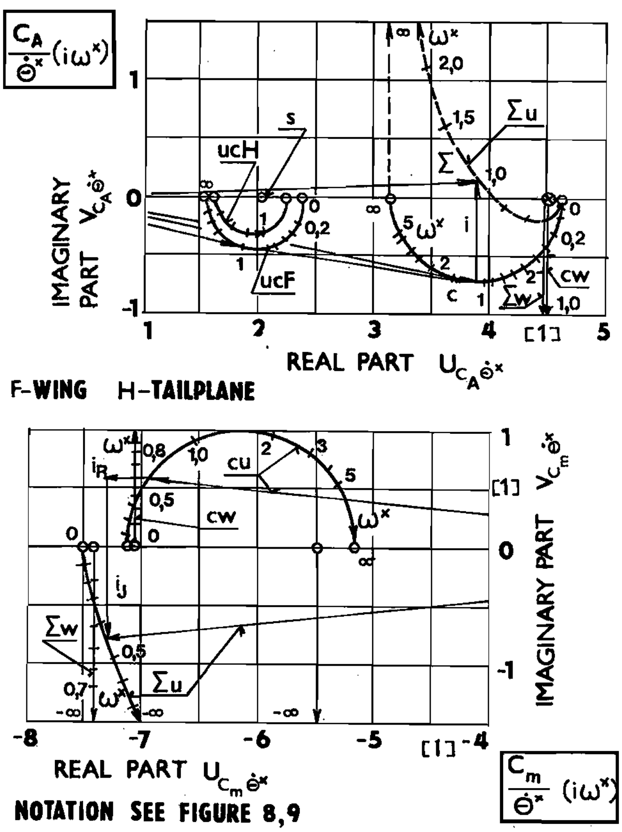


FIGURE 10 - FREQUENCY TRANSFERS $F_{C_A\dot\Theta^x}(i\omega^x)$, $F_{C_m\dot\Theta^x}(i\omega^x)$ OF THE A 145 AEROPLANE

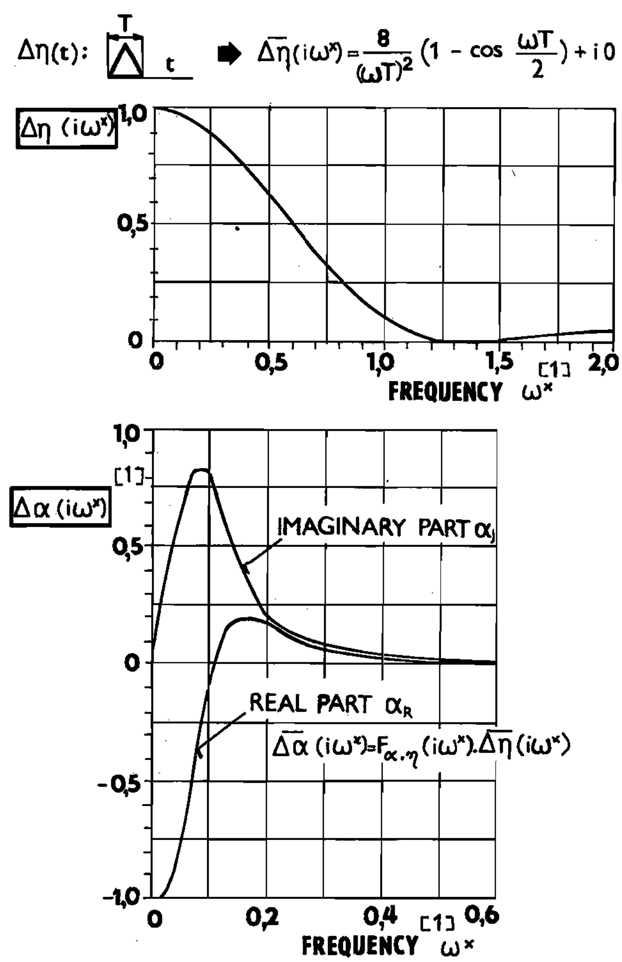


FIGURE 11 - EXAMPLES OF WEIGHT FUNCTIONS OF THE A 145 AEROPLANE

ments at steady flights. The efffective downwash angle of the tailplane was computed from measurements of aerodynamic forces on the tailplane at steady and unsteady flights by means of straingages. The time constants of its transfer were estimated from measurements by means of vane at unsteady flights, see ref. [1].

