THE ANALYSIS OF FATIGUE FAILURES

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Abstract

The post-failure analysis of fatigue cracking has developed during the last decade into a quantitative technique vital to both aircraft structural testing and accident investigations. The quantitative techniques used retrospectively to determine fatigue crack growth rates are explained and compared with complementary predictions based upon linear elastic fracture mechanics taking account of the limitations to both techniques. Examples include detailed analyses of fatigue cracking in undercarriages during structural testing, cracking from fastener holes in the wing of a military aircraft undergoing testing and the in-service failure of the tail-plane of a transport aircraft. The analyses consider in particular the causes of premature and unexpected fatigue failures with reference to current airworthiness philosophies.

1. Introduction

The history of aviation is beset with catastrophies involving failure of aircraft structures and components by fatigue cracking. This fact has long been recognized and has led to fatigue design requirements including the lifing of aircraft in terms of fatigue exposure and to a continual assessment of the airworthiness of aircraft based, in the main, upon the containment of fatigue problems. The assessment of airworthiness depends upon many factors reflecting the complexity of aircraft structures and working environments but at least four distinct processes can be recognized. Firstly, the design stage with the modern emphasis on finite element stress analysis and on demonstrating the tolerance of the design to damage, such as fatigue cracking, but with a continuing need to produce an efficient structure requiring little maintenance. Secondly, major structural testing which is regularly employed to demonstrate the fatigue properties of the design, again with an emphasis on damage tolerance. Current trends involve the more realistic simulation of real life stresses and the use of fracture mechanics in the interpretation of the failures. Thirdly, the regular inspection of aircraft in service is employed so that over the years of fleet operation a large quantity of data is collected concerning the incidence of failure in very large samples. This information is occasionally supplemented by a complete strip-down examination of an individual aircraft, perhaps the fleet leader. Finally, there is the post-failure analysis of the infrequent but significant quantity of catastrophic in-service failures.

There are many excellent papers[1-8] detailing current design techniques, including the collection and treatment of data, the application of fracture mechanics and the philosophy of damage tolerant design. There are also papers[9-10] revealing the studies of fleet histories and aircraft examinations although these tend to be proprietary. This paper attempts to show by quantitative examples why unexpected fatigue failures occur both in testing and in service use and to demonstrate the powerful combination of fracture mechanics and quantitative fractography in the assessment of the damage tolerance of the structures, that is their behaviour once cracked. An efficient design must never crack however and the examples also consider the factors affecting crack initiation in a quantitative manner.

2. The Analysis of Structural Test Fatigue Failures

Virtually all major aircraft, in UK service, are undergoing, or have undergone some degree of fatigue testing of their component parts ranging from the simple hydraulic tests of actuators to complete tests of whole wing assemblies including simulated real life loading patterns and environments. Frequently these tests produce fatigue cracks in unexpected places often in regions of structure hidden from inspection and consequently the cracking is hard to monitor during testing. Moreover, it is common practice to repair cracked structure to enable the test to proceed. Post-failure analysis is invaluable and the Materials Engineer is required to analyse the fracture on completion of the test to predict the number of fatigue loading cycles taken to grow cracks between pre-determined limits of crack depth so that inspection periodicities and safe lives can be assessed having applied appropriate factors for safety. The crack depth limits will typically be the largest crack that can escape detection by the prescribed technique and the crack size at which the residual strength is exceeded by the prescribed limit load. The purpose of this analysis is, of course to ensure the detection of cracking in service aircraft before it becomes dangerous. Most of the loading schedules used to date have involved the repeated applications of relatively simple variable amplitude programmes which are amenable to cycle-by-cycle analyses but which may require an empirical conversion from test fatigue damage rates to real life damage rates. It is frequently the case that the fatigue tests have produced unexpected or premature fatigue failures so that questions are raised as to the causes of the unexpected failure or of its premature nature.

The post-failure analysis is based upon two techniques, quantitative fractography and fracture mechanics predictions of the fatigue crack growth rates. Quantitative fractography relies upon the recognition of markings on the fatigue fracture surface and the correlation of the positions and spacings of such markings with the applications of the fatigue loadings. On a microscopic scale individual increments of crack extension are referred to as fatigue striations (Fig 1) and on a macroscopic scale larger abrupt increases in crack size may be discerned, these are referred to as crack jumps. A regular fatigue loading cycle produces regular striations (Fig 1)

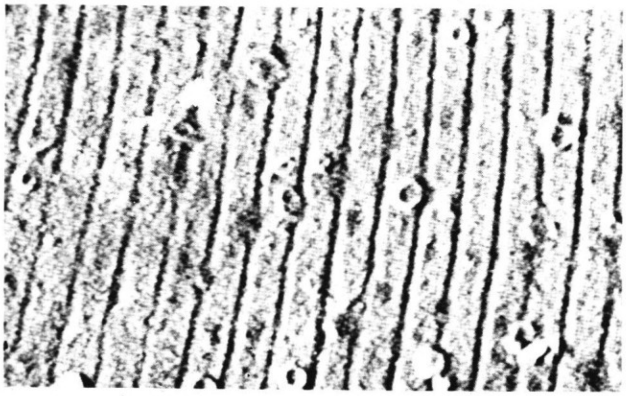

Fig 1a. Fatigue striations on a fracture surface formed under constant amplitude loading. X100000.

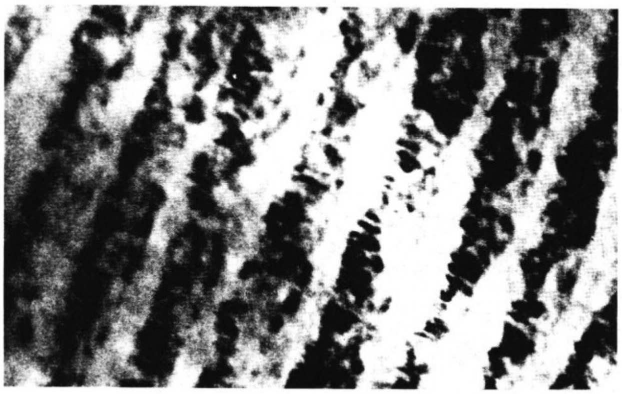

Fig 1b. Dislocation structure within the metal just below the fracture surface. The surface steps associated with these dislocation bands form the striations. X200000.

