OPTIMAL FLIGHT VEHICLE DESIGN AND LINEAR VECTOR SPACES

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Abstract

Flight vehicle design is a multi-disciplinary activity with complex interactions between the different areas. The paper presents a normed linear vector space approach to the design and optimization of a flight vehicle. The aircraft may be characterized by a set of parameters in the generalized performance hyper space, defining the manifold configuration and mission roles. The system configuration may be defined by a vector in the design hyperspace. Perturbations in the design vector and constraint vector caused by changes in technology, operational scenario and economic factors causes perturbations in the performance vector so that a weak derivative or a transfer matrix may be defined. The aircraft design process may be regarded as a problem in optimization of the design vector with multiple objective functions representing the different roles subject to a set of constraints specified by the constraint vector. As a measure of the objective function may be chosen the norm of the performance vector. An overall measure of the performance is provided by its metric. The performance space may be considered as an onto and one-one mapping of the design space so that there exists an optimum design for each given performance and specified technology. The performance space may be regarded further as a normed linear space or Banach space whereby the existence of continuous linear functionals required for optimization is ensured by the Hahn-Banach theorem. When some of the design, performance and constraint vectors governing the optimum are either unknown or known only imprecisely, the process may be characterized through probability measure or through membership functions of the theory of fuzzy sets. In this situation, optimization in the conventional sense is not possible and requires introduction of an uncertainty vector so that the objective function is no longer point valued but a set valued or interval valued function of the uncertainty vector. Through introduction of preference relations among the parameter values, a point valued generalized performance function may be defined to enable optimization in the conventional sense. Multivariate search procedures may then be applied to locate the design point.

1. Introduction

Flight vehicle design is a multi-disciplinary activity comprising of the synthesis of aerodynamics, configuration design, propulsion, structures, flight control systems and avionics to produce an aircraft of the required performance and economic sensibility whether it is for the civil or the military. The number of parameters involved in the preliminary design synthesis is large, varied and complex in their interactions. An understanding of the manifold complex interactions between the numerous technology areas and the specialist disciplines is essential for a successful realization of the objectives of a flight vehicle development program. The design process may appear ad hoc, empirical and oftentimes irrational to the uninitiated. However, a closer scrutiny of the various governing factors leading to a final decision of the configuration and the detailed configuration constituents reveals the essentially logical nature of the design process in a broad sense, whether it is for the choice of the aerodynamic configuration or engine choice or selection of an item of avionic equipment. Further, in the case of military aircraft, the user expects multiple roles for the aircraft, to provide for the unforeseen exigencies of a war, some of thich may be primary and the others of secondary or tertiary importance, the subsidiary roles being performed with minor or some major changes in the configuration and/or equipment standards. It would be profitable to translate this entire design process into a logical framework so that mathematical and other decision making operations may be systematized coherently. The use of generalized vector spaces appears to provide a suitable mathematical framework for this purpose. Such a formulation is attempted in this paper.

2. Mathematical Formulation of Design

A brief mathematical formulation of the above design process has been proposed by Ramachandra[1,2] earlier. The flight vehicle in the i-th mission/role may be characterized by a set of M linearly independent parameters $\mathcal{P}^i\{P_1^i, P_2^i, P_3^i, \ldots\ldots P_M^i\}$ representing the generalized aircraft aircraft performance regarded as a point in the M-dimensional performance hyperspace $\mathcal{P}(P_1, P_2, \ldots\ldots P_M)$ defined by $\mathcal{P} = \bigcup_{r=1}^{R} \mathcal{P}^r$ of the R-mission/roles of the given flight vehicle. The aircraft design process may be regarded as a problem of parameter optimization with multiple objective functions each representing one of the different roles r=1,2,3,...R, required of the aircraft. We shall assign the set of R importance or weight parameters $\lambda_i \ni 0 \leq \lambda_i \leq 1$ to the mission/roles $i \ni \sum_{i=1}^{R} \lambda_i = 1$. Since $\mathcal{P}^i \subset \mathcal{P}$, the aircraft in the i-th role may be described as a point in the generalized design space by a certain vector

$$\mathcal{D}^i = \mathcal{D}(D_1^i, D_2^i, D_3^i, \ldots\ldots D_L^i) \tag{1}$$

giving the performance

$$\mathcal{P}^i = \mathcal{P}(P_1^i, P_2^i, P_3^i, \ldots\ldots P_M^i) \tag{2}$$

in its i-th role. We shall assume the design space $\mathcal{D}$ of the linearly independent set of control variables $\{D_1, D_2, \ldots D_L\}$ to be convex. Typical of the design parameters $D_1, D_2, D_3, \ldots D_L$ are