The calculations were performed on the HP 9825 S table computor. The results are presented in table 2 together with aerodynamic derivatives and transfer functions of the other origin. For information the real and imaginary parts of the weighted complex derivatives are plotted in diagrams of aerodynamic frequency transfers, see fig. (8), (9), (10). Information about the convergence may be found in fig. (12), (13). There it is evident that starting from the limit frequency $\omega_w^x = 0{,}8$ the weighted derivatives are practically independent on the frequency interval used in the calculation.

One can see from table 2 and from the given diagrams that the real parts of deviations from the values of derivatives derived from steady flights are of the same sense as are the values deviations from unsteady flights measurements which were evaluated by the quasi-stationary model (as far as they could be estimated due to ill-conditioned equations). Some differencies of the real parts of the weighted derivatives from the derivatives derived from unsteady flights and estimated by means of the quasi-stationary model might be caused by the fact that the normalized dimensionless transfer function $C_F(i\omega^x)$ corresponds to the wing aspect ratio $\lambda = 6$ whilst in fact the aspect ratio is $\lambda_F = 8{,}78$. Besides, an uncertainty is in value of the time constant T_1^x of the transfer function $C_{\alpha_a}(i\omega^x)$, see ref. [1]. The greatest effect of the Strouhal number on the real parts of the weighted derivatives is seen at the derivative $(d\alpha_a / d\alpha)_w$, where the deviation is −33,6 per cent and the next one is the derivative $(C_{m.\dot{\alpha}^x})_w$ where the deviation

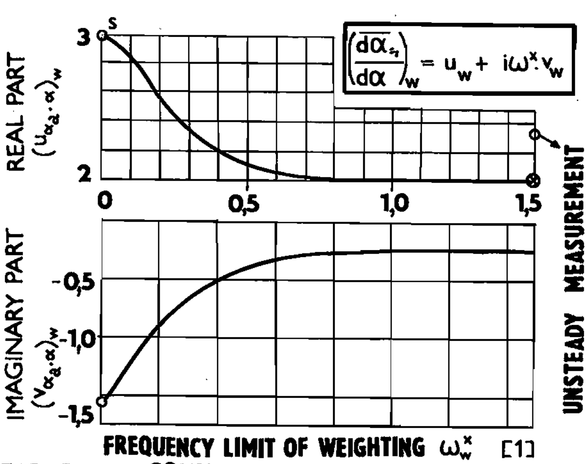


FIGURE 12-CONVERGENCY OF WEIGHTED DERIVATIVE $(d\alpha_a/d\alpha)_w$ OF A 145 AEROPLANE

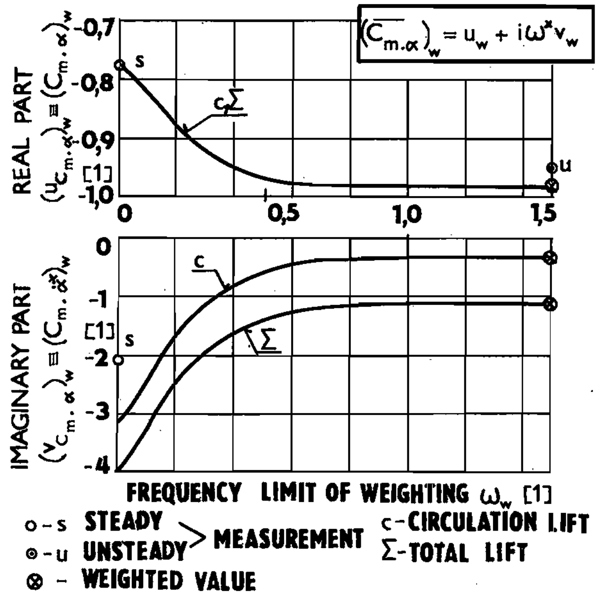


FIGURE 13-CONVERGENCY OF WEIGHTED DERIVATIVE $(C_{m.\dot{\alpha}})_w$ OF A 145 AEROPLANE

is +25,9 percent. At the real parts of other weighted derivatives the absolute values of deviations are in the extent of 1 up to 3 percent. The least deviations are found at the derivatives $(C_{A.\dot{\alpha}^x})_w$ and $(C_{A.\dot{\delta}_\Sigma})_w$.