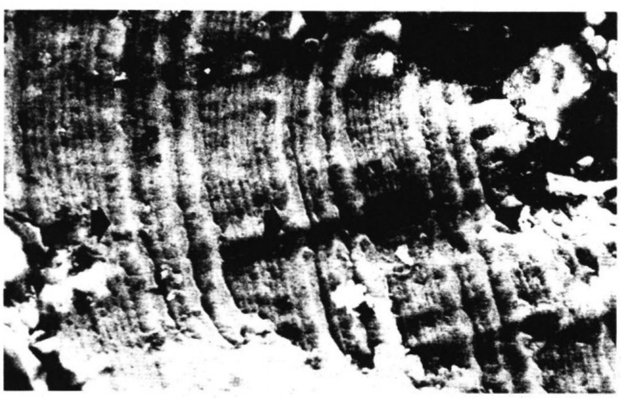

Fig 2. Fatigue striations formed by a complex loading programme used to test military aircraft undercarriages. The crack extension per flight is indicated. X1000.

whilst complex loading cycles produce complex striations (Fig 2). Striation spacings can be readily measured as a function of crack depth and, if a pattern of striations can be associated with a pattern of load cycling, then crack growth rates in terms of crack extension per load cycle can be determined directly from the fracture surface. This important relationship between fracture striations and load cycles has been confirmed by many researches[11-14]. Macroscopic crack jumping may also yield quantitative data, although the attribution of a single jump to any particular load cycle is more difficult. This aspect is considered later in the paper.

The complementary technique of prediction of crack growth rates from a knowledge of stress levels has received enormous attention and is now used as a basic design tool. The method used in these examples involves acquiring a knowledge of the load or stress cycles causing the fatigue cracking and the conversion of these loads into stress intensity factors at the crack tip. These are used to calculate crack growth rates by the summation of cycle-by-cycle increments of crack growth predicted by relatively simple physical laws which relate crack growth rate to the range in stress intensity factor. It should be appreciated that the stress intensity factor, K_1, partly quantifies the elastic stresses at the tip of a crack and is used to describe critical levels of these stresses at which catastrophic failure occurs (K_{1c} and K_c). K_1 should not be confused with the elastic stress concentration k_t which may be applied to macroscopic features of a sample, such as a hole or a cut-out, to describe the stresses at the boundary. The range in stress intensity factor ΔK, is the difference between the maximum and minimum levels of K_1, produced by the cyclic fatigue stress. A definition of stress intensity factor and a simplified example of its use in the prediction of fatigue crack growth are set out in Appendix A whilst the potential inaccuracies in the techniques are discussed in section 4.

Example A Undercarriage Fatigue Test

A fatigue substantiation test produced an early failure of an undercarriage fitting for a civil aircraft. The loading programme consisted of blocks of loads representing standard landings (Fig 3) and blocks of loads representing severe

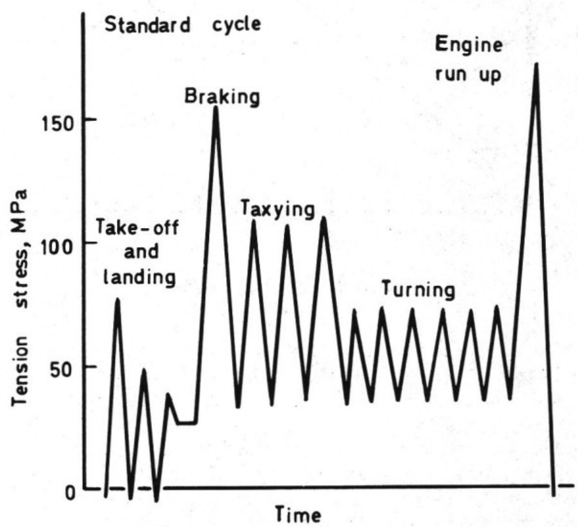

Fig 3. The standard severity loading programme used in testing an undercarriage for a civil transport aircraft. Note the large stress cycles associated with braking and engine run-up against the brakes.

ground handling conditions in which the minimum and maximum stress levels were increased by 10%. Fig 4 illustrates the striations formed, not by

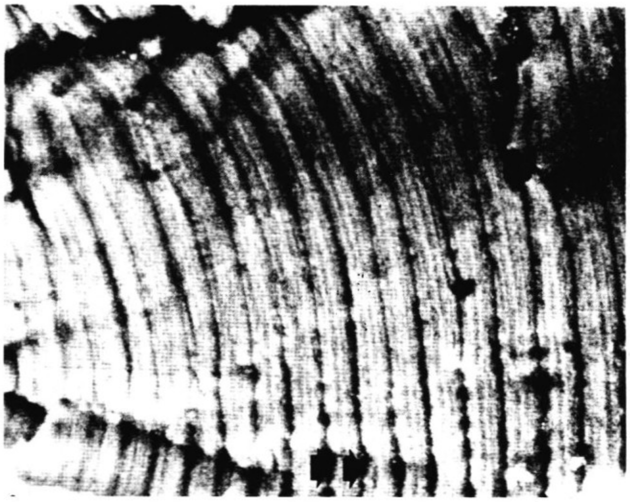

Fig 4. The striation pattern formed by the loading programme shown in Fig 3. The crack extension per landing is indicated. X5000.

individual loads but by complete landing duty cycles. The spacings of these striations increased with crack depth and changed abruptly when the loading severity changed (Fig 5).

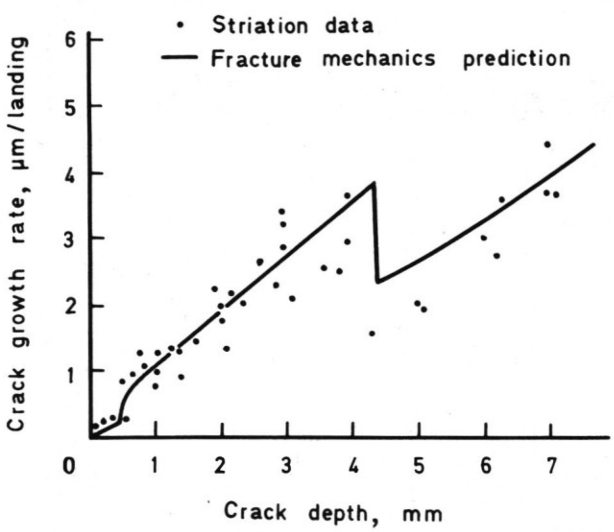

Fig 5. Crack growth rates in undercarriage A predicted by fracture mechanics and measured as the spacings of landing-by-landing striations.