KINEMATIC	**PERFORMANCE**
Wing Leading Edge Sweep	Cruise Speed
Wing Taper Ratio	Take-Off Weight-
Wing Thickness	Operating Weight
Wing Loading	Weight of Bombs & Missiles
Aspect Ratio	Range
Engine Thrust/Aircraft Weight	Turn Rate
Type of Flap	Specific Excess Power
Flap Chord/Wing Chord	Acceleration Time
Flap Span/Wing Span	Take-Off Distance
Take-Off Flap Setting	Landing Distance
Landing Flap Setting	Combat Radius
PROPULSION	**STRUCTURE**
Engine Thrust/Engine Weight	Wing Box Weight
Air Mass Flow	High Lift System Weight
Cruise SFC	Empennage Weight
Hold SFC	
Turbine Inlet Temperature	**AVIONICS**
Overall Pressure Ratio	Avionics Weight
By-Pass Ratio	Electronic Counter Measures
Number of Engines	Head-Up Display

Sometimes a clearcut distinction between two or more design variables may be difficult due to their mutual interaction If any two mission/roles i,j, $i \neq j$ of the aircraft in the performance hyperspace $\mathcal{P}$ are distinct or disjoint, $\mathcal{P}^i \cap \mathcal{P}^j = \phi$, $i \neq j$. However, very often the objectives of some of the mission/role of a flight vehicle may be different in only a minor manner retaining some measure of commonality in a large number of other areas so that we may write generally

$$\mathcal{P}^i \cap \mathcal{P}^j \neq \phi, \quad i \neq j \tag{3}$$

The different mission roles i may be achieved by a change in some of the basic internal or external armament, stores and avionic equipment standards or other flight hardware housed in the same basic airframe and may be represented as a change in a subset of the design vector $\mathcal{D}$. Thus, among the elements $D_1, D_2, D_3, \ldots D_L$, a change may be effected in only the s elements $D_{L-(s-1)}, D_{L-(s-2)}, \ldots\ldots D_L$, leaving the elements $D_1, D_2, D_3, \ldots\ldots D_{L-s}$, representing for example the basic engine-airframe configuration, unaffected to meet the requirements of the i-th role of the aircraft, If $(D^i_{L-s+1}, D^i_{L-s+2}, \ldots\ldots D^i_L)$ denote the set of armament, stores and avionics equipment standards required for the i-th role, we can represent the design space of the set of equipment standards by

$(D_{L-s+1}, D_{L-s+2}, \ldots\ldots D_L)$ where

$$D_{L-s+1} = \bigcup_{t=1}^{R} D^t_{L-s+1}; \ldots; D_L = \bigcup_{t=1}^{R} D^t_L \tag{4}$$

3. Design Optimization

Two approaches are now possible. One would consist of exploring the design space $\mathcal{D}$ with a view to obtain the best performance with the current or anticipated state-of-art technology to establish design goals for a hardware competition. For optimal experimental designs of regression problems, Smith[9] outlined a criterion called G-optimality by Kiefer and Wolfowitz[12]. Later, Wald introduced[10] a linear hypothesis testing criterion for the analysis of variance problems, which was also proposed by Mood[11] subsequently for obtaining weighing designs, a criterion called D-optimality by Kiefer and Wolfowitz[12], and extended to general regression models. Kiefer and Wolfowitz also established the equivalence of G- and D-optimality.

We therefore require a design measure of $\mathcal{P}$ on $\mathcal{D}$ for which we can define a variance function and verify whether a given design is either G- or D-optimal. As indicated by St.John and Draper[7], G-optimality is a design parameter estimation criterion whereas the D-optimality is a performance or response estimation criterion. In general, the performance of locally D-optimal designs may be obtained using G.E.P.Box and Lucas[8] local linearization method. Fedorov's algorithm for obtaining D-optimal designs is outlined by St.John and Draper[7]. However, due to the extremely large computational requirements of obtaining D-optimal designs, despite Fedorov's algorithm, D-optimality is difficult to implement. Nevertheless, D-optimal designs are of interest as they serve as a yardstick to gauge the efficiency of actual designs.