The effect of the Strouhal number on the imaginary parts of the weighted derivatives is substantially greater then on the real ones. A comparison with values stated at unsteady flights could not be effected as it was not possible estimate them reliably from an ill-conditioned equations system. At the imaginary part of the derivative $(d\alpha_a / d\alpha)_w$ the deviation is +83,6 percent and at the derivative $(C_{m.\dot{\alpha}^x})_w$ it is −89,6 percent. At the imaginary parts of the other weighted derivatives the absolute values of deviations lie in the extent of 13 to 37 percent. When considering the inertial component of the lift one can observe great deviations at the derivatives $(C_{A.\ddot{\alpha}^x\Sigma})_w$ and $(C_{A.\ddot{\delta}^x\Sigma})_w$ which are usually neglected in the motion equations when a quasi-stationary aerodynamic model is used.

5. Conclusions.

The identification and its mathematical-physical variant are considered here to be a method of recognition the substance of phenomena or of structure of systems. If at verifying the identity of a considered attribute of a system and of its model a significant difference is found out then it can be caused by the incorrect model just when by the identification process the absence of significant uncorrectable systematic errors has been proved. For this purpose a testing quantity for systematic er-

QUANTITY		CALCULATED								FLIGHT MEASUREMENTS(1)			
		FROM FREQUENCY TRANSFERS x)				WEIGHTED VALUES FOR $\omega^x \in \langle 0;1,5 \rangle$				STEADY		UNSTEADY	
		$U_{y.x}(0)$		$V'_{y.x}(0)$		$(u_{y.x})_w$ xxx)		$(v_{y.x})_w$ xxx)		$C_{y.x}$ ‡	$C_{y.x}$ ‡	$C_{y.x}$ ‡ +)	$C_{y.x}$ ‡ +)
y	x	c xx)	Σ xx)	c	Σ	c	Σ	c	Σ				
α_a		+0,304	—	-1,557	—	+0,202	—	-0,255	—	+0,304	-1,058	+0,234	—
C_A	α	+5,139		-1,398	+0,013	+5,081		-1,064	+0,347	+5,140	+0,615 ++	+5,... +++	—
C_m		-0,777		-3,091	-3,930	-0,978		-0,322	-1,161	-0,777	-2,052 ++	+0,946	× +++
C_A	$\dot{\Theta}^x$	+4,614		-1,553	-0,691	+4,481		-0,983	-0,122	—	+2,022 ++	—	—
C_m		-7,164	-7,517	+1,280	-1,462	-7,083	-7,435	+1,022	-1,720	—	-7,423 ++	—	× +++
C_A	η	+0,341	—	-0,065	—	+0,333	—	-0,056	—	+0,341	—	+0,318	—
C_m		-1,138	—	+0,216	—	-1,113	—	+0,187	—	-1,201	—	-1,060	—

x) $F_{y.x}(i\omega^x) = U_{y.x}(\omega^x) + i\omega^x V'_{y.x}(\omega^x)$ FOR $\omega^x = 0$; xx) c – CIRCULATION PART, Σ – TOTAL INCL. INERTIAL PART

xxx) $(\bar{C}_{y.x})_w = (u_{y.x})_w + i\omega^x (v_{y.x})_w$; +) $\bar{C}_{y.x} = C_{y.x} + i\omega^x C_{y.\dot{x}^x}$, $C_{y.\dot{x}^x} \equiv V'_{y.x}(0)$; ++ CALCULATED BY STEADY AERODYNAMIC MODEL FROM $(a_1 k_H)$ MEASURED AT STEADY FLIGHTS +++) ILL CONDITIONED EQUATIONS – $C_{m.\dot{\alpha}^x} + C_{m.\dot{\Theta}^x} = -8,753$; $(C_{m.\dot{\alpha}^x} + C_{m\dot{\Theta}^x})_{\Sigma w} = -7,435 - 1,161 = -8,596$ ‡ $y = \alpha_a, A, m$; $x = \alpha, \dot{\Theta}^x, \eta$.

TABLE 2 - SUMMARY OF DERIVATIVES VALUES OF THE A 145 AEROPLANE

rors and a global test of identity or of closeness are suggested.