Striations could not be observed over the entire crack depth but in the region 0.5 mm to 8 mm crack depth the striation spacings are in good agreement with crack growth rates, predicted by fracture mechanics. This indicates that the stresses that were applied were correct, ie at the intended level. Integration of the reciprocal of the growth rates enabled the crack growth period to be accurately defined (Fig 6) for the subsequent definition of safe lives or inspection intervals. In this example, the life obtained

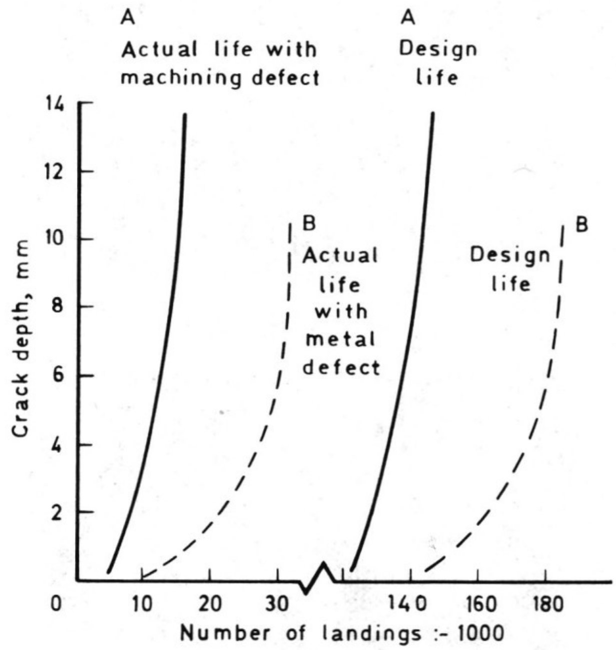

Fig 6. The actual fatigue life of undercarriage A compared to the expected design life. A second undercarriage test, B , is included.

was much less than that required by the designers and a detailed analysis of striation spacings close to the point of crack initiation revealed that the short cracks initiated and grew under the influence of stress concentrations associated with a change in section and, very close to the surface, under the influence of coarse machining marks (Fig 7). The effective local stress concentration

Fig 7. The surface of undercarriage A at the fatigue crack origin showing coarse machining. X100.

could be predicted from the striation spacings (Fig 8) and this indicated that the combined effects of the change in section ($k_t \sim 2$) and rough machining ($k_t \sim 2.5$) produced a stress concentration factor in excess of 4 reducing the crack initiation period significantly, that is moving the whole crack growth curve to the left in Fig 6. In summary the combination of fracture mechanics analysis and quantitative fractography confirmed the correct application of the fatigue loads and indicated that premature failure was associated with surface stress concentrations.

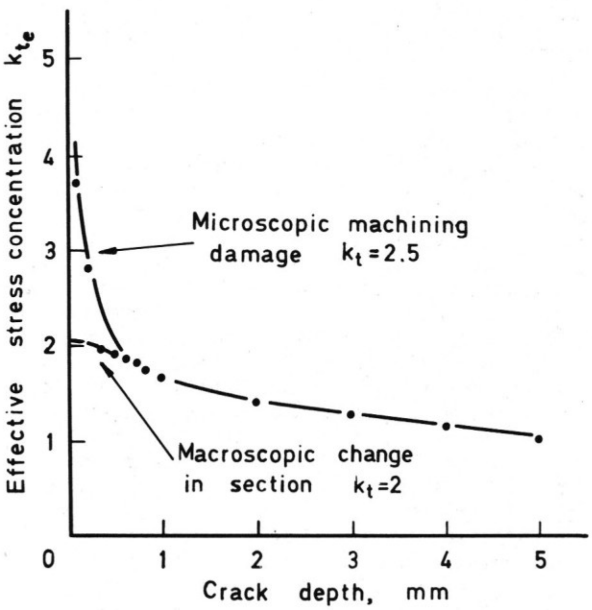

Fig 8. The effective stress concentration factor, k_{t_e}, predicted by analysis of striation spacings close to the crack origin. Note the combined effects of the macroscopic change in section and the coarse machining.

A similar effect was obtained in a different undercarriage test in which a large metallurgical defect initiated premature failure (Fig 6 broken lines) producing a very short fatigue life. Similar loss in fatigue life can be caused by the presence of corrosion pits[15]. Whilst not necessarily typical of all undercarriage designs, these results illustrate a common feature that is a safe-life design with a long initiation period followed by very rapid crack growth. Clearly, quality control is of great importance if metallurgical and manufacturing defects are to be avoided.

Example B Military Aircraft Wing Test

The fatigue substantiation of military aircraft follows a similar pattern to that of civil aircraft, except that the loadings applied are generally more severe and the service fatigue lives shorter[7]. In this example, a matrix of load cycles representing flights of three different types has been applied to an aircraft wing (Fig 9). After a due period of testing fatigue cracks initiated at several fastener holes and grew until the test was stopped.

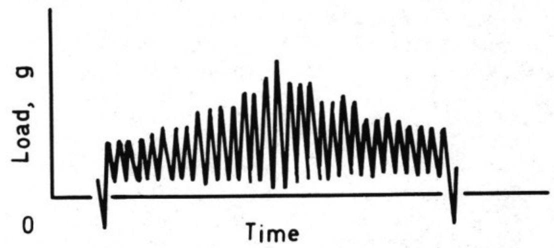

Fig 9. A medium severity flight applied to the military aircraft wing (Example B). One of a hundred flights comprising a complete loading programme.

The complex loadings produced a repeating pattern of striations on the fracture surfaces (Fig 10) and striation pattern spacings were used as a measure of fatigue crack growth rates (Fig 11).

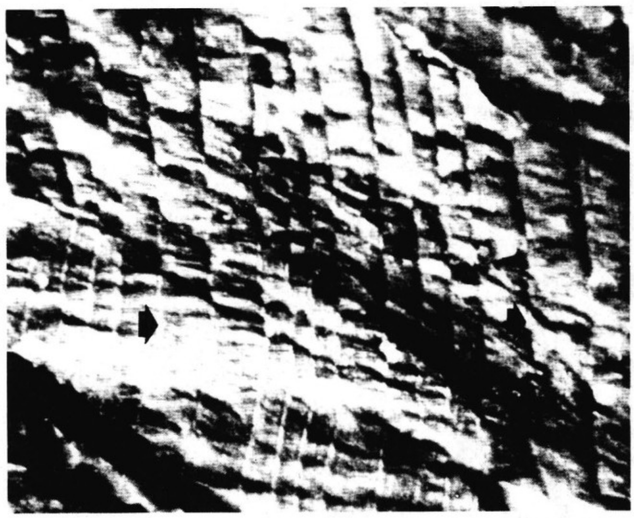

Fig 10. The striation pattern on the fracture surface of the military aircraft wing. A complete programme of 100 flights is indicated between the arrows. X1500.

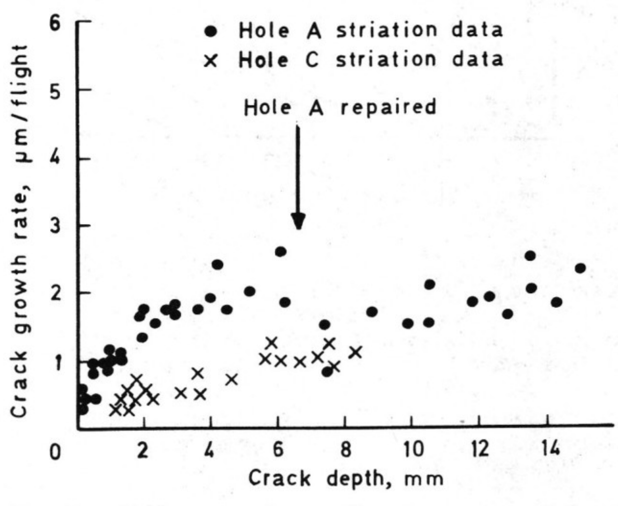

Fig 11. Fatigue crack growth rates measured for cracks at two holes A and C in the military aircraft wing.

