METHOD FOR THE EVALUATION VERTICAL TAIL SURFACES OF A LIGHT AIRCRAFT BASED ON LATERAL FLYING QUALITIES

Memon Goran[1]
Aeronautical institute, Belgrade, Federal Republic of Yugoslavia

Abstract

The flying qualities requirements defined yields upper and lower limits on stability mode frequencies, damping ratios and time constants. It is relatively straight forward and less time for method to establish mode characteristics limits apropriate to the aircraft class, his flight phase and required level of flying qualities.Vertical tail surfaces design method based on the airplanes flying qualities is presented in this paper. It yields the explicit solution in the terms of vertical tail area and position. Simple and fast analisys, using Dutch roll mode solution, yields the explicit solution in the terms of the vertical tail area and its position relative to the wing, in less time and steps than the classical, iterative, vertical tail surface evaluation methods. Finally, method is tested in numerical example in case of an light airplane. Olso, results are compare with other light aeroplanes of same category and same flight phase.

Nomenclature

q	= dynamic pressure,N/m^2
m	= mass,kg
ρ	= air density,kg/m^3
b	= wing span,m
S_w	= wing planform area,m^2
A_w	= wing aspect ratio
A_v	= vertical tail aspect ratio
I_x,I_y,I_z	= moments of inertia about X,Y,Z axes respectively,kgm^2
I_{xz}	= products of inertia,kgm^2
ω_{nd}	= natural frequency of Dutch roll mode.rad/s
ζ	= damping ratio of Dutch roll mode
β	= angle of sideslip,rad
ψ	= angle of yaw,rad
σ	= sidewash angle,rad
$\frac{d\sigma}{d\beta}$	= variation of sidewash angle with angle of sideslip
ω_ϕ	= frequency of rotational velocity about Y axe
$\lambda_{\frac{c}{4}}$	= sweep angle on 25% mean aerodynamic cord (MAC)
u,v,w	= pertubed velocity along X, Y, Z axes respectively, m/s
$\overline{V}_v$	= vertical tail volume coefficient
$C_{n_{\delta_r}}$	= variation of yawing moment coefficient with rudder angle, rad^{-1}
$C_{n_{\delta_r}}$	= variation of side force coefficient with rudder angle, rad^{-1}
$C_{l_{\delta_r}}$	= variation of rolling moment coefficient with rudder angle, rad^{-1}
C_{l_p}	= variation of rolling moment coefficient with roll rate, rad^{1}
C_{l_r}	= variation of rolling moment coefficient with yaw rate, rad^{-1}
C_{y_r}	= variation of side force coefficient with yaw rate, rad^{-1}
C_{y_p}	= variation of side force coefficient with roll rate, rad^{-1}
C_{y_β}	= variation of side force coefficient with angle of sideslip, rad^{-1}

Subscripts

d	= Dutch Roll mode
v	= vertical tail
w	= wing
x,y,z	= X, Y, Z axes respectively
xz	= products of inertia

[1] Senior Research Engineer, Aeronautical Institute

n = naturall
ϕ = angle of roll

Introduction

The lateral characteristics of the aircraft are important in determinig flying qualities. More dificultis in solving problem of lateral motion is because the modes of lateral motion are not so well separeted and because they involve all of the perturbation variables. Some important that the dutch roll mode, which has a frequency in the same band as human pilot frequency, should be well damped in order to avoid handling problems. Similarity in lateral and longitudinal modes is represent in short period modes, should have required damping, but othervise create several problems. First of all, is similarity of natural frequency of human pilot and time constant or frequency of the modes. Main reuquirement in designing an aircraft vertical tail surfaces is to choose proper location and area based on requirements for the flying and handling qualities.This important parametars are function of many aerodinamic, inertial and mass charcteristics. Foloving formulation present one simply algorithm in evaluation process.