To facilitate analysis of aerodynamic models a classification of nonstationary aerodynamics to two criteria was given. To improve the physical clearness there was suggested to distinguish "attitude" and "path" changes of angle of attack. For these two sorts of angle of attack comparable expressions have been deduced for aerodynamic frequency transfer functions of the whole aeroplane. To this aim , normalized dimensionless transfers have been used for the wing, for the tailplane and for the interaction of the wing on the tailplane. From the expressions one can see the frequency transfer for the "attidude" changes of angle of attack to be given by a sum of the frequency transfer for the "path" changes of angle of attack and of the damping frequency transfer for a time change of the attitude angle of attack. From coefficients in these expressions the participation of the wing and of the taiplane including the interaction is evident on the global frequency transfer of the aeroplane. From these expressions one can also easily derive the formulae for complex derivatives of an aeroplane in a form which is more correct than that customary used at a quasi-stationary model.

To prove the effect of the Strouhal number on complex aerodynamic derivatives the weighted values of these derivatives have been introduced. The appertaining weight functions are determined by the frequency spectrum of a time history of the elevator deflection (e.g. a triangular impuls) and by the frequency transfer functions of the relevant responses of an aeroplane. The weighted complex derivatives show in which direction and to what extent their values measured at unsteady flights when using a model of quasi-stationary aerodynamics may be expected to differ from the derivatives values stated from measurements at steady flights. They make therefore possible to decide at the identification whether a cause of the significant difference of the aerodynamic derivatives values stated from measurements at steady or unsteady flights may be the Strouhal number effect. Further the weighted complex derivatives make possible to estimate flying qualities by means of a quasi-stationary model but with the derivatives values changed by the Strouhal number effect according to the spectral contents of the time history of the elevator deflection.

Acknowlegement.

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6. Appendix

The data of the A 145 aeroplane are taken from ref. [1] and are given in the table 3 .

m	kg	1530	v	m / s	54,20
$\tilde{i}_H^2$	1	0,859	H	m	1390
$\tilde{x}_S$	1	0,247	ϱ	kg/m³	1,070
μ	1	112,96	τ_A	s	0,0273
$\tau = \mu\tau_A$	1	3,084	$1/\tau_A$	s^{-1}	36,622
$\tilde{x}_{N_F}$	1	0,126	$\tilde{x}_{SN_\Theta}$	1	0,503
$\tilde{x}_{N_F S}$	1	0,121	$\tilde{\xi}_H$	1	2,373
l	m	1,480	l_H	m	1,030
b	m	12,25	b_H	m	3,39
λ	1	8,78	λ_H	1	3,47
S	m²	17,09	S_H	m²	3,31
r	m	5,119	r_H	m	4,940
ξ	m	3,770	ξ_H	m	3,512
$\tilde{l}_H$	1	0,696	$\tilde{\tilde{l}}_H$	1	0,726
$\tilde{r}_H$	1	3,338	$\tilde{\tilde{r}}_H$	1	4,796
$\tilde{S}_H$	1	0,194	$\tilde{\tilde{r}}_H + 0{,}25$	1	5,046
$\tilde{S}_H\,\tilde{r}_H$	1	0,646	$\tilde{\tilde{r}}_H + 0{,}5$	1	5,296

TABLE 3 – CHARACTERISTICS OF THE A 145 SMALL TWIN-ENGINED AEROPLANE AND OF STEADY FLIGHTS

The aerodynamic normalized dimensionless transfer functions C_F , C_H , H_H according to eq. (17) are given by a formulae:

$$C(i\omega^x) = 1 - \sum_i {}^\circ C_i \frac{i\omega^x T_i^x}{1 + i\omega^x T_i^x} \qquad (44)$$

The function h_H is calculated to the point located in 0,25 l_H by the expression:

$$h_H(i\omega^x) = H_H(i\omega^x) \cdot e^{+i\omega^x T_{H0,25}}, \qquad (45)$$

$$T_{H0,25} = 0{,}25\, \tilde{l}_H / \sqrt{k_H} \;.$$

For the normalized dimensionless function for the downwash angle the formula is taken from ref. [1] in the form:

	λ	i	${}^\circ c_i$ [1]	b_i^* [1]	T_i^* [1]
C_F	6	1	0,361	0,762	1,312
C_F	3	1	0,283	1,080	0,926
C_H	3	1	0,283	1,488	0,672
H_H	3	2	0,679	1,538	0,650
			0,227	8,821	0,113
h_H	$T^*_{H\,0,25} = 0{,}25 \cdot l_H = 0{,}182$				
C_{α_a}	$T_1^* = 0{,}075/\tau_A = 2{,}747$				
	$\tau_W^* = 0{,}065/\tau_A = 2{,}380$				
$K_{\lambda F}$	0,267			$K_{\lambda H}$	0,351