In this complicated loading programme, the striation pattern between the indicated points contains 100 simulated flights of which the most severe are recognizable as the irregularly spaced striations. As in the previous undercarriage example, an abrupt change in growth rate was detected but, on this occasion, it was associated with a deliberate repair of the cracked test wing which enabled the test to proceed. Integration of reciprocal rates enabled the prediction of safe lives and illustrated the benefits of a strap repair to the wing. Several cracks at identical fastener holes were analysed and three different types of behaviour were found (Fig 12). Firstly, there was rapid crack growth at a highly stressed hole (A in Fig 12). Repair of this hole had produced a slightly beneficial effect. Secondly, there was slow crack growth at a lowly stressed hole (B in Fig 12). It can be seen that, for both cracks A and B, the initiation period had been substantial but for the third crack C initiation had been early, although the hole was lowly stressed. The geometry of the three holes was similar so that it can be seen that the effect of an increase in stress level is to induce earlier crack initiation and more rapid crack growth. However, early crack initiation can also be induced by defective material, a poorly machined hole in the case of crack C, in which case crack initiation can occur very rapidly perhaps even during the first load cycle and the disastrous combination of a high stress level and a poor quality hole must be allowed for (speculatively represented as curve D).

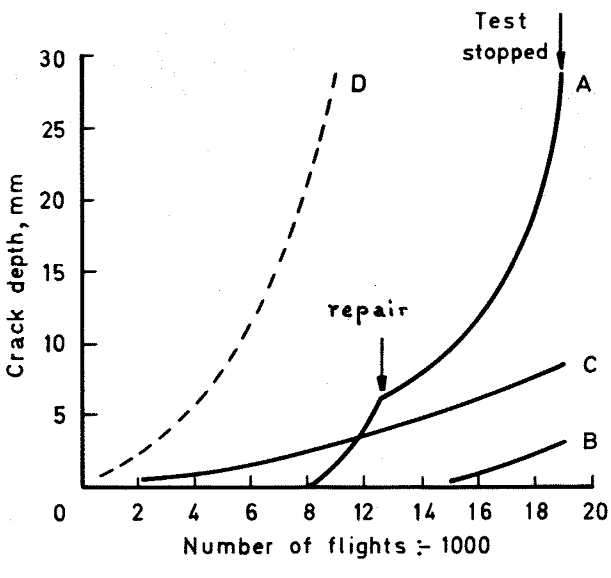

Fig 12. The fatigue lives of cracks from a highly stressed hole (A) and two lowly stressed holes (B and C). Note the early initiation at hole C and the potentially dangerous combination of early initiation and high stress at hole D.

This example illustrates the differences in fatigue design philosophy. The traditional safe life philosophy would have limited the aircraft life to that of crack A (with final failure defined as a prescribed residual strength) divided by a safety factor of perhaps five. Fail safe philosophy would define a period of crack free life followed by a period of regular inspections defined by the crack growth rates of curve A. Clearly, because of curve C, ie premature initiation at a metallurgical defect, the definition of a crack free life is difficult and current damage tolerant design would consider the combination of a high stress level and initiation at a pre-existing defect, curve D. Statistically, it may be more probable that early crack initiation will occur in a relatively lowly stressed region, there being more of these, and damage tolerant philosophies cover this feature by considering the consequences of pre-existing defects occurring at every fastener hole.

3. The Analysis of Service Fatigue Failures

Approximately two-thirds of all the examples of cracking in service that have been examined in our laboratory have occurred by fatigue failure. Superficial analyses of such failures abound with emphasis on metallurgical features which may have induced premature failure. Currently, the pressures to produce quantitative analyses of critical failures have produced a limited success. The techniques are similar to those outlined in the previous section but the problems are greater. For example, recognizable striations or crack jumps are less likely to be present, partly because of the more irregular service loadings partly because of the damage caused to the fracture surfaces by the working environment. Moreover, it is unlikely that adequate loading data will be available in the event of a catastrophic failure and almost certain that the pattern and distribution of stresses will not be available for the cracked structure. Both quantitative analysis of fracture surfaces and fracture mechanics predictions are thus very difficult. Nevertheless certain parts of the aircraft structure do receive a consistent repetitive loading. This is particularly the case with transport aircraft in which, for example, the fuselage undergoes regular pressurisation and the wing a ground-air-ground cycle on a one per flight basis. Higher frequency loadings occur during landing and take-off and whilst undergoing atmospheric turbulence. Combat aircraft suffer, in addition, high manoeuvre loads which will dominate the situation in many cases. Aircraft components undergoing a regular loading pattern are likely to show regular fatigue striations and examples are included of service failures of a tail plane and a main wing from civil transport aircraft and of a helicopter rotor blade (Figs 13-15). Here the striations are seen as bands containing higher frequency striations. The high frequency loads are irregular but the flight-by-flight bands may be sufficiently well formed for counting purposes, their width varying from striation to striation with severity of flight.

The analysis of components from service combat aircraft has proved extremely difficult because the dominant manoeuvre loadings and hence striations are irregular and cannot be readily related to service experience on a flight-by-flight basis. On the other hand the test simulations of combat aircraft loadings, however randomized, are still sufficiently repetitive to enable programme repeats to be recognized and counted.

Fig 13. Flight-by-flight striations on the fracture surface of a wing joint of a commercial service aircraft X500.

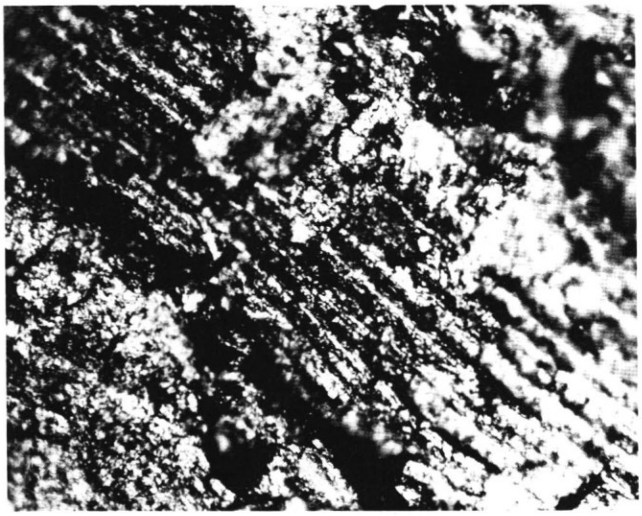

Fig 14. Flight-by-flight striations on the fracture surface of a tail-plane spar of a commercial service aircraft (Example C) X500.