Thus, as a measure of the objective function $\mathcal{F}$ to be extremalized in the optimization process may be chosen the maximum of the norm $\|\mathcal{P}\|$. In the case of multiple roled aircraft, a composite objective function may be formed as a weighted norm

$$\mathcal{F}_1 = \sum_{i=1}^{R} \lambda_i \|\mathcal{P}^i\| \tag{5}$$

An alternative form of the objective function may be defined by

$$\mathcal{F}_2 = \sum_{i=1}^{R} \|\lambda_i \mathcal{P}^i\| \tag{6}$$

Whereas the second form of the objective function represents as quadratic composite functional, the first is a linear composite functional ofthe individual role performances.

A second approach consists of an exploration of the design space $\mathcal{D}$ to obtain a performance $\mathcal{P}^i$ approaching the customer desired performance $\mathcal{P}_o^i$ as closely as possible subject to the set of customer specified and/or technological, operational or economic constraints denoted by the constraint vector. Thus, if the performance desired by the customer be denoted by the vector

$$\mathcal{P}_o^i = \mathcal{P}_o^i(P_{1o}^i, P_{2o}^i, P_{3o}^i, \ldots\ldots P_{Mo}^i) \tag{7}$$

and

$$\epsilon_j^i = |P_{j0}^i - P_j^i| \qquad (8)$$

is the j-th component of the deviation vector

$$\mathcal{E}^i = \mathcal{E}^i(\epsilon_1^i, \epsilon_2^i, \cdots \epsilon_M^i) \qquad (9)$$

of the candidate aircraft performance $\mathcal{P}^i$ from $\mathcal{P}_o^i$ defined by the vector

$$\mathcal{E}^i = \mathcal{P}_o^i - \mathcal{P}^i \qquad (10)$$

then, a measure of our design objective may be projected as minimization of the norm $\|\mathcal{E}\|$. For a multiple -roled flight vehicle we may use the importance parameters λ_i to form the deviator functional

$$\mathcal{F}_3 = \sum_{i=1}^{R} \lambda_i \|\mathcal{E}^i\| \qquad (11)$$

or the alternative

$$\mathcal{F}_4 = \sum_{i=1}^{R} \|\lambda_i \mathcal{E}^i\| \qquad (12)$$

The importance parameters λ_i , $0 \leq \lambda_i \leq 1$ $\ni \sum_{i=1}^{R} \lambda_i = 1$ may be chosen on heuristic grounds or by the Delphi method. Thus, the design process may be described mathematically as the determination of the design vector $\mathcal{D}$ subject to the setof equality or inequality constraints$\{C_1, C_2, \cdots C_N\}$ specified by the constraint vector

$$\mathcal{C} = \{C_1, C_2, C_3, \cdots C_N) \qquad (13)$$

to minimize $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ as the case may be.

Typical of the constraints of military aircraft are the unit aircraft cost, the aircraft or the fleet life cycle cost, aircraft weight, development cost and time, technology availability, radar cross section, armament and avionic fit etc. representing both qualitative and quantitative limitations. Besides, perfromance constraints in the form of maximum available field lengths, maximum stall speed, minimum rate of climb, minimum acceptable turn rate etc. may also appear as inequality or equality constraints. Many of the elements of the performance vector $\mathcal{P}$ do not, in general, form a dimensionally homogeneous set so that they must be normalized through appropriately chosen measures to form a suitable metric in the $\mathcal{P}$-space. We may therefore regard $\mathcal{P}$-space as a normed linear space or a Banach space so that the Hahn-Banach theorem assures the existence of continuous linear functionals of importance to us in the optimization work.