TABLE 4 – COEFFICIENTS OF NORMALIZED AERODYNAMIC FREQUENCY TRANSFERS

$$C_{\alpha_a}(i\omega^x) = \frac{1}{1 + i\omega^x T_1^x} \cdot e^{-i\omega^x \tau_w^x} \qquad (46)$$

The coefficients values in eq. (44) up to (46) for the aeroplane A 145 according to the ref. [14] are given in the table 4 .

The motion equations system for a quasi-stationary aerodynamic model has the following dimensionless form:

$$\left\{(i\omega)^2 \mu \begin{bmatrix} 0, & 0 \\ 0, & \tilde{i}_y^2 \end{bmatrix} + i\omega^x \mu \begin{bmatrix} +1, & -1 \\ 0, & 0 \end{bmatrix} + \begin{bmatrix} \cancel{C_{Wo}}, & -\cancel{C_{Wo}} \\ 0, & 0 \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} (C_{A.\alpha} + i\omega^x C_{A.\dot{\alpha}^x}), & i\omega^x \cancel{C_{A.\dot{\Theta}^x}} \\ (C_{m.\alpha} + i\omega^x C_{m.\dot{\alpha}^x}), & i\omega^x C_{m.\dot{\Theta}^x} \end{bmatrix} \right\} \cdot \begin{bmatrix} \Delta\bar{\alpha} \\ \Delta\bar{\Theta} \end{bmatrix} =$$

$$= - \begin{bmatrix} C_{A.\eta} + i\omega^x \cancel{C_{A.\dot{\eta}^x}} \\ C_{m.\eta} + i\omega^x \cancel{C_{m.\dot{\eta}^x}} \end{bmatrix} \cdot \Delta\bar{\eta} \qquad (47)$$

By solving the equations system (47) one obtains expressions for frequency transfer functions for the aeroplane responses of the form:

$$F_{r.\eta}(i\omega^x) = \frac{\tilde{K}_{ro} + i\omega^x \tilde{K}_{r1}}{\tilde{K}_o + i\omega^x \tilde{K}_1 + (i\omega^x)^2} \qquad (48)$$

where $r = \alpha, \dot{\Theta}^x$

The transfer coefficients in the dimensionless form are expressed as follows:

$$\mu^2 \tilde{i}_y^2 \cdot \tilde{K}_o = -\mu C_{m.\bar{\alpha}} C_{A.\alpha} \cdot C_{m.\dot{\Theta}^x} \qquad (49a)$$

$$\mu.\tilde{K}_1 = C_{A.\dot{\alpha}} - (C_{m.\dot{\alpha}^x} + C_{m.\dot{\theta}^x})/\tilde{i}_y^2 \quad (49b)$$

$$\mu^2 \tilde{i}_y^2.\tilde{K}_{\alpha 0} = -C_{A.\eta}(\mu \tilde{r}_H - C_{m.\dot{\theta}^x}) \quad (50a)$$

$$\mu.\tilde{K}_{\alpha 1} = -C_{A.\eta} \quad (50b)$$

$$\mu^2 \tilde{i}_y^2.\tilde{K}_{\dot{\theta}^x 0} = -\tilde{r}_H C_{A.\eta}(C_{A.\alpha} + C_{m.\alpha}/\tilde{r}_H) \quad (51a)$$

$$\mu^2.\tilde{i}_y^2.K_{\dot{\theta}^x 1} = -\tilde{r}_H C_{A.\eta}(\mu + C_{m.\dot{\alpha}^x}/\tilde{r}_H) \quad (51b)$$

The transfer coefficients values in the dimensionless form are given in table 5 .