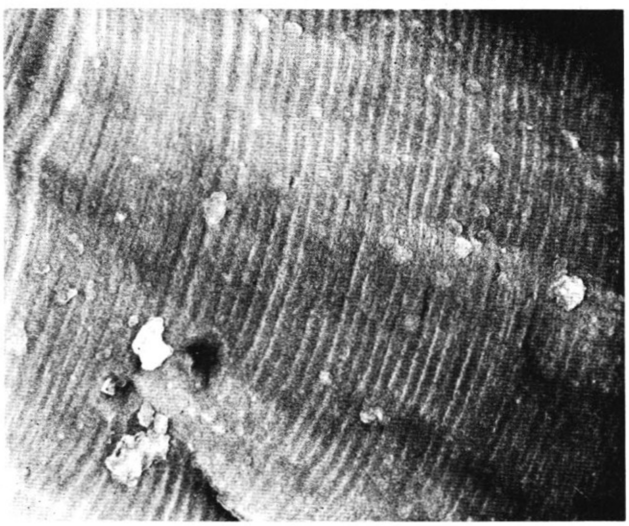

Fig 15. Flight-by-flight striations on the fracture surface of a service helicopter main rotor blade. X5000.

Example C The Failure in Service of a Tail Plane of a Civil Transport Aircraft

A large fatigue crack in the spar and associated tail-plane structure of a civil transport aircraft caused its crash (Fig 19). Striations were observed on the surface of the fatigue failure as were macroscopic jumps. Measurement of the striation spacings allowed the crack depth to be determined retrospectively as a function of numbers of aircraft flights making the assumption that the striations and jumps occurred on a one-per-flight basis. This assumption was partly confirmed by a fracture mechanics analysis using a fatigue stress programme to represent a typical aircraft flight (Figs 16 and 17).

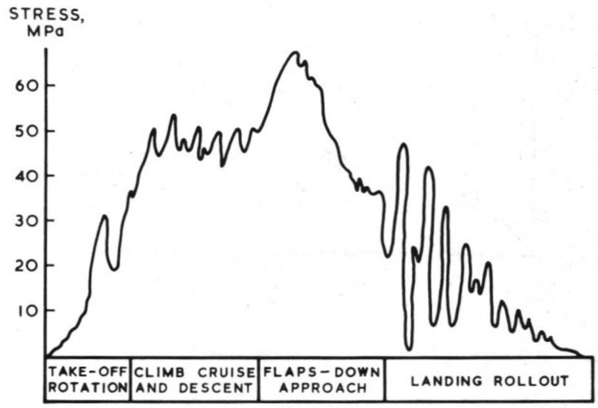

Fig 16. The stresses in the tail plane during a typical flight of two hours.

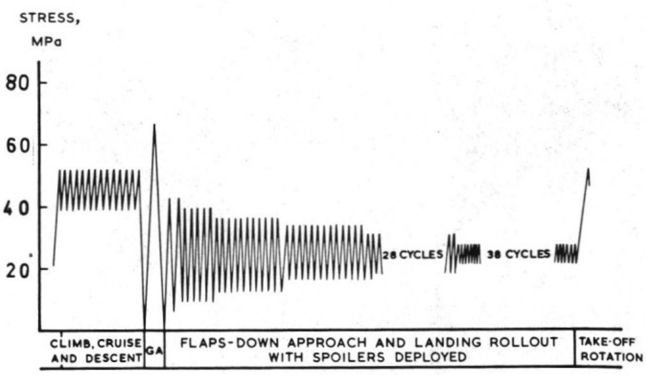

Fig 17. The stress programme used to represent a typical aircraft flight for testing and calculation purposes.

An important feature in this analysis was the representation of the ground-air cycle as a single large cycle with a zero stress ratio. It can be seen that the fracture mechanics analysis predicted the growth rates for short cracks but apparently over-predicted the rates for deeper cracks (Fig 18). It was found that the actual crack growth rate had been faster than that measured as striation spacings because of a significant contribution of crack jumping (Fig 19) and that the fracture mechanics analysis failed to account for problems associated with load shedding during crack growth and possibly with beneficial load interaction effects that may have occurred during service.

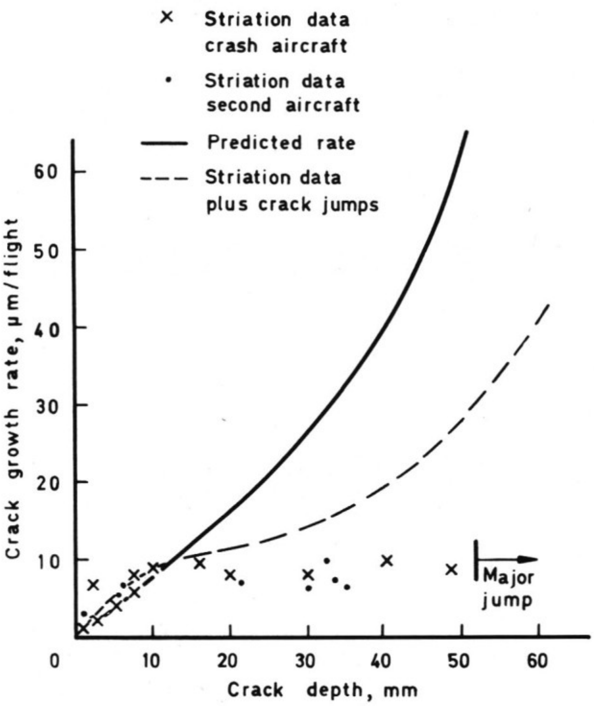

Fig 18. Crack growth rates measured as striation spacings on the fracture surface of example C and predicted by fracture mechanics. The dashed curve makes allowance for a crack jump contribution.

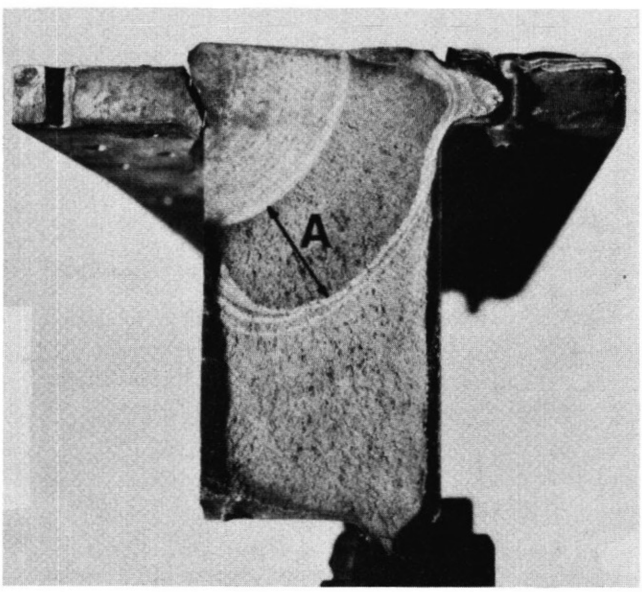

Fig 19. The fatigue crack in the tail-plane spar of example C. A large crack jump is indicated A. X½.