Formulation

Decoupling of the six complete equations of motion for the aircraft represent in [1], the three lateral equations are :

$$\sum \Delta F_y = m*(\dot{V}+U*R-W*P) \tag{1.1}$$

$$\sum \Delta L = \dot{P}*I_x-\dot{R}*I_{xz}+Q*R*(I_z-I_y)-P*Q*I_{xz} \tag{1.2}$$

$$\sum \Delta N = \dot{R}*I_z-\dot{P}*I_{xz}+Q*P*(I_y-I_x)-R*Q*I_{xz} \tag{1.3}$$

If the flight path straight and the aircraft is slippping with its wing level, then $\beta=-\psi$. By requiring tat the equilibrium direction of the X axis belong the flight path and that there be no sideslip while in equilibrium, then $\dot{V}=\dot{v}, \dot{w}=0, \dot{U}=\dot{u}$. As the aircraft initialy is in unaccelerated flight $P_o, R_o=0$. Making these substitutions equations (1.1),(1.2),(1.3) becomes :

$$\sum \Delta F_y = m*(\dot{v}+U_0*r+u*r) \tag{2.1}$$

$$\sum \Delta L = \dot{p}*I_x-\dot{r}*I_{xz} \tag{2.2}$$

$$\sum \Delta N = \dot{r}*I_z-\dot{p}*I_{xz} \tag{2.3}$$

However, since the perturbations are asumed small, the products of the perturbations can be neglected, and equations (2.1),(2.2),(2.3) can be reduced to :

$$\sum \Delta Fy = m*(\dot{v}+U_0*r) \tag{3.1}$$

$$\sum \Delta L = \dot{p}*I_x-\dot{r}*I_{xz} \tag{3.2}$$

$$\sum \Delta N = \dot{r}*I_z-\dot{p}*I_{xz} \tag{3.3}$$

Using relation $\frac{\dot{v}}{U_0} \cong \dot{\beta}, U_0 \cong U$, for small perturbations equations (3.1), (3.2), (3.3), becomes :

$$\sum \Delta F_y = m*U_0*(\dot{\beta}+\dot{\psi}) \tag{4.1}$$

$$\sum \Delta L = \ddot{\phi}*I_x-\ddot{\psi}*I_{xz} \tag{4.2}$$

$$\sum \Delta N = \ddot{\phi}*I_z-\ddot{\theta}*I_{xz} \tag{4.3}$$

After the expand the forces and moments in terms of total derivative form :

$$\sum dF_y = \frac{\partial F_y}{\partial \beta}*d\beta + \frac{\partial F_y}{\partial \psi}*d\psi + \frac{\partial F_y}{\partial \phi}*d\phi + \frac{\partial F_y}{\partial \dot{\phi}}*d\dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}}*d\dot{\psi} \tag{4.2}$$

where forces in the Y direction are function of following parameters $(\beta, \phi, \psi, \dot{\phi}, \dot{\psi})$:

After that, and dividing trough by (S_w*q*b), and going to coefficient form equations (4.1), (4.2), (4.3) becomes :

$$\frac{I_x}{S_w*q*b}*\ddot{\phi} - \frac{b}{2*U}*C_{l_p}*\dot{\phi} + \frac{I_{xz}}{S_w*q*b}*\ddot{\psi} - \frac{b}{2*U}*C_{l_r}*\dot{\psi} - C_{l_\beta}*\beta = \frac{L_a}{S_w*q*b} = C_{l_a} \tag{5.1}$$

$$-\frac{I_{xz}}{S_w * q * b} * \ddot{\phi} - \frac{b}{2*U} * C_{n_p} * \dot{\phi} + \frac{I_z}{S_w * q * b} * \ddot{\psi} - \frac{b}{2*U} * C_{n_r} * \dot{\psi}$$

$$-C_{n_\beta} * \beta = \frac{N_a}{S_w * q * b} = C_{n_a}$$

(5.2)