4. Regression Functions

The bridge between the design space $\mathcal{D}$ and the performance space is the technology space $\mathcal{T}$ which is intimately connected with and represents the technology level injected into the realization of the ultimate hardware. The flight vehicle performance vector $\mathcal{P}$ may be represented in terms of the design vector $\mathcal{D}$ through the mapping $\mathcal{M} \ni$

$$\mathcal{P} = \mathcal{M}(\mathcal{D}) \qquad (14)$$

which may be written in the form of a set of functions

$$\{P_i = P_i(D_1, D_2, D_3, \cdots D_L)\} \qquad (15)$$

the P_i being either uniformly or piecewise continuous functions mapping $\mathcal{D}$ into $\mathcal{P}$. A reasonable domain of the mapping $\mathcal{M}$ may consist of the set of all aircraft of a given type. We shall assume on purely intuitive grounds that the mapping $\mathcal{M}$ is onto and one-one so that the inverse mapping $\mathcal{M}^{-1}$ also exists from which it follows that there exists an optimum aircraft design for each given performance and vice versa, for a specified technology level. The graph $\mathcal{P} = \mathcal{M}(\mathcal{D})$ is somewhat complex and involves the technology areas like aerodynamics, structures, electromagnetics, materials science or other relations and is therefore, not definable as a simple direct transformation The mapping functions of the graph $\mathcal{P} = \mathcal{M}(\mathcal{D})$ are expressed throughregression functions.

Aerospace regression models are generally non-linear. Also, the set of regression functions obtained through statistical/analytical processes on the basis of a limited sample of currently available aircraft may not be unique even at a point of time, varying from one manufacturer to another and one aircraft type to another. Thus, Czysz, Dighton and Murden[3] represent the logarithm of the empty weight of combat aircraft as a quadric surface in an 8-dimensional manifold. Silver[4] numerous complex regression equations applicable to light general aviation aircraft. Greenway and Koob[5] represent the aircraft take-off,gross weight take-off distance, sustained load factor and acceleration time as a quadric surface in a five-dimensional design manifold. Thus, keeping in mind simplicity of representation, we may write the performance parameter P_K as a quadratic function

$$P_K = P_K(D_1, D_2, D_3, \cdots D_L) = A_o^{(K)} + \sum_i A_i^{(K)} D_i + \sum_i^L \sum_j A_{ij}^{(K)} D_i D_j \qquad (16)$$

where the $A^{(K)}$ are regression coefficients and D_i the design variates. Cubic, quartic and higher order surfaces may also be used whenever required still retaining the relatively simple representation for computational purposes. Both the performance vector $\{P_i\}$ and the regression coefficients $A^{(K)}$ in any of these regression equations are strong functions of the technology level used on the aircraft and the technology strides with the passage of time. Besides, these regression equations may not be differentiable with respect to some of the design and technology variates and . . differentiable with respect to theothers.

Technology is defined here in a generalized sense to include the set of aerodynamics/aerothermodynamics/configuration /structures/materials/systems concepts/

generalized component efficiencies; analysis/measurement/detection techniques. Consequently, technology level bears strongly on the flight vehicle service introduction date. The prevailing materials and other technology levels like the stress and creep properties, turbine entry temperature etc. in the $\mathcal{T}$-space govern the aerothermodynamic and mechanical design parameters in $\mathcal{D}$-space, which, in turn, govern the overall engine-airframe performance indices like the specific fuel consumption, thrust/weight ratio, noise level etc. The avionic technology required/available governs the size, location, performance and cost, which, in turn, governs the flight vehicle system design parameters and, thereby, affect the overall performance in terms of the radar cross section, survivability, autonomous/vectored operabilities etc. For a given engine thrust, the maximum Mach number of an aircraft in $\mathcal{P}$ depends on the drag coefficient in $\mathcal{T}$-space which, in turn, depends on the aircraft geometric layout in $\mathcal{D}$. Again, another example is the spin performance of an aircraft in $\mathcal{P}$-space which depends onits aerodynamic and inertial characteristics in $\mathcal{T}$-space which, turn, depend on the geometric and layout characteristics in $\mathcal{D}$-space.