For example, it can be seen (Fig 20) that the stress programme used for the prediction contains none of the larger infrequent stresses that might be encountered in service, represented (in Fig 20) as a current programme for transport aircraft. These large infrequent stresses can be shown to produce little crack extension, provided K_{max} is not exceeded, and to produce beneficial retardation effects[16]. The actual stress programme used was therefore conservative, over-predicting the crack growth rates and, as such, was typical of many programmes used for structural testing. To summarize, the actual crack growth rate should have been between the measured striation spacings and the fracture mechanics predictions. Integration of the two rate curves (Fig 21) showed that, despite the discrepancy between the two curves, a reasonable prediction of the initiation and crack growth periods had been achieved by either technique.

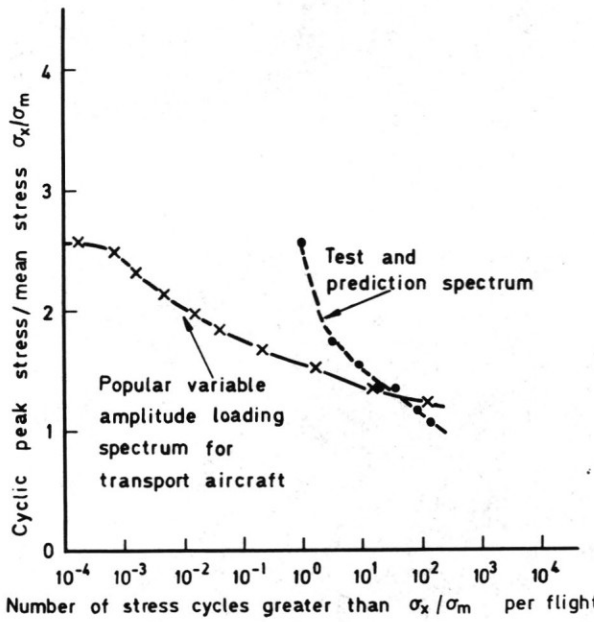

Fig 20. The frequency of occurrence of stresses greater than σ_x/σ_m (σ_x being peak stress, σ_m mean stress) for the stress programme (Fig 17) and for a current variable amplitude programme for transport aircraft.

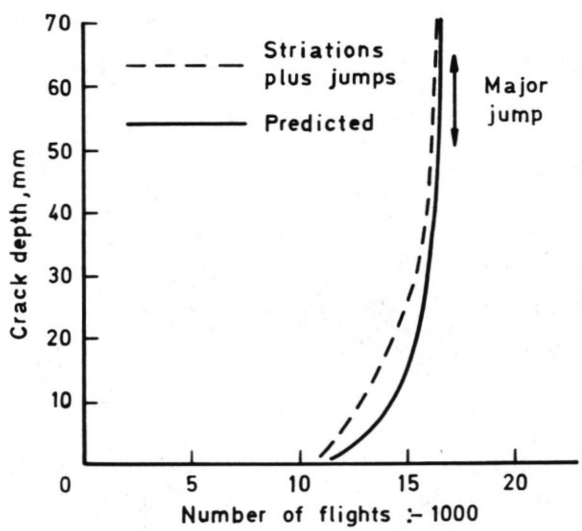

Fig 21. The measured and predicted crack growth periods for example C.

The failure analysis indicated that cracking had been missed during inspection perhaps because it initiated in a region where cracks were not anticipated. This is a major problem in the containment of aircraft fatigue, the fact that cracks of catastrophic size may form in areas where the initial predictions based upon fatigue stress levels alone indicate a low probability of failure. It follows that cracks in highly stressed regions, although potentially more dangerous, are more likely to be detected because that region is more likely to be inspected.

4. Discussion of the Analysis Techniques

4a. Limitations to the Fracture Mechanics Analysis

The prediction of fatigue crack growth rates by fracture mechanics involves several discrete processes each with its associated problems. Firstly, the description of the working loads in the structural component. This subject is beyond the scope of the present paper but one point is obvious. Any schedule of loads used for testing or calculation must be related to those occurring during service as accurately as possible. The complexity of aircraft structures and the variations in service usage make this difficult enough in civil transport aircraft but with military aircraft the continual changes in aircraft role, operating procedures and aircraft configuration multiply the problems and improved data acquisition techniques are having to be developed.

Secondly, the working loads must be converted to stresses in the region of the component under consideration. Finite element stress analysis methods are now widely used in the aircraft industry. Currently it is possible in the UK(7) to analyse a major component of an aircraft to predict the stress levels in the major parts of the component with errors of less than 5%. Limitations again arise because of the complexity of aircraft structures, where multi-axial loading systems are common as are complicated combinations of different materials and shapes joined by a profusion of techniques.

Thirdly, the stress intensity factor for a growing crack must be calculated from the prescribed stresses. This involves the choice of correction factors which allow for the effects of component geometry and for effects of curvature of the fatigue crack front. Design handbooks(17) are now available outlining a great variety of calibrations for the stress intensity factors in engineering components. Superficially prediction of the stress intensity factor with errors of less than 10% appears readily possible, but in practice other more nebulous variables such as levels of residual stresses, load shedding, wear in joints and plastic relaxation make this level of accuracy hard to achieve.

Finally, the rates of fatigue crack growth may be predicted using one of the relationships between crack growth rates and the range in stress intensity factor ΔK. The predictions all require a factor, usually derived from constant amplitude testing, to allow for material variations from alloy to alloy and with heat treatment and crack growth direction. The material factor should also include the effects of different environments, in particular taking account of the effects of water vapour (Fig 22).

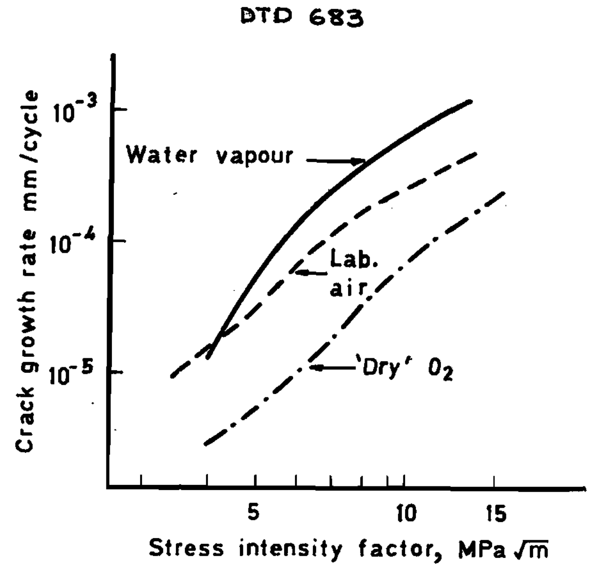

Fig 22. Crack growth rates30 in water vapour saturated air, laboratory air and dry oxygen. Tested under constant amplitude at 100 Hz.