In final form equation (4.1), (4.2), (4.3) and (5.1), (5.2) becomes :

$$\frac{-b}{(2*U)} * C_{y_p} * \dot{\phi} - C_{y_\phi} * \phi + (\frac{m*U}{S_w * q} - \frac{b}{2*U} * C_{y_r}) * \dot{\psi} - C_{y_\psi} * \psi +$$

$$\frac{m*U}{(S_w * q)} * \dot{\beta} - C_{y_\beta} * \beta = C_{y_a}$$

(6.1)

$$\frac{I_X}{(S_W * q * b)} * \ddot{\phi} - \frac{b}{(2*U)} * C_{l_p} * \dot{\phi} + \frac{I_{XZ}}{(S_W * q * b)} * \ddot{\psi} -$$

$$\frac{b}{(2*U)} * C_{l_r} * \dot{\psi} - C_{l_\beta} * \beta = C_{l_a}$$

(6.2)

$$-\frac{I_{XZ}}{(S_W * q * b)} * \ddot{\phi} - \frac{b}{(2*U)} * C_{n_p} * \dot{\phi} + \frac{I_Z}{(S_W * q * b)} * \ddot{\psi} -$$

$$\frac{b}{(2*U)} * C_{n_r} * \dot{\psi} - C_{n_\beta} * \beta = C_{l_a}$$

(6.3)

These equations are uncoupled, linearized, lateral equatins of motion.

Linearized eqations (6.1), (6.2), (6.3) represent lateral motion.By examine transfer function $\beta(s) = f(\delta_r)$, shows that the roll subsidence pole is effectively canceled by the zero in the numerator. Thus it is considered that the Dutch roll mode consists of only sideslip and yaw. In the case of pure sideslip $\beta = -\psi$, yields $\phi = 0$. Considering only a δ_r input with $\theta = 0$, and neglectic Cy_r derivatives, equations (6.1), (6.2), (6.3) becomes :

$$(\frac{m*U}{S_w * q}) * \Psi(s) + \left[\frac{m*U}{(S_w * q)} * (s) - C_{y_\beta}\right] * \beta(s) =$$

$$C_{y_{\delta_r}} * \delta_r(s)$$

(7.1)

$$\left[\frac{-J_{xz}}{(S_w * q * b)} * (s) - (\frac{b}{2*U} * C_{l_r})\right] * \psi(s) - C_{l_\beta} * \beta(s) =$$

$$C_{l_{\delta_r}} * \delta_r(s)$$

(7.2)

$$\left[\frac{-J_z}{(S_w * q * b)} * (s) - (\frac{b}{2*U} * C_{n_r})\right] * \dot{\psi}(s) - C_{n_\beta} * \beta(s) =$$

$$C_{n_{\delta_r}} * \delta_r(s)$$

(7.3)

The yawing moment equation with $\beta = -\psi, \phi$ equation (7.3) can be rewriten :

$$\frac{-Jz}{(Sw * q * b)} * (s) - (\frac{b}{2*U} * Cn_r) * \psi(s) - Cn_\beta * \beta(s)$$

$$= Cn_{\delta r} * \delta r(s)$$

(8.3)

Taking the characteristic equation and dividing trough $\frac{J_z}{S_w * q * b}$ equations (8.3) becomes :

$$\left[(s)^2 - (\frac{S_w * q * b}{I_z}) * \frac{b}{2*U} * C_{n_r}(s) + (\frac{S_w * q * b}{I_z}) * C_{n_\beta}\right] *$$

$$\beta(s) = 0$$

(8.4)