5. Design Sensitivity

Frequently, it becomes important to assess changes in the performance vector $\mathcal{P}$ due to perturbations in one or more elements of the design and/or the technology space for enabling decision making processes. To study the perturbations in the design vector $\mathcal{D}$ caused by perturbations in the performance vector $\mathcal{P}$ we may define a sensitivity or influence or transfer coefficient $\mathcal{S}_{ij}$ defined by the derivative

$$\mathcal{S}_{ij} = (\partial P_i/\partial D_j), \quad \begin{matrix} i = 1,2,3,\ldots\ldots M \\ j = 1,2,3,\ldots\ldots L \end{matrix} \qquad (17)$$

forming an element of the (M x L) sensitivity or transfer matrix $S = [\mathcal{S}_{ij}]$ of the technology manifold. These variations may be caused through alterations in the hardware geometry, materials changes, changes in equipment standards etc. impacting on one or several elements of the performance set $\{P_i\} \in \mathcal{P}$. The transfer coefficients relating the $\mathcal{D}$ and $\mathcal{P}$-space may be related through the technology interface set $\{T_i\}$ in the technology space $\mathcal{T}$ of aerodynamics, configuration, structures, aeroelasticity, propulsion, materials, electrical, electronics, armament etc. comprising the flight vehicle hardware, as

$$\mathcal{S}_{ij} = \sum_{k=1}^{T} (\partial P_i/\partial T_k)(\partial T_k/\partial D_j), \quad \begin{matrix} i = 1,2,\ldots\ldots M \\ j = 1,2,\ldots\ldots L \\ k = 1,2,\ldots\ldots T \end{matrix} \qquad (18)$$

It can be seen from the examples mentioned above that the functional relationships involved are, in general, non-differentiable so that the derivatives in the transfer matrix $S = [\mathcal{S}_{ij}]$ must be regarded as symbolic or weak derivatives.

6. Optimization With Uncertainty

In practice, many of the important parameters of design and performance space which govern the optimum are either unknown or known only imprecisely in the beginning, especially. This renders a techno-economic evaluation of a development program difficult. Thus, for a combat aircraft, the overall aircraft weight, internal fuel contents, aerodynamc drag, fatigue life, aircraft cost, maintainability, etc. are some of the uncertain parameters belonging to the $\mathcal{D}$ or $\mathcal{P}$-space and in some cases even the $\mathcal{T}$-space. In such cases, we may specify the uncertainties through probability measures or through membership functions of the theory of fuzzy sets. Hence, optimization in the usual sense may not be possible and requires the introduction of an uncertainty parameter ω, which is typically a vector variable and whose components are the uncertainties of the various elements of the spaces considered. The objective function expressed through the norm $\|\mathcal{P}\|$ or $\|\mathcal{E}\|$ is therefore no longer point valued but a set valued function of the control parameters. Under this condition, Dresnick[6] suggests introduction of a preference relation $u_1 \prec u_2$ among the control parameter values as a first step. As a second step, a new point valued performance $\Psi(u)$ must be introduced with the property that

$$\Psi(u_1) < \Psi(u_2) \text{ iff } u_1 \prec u_2 \qquad (19)$$

If $\psi(u,\omega)$ be a point valued function of u and the uncertainty parameter ω, the conditions for the feasibility of the step may be fulfilled sometimes, so that, we can justify the relation

$$u_1 \prec u_2 \quad \text{iff} \quad \min_{\omega} \psi(u_1,\omega) < \min_{\omega} \psi(u_2,\omega) \qquad (20)$$

which may be stated in words as " u_2 is preferred if and only if the worst performance under u_2 is better than that under u_1 ". Hence, $\Psi(u) = \min_{\omega} \psi(u,\omega)$ is point valued and an optimum defined relative to it so that it is the minimum value of u. Consequently, due to an introduction of preference relations among parameter values the notion of optimality must be revised in the sense suggested above.

Extremalization of the weighted norm $\|\mathcal{E}\|$ or $\|\mathcal{P}\|$ can be carried out through multi-variate search programs. Hence, the design problem resolves itself into a multi-variate search in the L-dimensional design space giving an extremum of $\mathcal{P}$ or $\mathcal{E}$ depending on the formulation. When the number of variates is small, upto three or four, the search can be done manually whereas when L is large, as is often the case in flight vehicle design, manual search becomes difficult and cumbersome. Multi-variate search procedures may be applied using high speed computers to locate the design point $\mathcal{D}$ for optimum $\mathcal{P}$ or $\mathcal{E}$. The search for an optimum design requires a baseline design representing in some sense the nature of the ultimate hardware in the $\mathcal{X}$ -space. The baseline design can then be used to establish baseline technology and performance points in the $\mathcal{T}$ and $\mathcal{P}$ hyperspaces.

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