Furthermore a salt laden environment can produce fatigue crack growth rates of up to one order of magnitude greater than dry laboratory air(7). If the loading programme involves mixtures of high and low load levels, account may also have to be taken of interaction effects. It has been demonstrated(14,16,18,19) that large tensile loads make subsequent smaller loads less damaging whilst the reverse is true for large compressive loads. Some success has been achieved in the development of more complicated models(20,21) to predict load interaction effects. A good example of a beneficial load interaction effect can be seen in Fig 5 where the 10% reduction in loading severity between the severe and standard duty cycles produced a greater retardation than predicted by the simple fracture mechanics model. Unfortunately, the high stress dependence of the fatigue cracking phenomenon ($\sim\sigma^3$) results in $\pm 30\%$ errors in predicted crack growth rates from $\pm 10\%$ errors in stress intensity factor before any consideration is given to the effects of environment or load interaction. Conversely, if fatigue stresses are being predicted from measurements of striation spacings a large scatter in measured rates may still produce an accurate measure of stress level(22).

4b. Limitations to the Quantitative Fractographic Techniques

It has already been shown that the fractographic technique depends upon the recognition of fatigue striations or crack jumps upon the fracture surface. Superficially, these two features are versions of the same phenomenon but on different scales. In fact the mechanisms of their formation are different. The microscopic striation is delineated by co-ordinated dislocation activity on slip planes (Fig 1), thus limiting its appearance to those metals with a suitable crystal structure and unfortunately excluding many engineering steels. The crack jump is a burst of overload failure bounded and hence delineated by periods of more stable fatigue crack growth. It has been found, if a mixture of fatigue striations and crack jumps is observed, both must be taken into account in the measurement of fatigue crack growth rates since the striation spacings alone will underestimate the growth rate. It has been found[23] that a crack will jump when the maximum stress intensity factor of the fatigue cycle (K_{max}) reaches a critical value (K_{1c} or K_c) appropriate for the section thickness. This critical condition is sensitive to changes in crack front shape so that a jumping crack can be stabilized by an increase in its crack front length with an associated reduction in stress intensity factor, essentially bowing as it jumps and re-straightening during subsequent stable growth. This occurs both under uniform loading conditions at high values of K_{max} and particularly at high mean stresses and under complex loading conditions, so that it cannot be immediately assumed that a crack jump is evidence for the occurrence of a high load in the fatigue spectrum. It has been found that crack jumps can provide useful additional information, ie under uniform or near-uniform loading conditions one crack jump occurs per loading cycle and under complex loadings a crack jump may be related to a particular load providing attention is paid to the fracture mechanics conditions governing the jumping.

The quantitative fractographic technique may also be limited at very slow crack growth rates by an inability of the fractographic equipment to resolve striations. The best electron optical equipment available to us has a resolution of 3 nm in the fractographic mode but the discrimination of individual striations with a spacing of less than 100 nm is rare because of the likelihood of surface contamination. It should also be pointed out that other mechanisms of fatigue crack growth occur without the formation of striations. The value of extrapolation techniques based upon fracture mechanics is obvious in these circumstances.

5. Discussion. The Analysis of Fatigue Problems

The fatigue failures that have caused us most concern have been unexpected initially. That is they have occurred at places in the structure where no cracking has been predicted by stress analysis or detected in a major structural test. Alternatively cracking may have occurred prematurely but where expected. Two features of the fatigue phenomenon can explain these unexpected failures. Firstly, the strong dependence of both initiation and crack growth upon stress level. In general terms the fatigue life, initiation and growth, will be inversely proportional to the design stress (stress in MPa per 'g') to a power between 6 and 8. Stresses 10% higher than anticipated could result in half the expected fatigue life. The problems of predicting fatigue performance from a knowledge of aircraft loadings have already been addressed. The occasional failure of a fatigue substantiation test to predict service failures may also be attributed, in some cases, to an inadequate simulation of the real stress and environmental conditions. However the second, less predictable, feature of the fatigue phenomenon is the effects of surface finish and material quality on the initiation period. Provided the stress level is above a threshold value the damaging effects of surfaces and material of poor quality will occur almost independent of the level of stress. This feature has already been demonstrated but is confirmed in Fig 23 where the fraction of the total fatigue life, N_i/N_f, taken to initiate a crack is plotted for various notches as a function of stress level[24]. A sharp notch, metallurgical defect, corrosion pit, or damaged fastener hole will all produce a low value of N_i/N_f and the loss of the large initiation period. This of course is disastrous in highly stressed regions of structure but more insidiously will produce unexpected cracks in lower stressed regions.

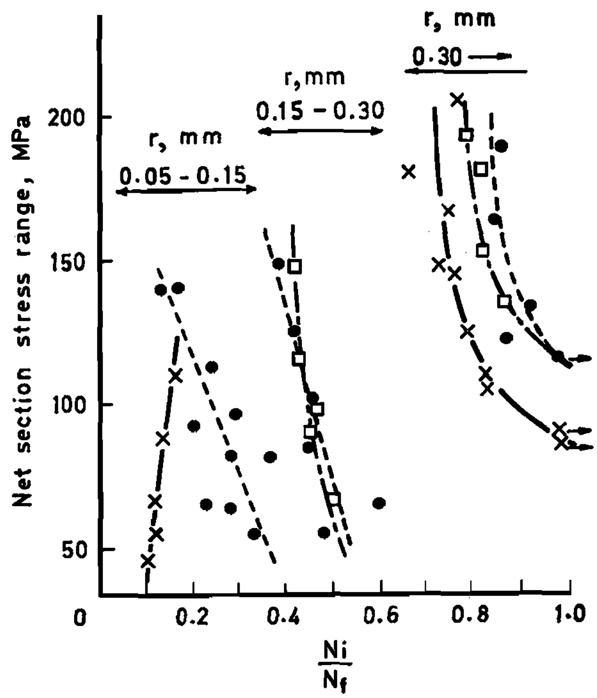

Fig 23. The fraction N_i/N_f of the total fatigue life, N_f, taken to initiate fatigue cracks 0.125 mm deep at notches of different radii (r) and depths[24]. (X 6.4mm deep, ● 3.2mm deep, □ 1.3mm deep.)

6. Concluding Remarks

The quantitative analysis of fatigue failure has made an enormous advance in the last decade with the emergence of techniques based upon fracture mechanics. At the same time quantitative fractographic techniques have developed to the extent that extremely detailed post-failure analyses are now routine. New materials and improved aircraft designs seem likely to reduce the incidence of fatigue failure in the future but for current aircraft it seems that safety can only be assured by assuming that the worst imaginable combinations of stress level and material quality exist in service and in the worst cases will occur in combination.