		$\tilde{K}_0$ [1]	$\tilde{K}_1$ [1]	$\tilde{K}_{r0}$ [1] x)	$\tilde{K}_{r1}$ [1] x)
$F_{\alpha.\eta}(i\omega^x)$	† c	+0,011 53	+0,146 06	-0,011 96	-0,003 012
$F_{\dot{\theta}^x.\eta}(i\omega^x)$				-0,000 510	-0,011 65
$F_{\alpha.\eta}(i\omega^x)$	‡ m	+0,009 947	+0,135 17	-0,011 56	-0,002 812
$F_{\dot{\theta}^x.\eta}(i\omega^x)$		+0,008 940	+0,124 79	-0,000 393	-0,010 51

† CALCULATED FROM STEADY FLIGHT MEASUREMENTS
‡ UNSTEADY FLIGHT MEASUREMENTS
x) $r = \alpha, \dot{\theta}^x$

TABLE 5 - COEFFICIENTS OF FREQUENCY TRANSFERS OF RESPONSES $\alpha, \dot{\theta}^x$ OF THE A 145 AEROPLANE

7. Symbols

A , A_F, A_H — Lift of aeroplane, of wing or tailplane respectively

$A_{y.x}(s)$ — Unit step admittance- response of the y quantity on a unit step change of the x quantity

$C_A = A/qS$ — Lift coefficient of aeroplane

$C_m = M/qSl$ — Pitching moment coefficient

$C_{y.x} = \frac{\partial C_y}{\partial x}$ — Aerodynamic derivative, $y = A, m$ or A_F, A_H ; $x = \alpha, \alpha_\gamma, \Theta, \eta ; \dot{\alpha}^x, \dot{\theta}^x, \dot{\eta}^x ; \alpha_F, \alpha_H$.

C_W — Drag coefficient

$C(i\omega^x)$ — Aerodynamic normalized nondimensional transfer functions of the Theodorsen type

$C_{\alpha_a}(i\omega^x)$ — Normalized nondimensional transfer functions for the downwash angle

$F_{y.x}(i\omega^x) = U_{y.x}(\omega^x) + iV_{y.x}(\omega^x)$ — Frequency transfer function of the response y on the input x or u

$F_{C_{y.x}}(i\omega^x) = U_{C_{y.x}}(\omega^x) + i\omega^x V'_{C_{y.x}}(\omega^x)$ — Aerodynamic transfer function , $y = A, m, A_F, A_H$; $x = \alpha, \alpha_\gamma, \Theta, \eta ; \dot{\theta}^x$.

$H(i\omega^x)$, $h(i\omega^x)$ — Aerodynamic normalized nondimensional transfer functions of the Sears type

$\tilde{i}_y = \sqrt{J_y/ml^2}$ — Nondimensional moment of inertia about the y-axis

$k_H = q_H/q$

l — Length of aerodynamic mean chord - aeroplane reference length , m

m — Aeroplane mass , kg

$q = \rho V^2/2$ — Kinetic pressure , N/m

r_H — Distance between aeroplane c.g. and tail aerodynamic centre, m

$s = Vt/l$ — Strouhal number

S , S_H — Wing or tailplane area, m

V — True velocity of an aeroplane, m/sec

$\alpha, \alpha_F, \alpha_H$ — Angle of attack of aeroplane, wing or tailplane respectively

α_a — Downwash angle (positive in oposite sign of α)

$\Delta\alpha_\gamma, \Delta\alpha_\Theta$ — "Path" or "attitude" change of angle of attack

γ — Flight path inclination angle

Θ — Aeroplane inclination angle

η — Elevator angle

$\varepsilon_r, \varepsilon_S, \varepsilon_\Sigma$ — Random, systematic or total error

ε_E , ε_J — Experimental or identification error

λ — Aspect ratio

$\mu = 2m/\rho Sl$ — Aeroplane normalized mass

ρ — Air density, kg/m^3

$\tau = \mu\tau_A$ — Dynamic unit of time, sec

$\tau_A = l/V$ — Aerodynamic unit of time, sec

ω — Circular frequency, sec

$\omega^x = \omega\tau_A$ — Strouhal number - reduced frequency

Denominations

$\bar{x}(i\omega^x)$ — Fourier transform of the x(t) quantity

$\hat{x}$ — Estimate of x by optimization method

$\check{x}$	Estimate of x from corrected measurements

$\tilde{x} = x/x_{ref}$; $\tilde{\dot{x}} = \mu . \dot{x}^{\times}$; $\dot{x}^{\times} = \tau_A . \dot{x}$

$^{\circ}x = x/x(0)$ Normalized quantity

$(x)_w$ weighted value; $(x)_s$ steady value

Indexis

R , I	Real or imaginary part
c , i , Σ	Circulation or inertial part; sum of the preceding parts
F , H	Wing, tailplane

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