Appendix A
A Simplified Analysis by Fracture Mechanics

Most analyses are based upon the concepts[4] of Linear Elastic Fracture Mechanics (LEFM) in which the elastic solutions for the stress fields around cracks produce stress components, σ_{ij}, recognizably of the same form. Here in polar coordinates:

$$\sigma_{ij}(r, \theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta),$$

if terms with small effects are neglected. This elastic solution predicts infinite stresses at the crack tip ($r = 0$) which are alleviated in practice by a small zone of plastic deformation. It is basic to LEFM that this plastic zone is small in size compared to the extent of the perturbations in the elastic stress field caused by the presence of the crack. The stress intensity factor K is a function of the loading, of the size and shape of the crack and of the geometry of the cracked component. K has the dimensions of stress x $\sqrt{\text{crack depth}}$. Three discrete cracking modes occur (K_1, K_2, and K_3) of which that most commonly encountered is K_1 for the crack opening mode as opposed to the sliding and tearing modes. Solutions for the stress intensity factor take account of the loading and geometrical effects in different ways. For preference we follow the method of Rooke and Cartwright where a normalizing factor, K_o, is employed. K_o describes K in the absence of geometrical boundaries. For example, for a small edge crack of depth 'a' in a sheet of infinite size:

$$K_o = \sigma\sqrt{\pi a}$$

where σ is the applied uniaxial tensile stress. For the same crack in a sheet of finite size:

$$K_1 = \frac{K_1}{K_o} \sigma\sqrt{\pi a}$$

K_1/K_o contains correction factors for the effects of component boundaries based upon assumptions such as the length to width ratio of the sheet and whether bending of the sheet has been restricted. Values of K_1/K_o have been plotted as a function of the crack depth to sheet width ratio. Solutions for K_1/K_o for many other specimen geometries are available. Corrections for the shape of the crack front ϕ may need to be included in K_1/K_o and the crack depth a may be increased by addition of the plastic zone size r_p.

A2. Fracture Toughness and Residual Strength

Extensive testing has established that, for a particular alloy and heat treatment, there is a critical value of stress intensity factor at which a cracked component will fail completely. For plane strain situations this critical value is independent of component size and is referred to as the plane strain fracture toughness K_{1c}. It follows that, if K_{1c} is known for an alloy, then the relationship between failure stress σ_c or residual strength and critical crack depth a_c is known provided the geometrical correction factor K_1/K_o is also known:

$$K_{1c} = \frac{K_1}{K_o} \sigma_c\sqrt{\pi a_c}$$

Commonly, the residual strength will be prescribed as some fraction of the limit load and the critical crack depth will be calculated. For plane stress situations the fracture toughness K_c is dependent upon the thickness of the component.

A3. Prediction of Fatigue Crack Growth Rates

Several laws for fatigue crack propagation have been presented in recent years but the tentative suggestion of Paris and Erdogan[25] that:

$$\frac{da}{dn} = \frac{\Delta K^4}{M}$$

has possibly proved the most popular. In this formula the crack extension per load cycle, da/dn, is related to the range in stress intensity factor ΔK through the fatigue cycle. M is a constant dependent upon the material. Important modifications to this formula were made by Pearson[26], who normalized the curves for different materials using the Young's modulus, E:

$$\frac{da}{dn} = M_1\left(\frac{\Delta K}{E}\right)^n$$

and by Forman, Kearney and Engle[27] who incorporated the effects of the fracture toughness K_c of the material and the ratio R of minimum to maximum stress in the fatigue cycle:

$$\frac{da}{dn} = \frac{M_2(\Delta K)^n}{(1-R)K_c - \Delta K}$$

The examples of fracture mechanics predictions of growth rates in this paper have been based upon a slight empirical modification of the Forman equation (Fig 24):

$$\frac{da}{dn} = \frac{M_3(\Delta K)^n}{[(1-R)K_c - \Delta K]^{\frac{1}{2}}}$$

In the examples, individual contributions to the total crack growth rate have been calculated for each stress cycle in the complex programmes of load cycles representing aircraft flights assuming that no interactions have occurred between successive large and small loads. This approach is more than adequate for the purposes of the present demonstration and may be generally valid when the plastic zone size of the largest loads is of the order of the crack extension per flight. However, longer range load interaction effects do occur and yet more complicated formulae are available to account for them with varying degrees of success. The models of Willenborg[20] and Wheeler[21] have received much attention and the concepts(28,29) of crack closure may prove to be a suitable basis for crack growth rate predictions. The prediction methods considered in this paper have involved the cycle-by-cycle addition of increments of crack growth individually calculated from a preferred crack growth law. A second technique involves the prediction of growth rates by correlation with laboratory growth rate data essentially using the stress intensity factor to scale crack growth rates for stress level and crack depth, but taking no account of the individual components of the loading programme. This subject is well reviewed by Schütz(31).

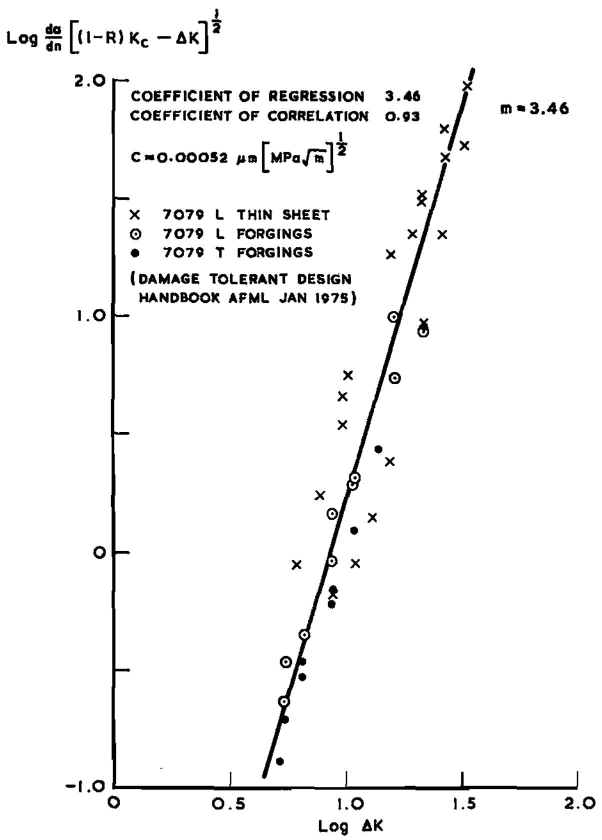

Fig 24. Laboratory data for fatigue crack growth in 7079 aluminium alloy in sheet and forging form. The growth rate, da/dn, is weighted in accordance with the modified Forman formula to allow the constants m and c to be determined.

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