From an examination it can be seen that for a given altitude the natural frequency :

$$\omega_{nd}^2 = \frac{C_{n_\beta} * S_w * q * b}{J_z}$$

(9)

evaluating, it becomes :

$$\omega_{nd} = U * \sqrt{\frac{Cn_\beta * Sw * \rho * b}{2 * Jz}}$$

(10)

products natural frequency and damping ratio of Dutch roll mode are:

$$2 * \zeta_d * \omega_{nd} = -\frac{S_w * \rho * U * b^2}{4 * I_z} * C_{n_r}$$

(11)

and damping ratio only :

$$\varsigma_d = -\frac{C_{n_r}}{8}\sqrt{\frac{2*S_w*\rho*b^3}{I_z*C_{n_\beta}}} \tag{12}$$

Military specification MIL-F-8785C defines characteristic of Dutch roll mode in 3.3.1.1 (**Lateral-directional oscilations- Dutch roll**) . This document present some view of design methods an light airplane, considering class I airplanes, and requirements for Level 1 Flying qualities, the frequency, ω_{nd}, and damping ratio, ζ_d, of the lateral -directional oscillations following a yaw disturbance input shall exceed the minimum values in **Table1.**

Table 1 Dutch roll mode requirements

Flight Phase	Min ζ_d	Min $\zeta_d*\omega_{nd}$ (rad/s)	Min ω_{nd} (rad/s)
A	0.19	0.35	1.0
B	0.08	0.15	0.4
C	0.08	0.15	1.0

Derivatives equations becomes :

$$C_{n_\beta} = C_{n_\beta}(wing+body) + C_{n_\beta}(vertikal \cdot tail) + C_{n_\beta}(ventral \cdot fin) \tag{13}$$

$$C_{n_\beta}(vertikal \cdot tail) = C_{L\alpha_v}*(1-\frac{d\sigma}{d\beta})*\eta_v*\frac{X_v}{S_w*b} \tag{14}$$

$$C_{n_r} = C_{n_r}(wing+body) + C_{n_r}(vertical \cdot tail) + C_{n_r}(ventral \cdot fin) \tag{15}$$

$$C_{n_r}(vertical \cdot tail) = C_{L\alpha_v}*2*X_v^2*\eta_v*\frac{S_v}{Sw*b^2} \tag{16}$$

In the analysis equation **(14)** and **(16)**, some important is distribution of parametars Xv(location of a.c. vertical tail due to c.g.) and Sv (surface of vertical tail). This parameters are function of aerodynamic derivatives.Analysis equations **(14)** and **(16)** shows that will be posible create relations betwen position of aerodynamic center of vertical stabilizer due to center of gravity(c.g.)., and planform area of vertical tail(S_v). This parametars are functions of aerodynamic derivatives **Cn**$_\beta$, **Cnr**. In the same time aerodynamic derivatives **Cn**$_\beta$, **Cnr** are functions of flying qualities requirements, dinamic presure and free stream velocity

From equation **(10)** becomes:

$$C_{n_\beta} = \frac{2*I_z*\varpi_{nd}^2}{U^2*S_w*\rho*b} - \tag{17}$$

and from equation **(12)** becomes:

$$C_{n_r} = -\frac{8*I_z*\zeta_d*\varpi_{nd}}{U*S_w*\rho*b^2} \tag{18}$$

If in the first iteration we have alredy defined configuration wing-body without ventral fin , equations (13) and (15) must be rewriten :

$$C_{n_\beta}(vertical \cdot tail) = C_{n_\beta} - C_{n_\beta}(wing+body) \tag{19}$$

$$C_{n_r}(vertical \cdot tail) = C_{n_r} - C_{n_r}(wing+body) \tag{20}$$

Includimg equations **(16)** and **(17)** in equations **(11)**, **(13)**, **(14) i (15)** becomes :

$$C_{n_\beta} - C_{n_\beta}(wing+body) = C_{L\alpha v}*(1-\frac{d\sigma}{d\beta})*\eta_v*\frac{Xv}{Sw*b} \tag{21}$$

$$C_{n_r} - C_{n_r}(wing+body) = \frac{2*C_{L\alpha_v}*\eta_v*S_v}{S_w*b^2} \tag{22}$$

From equations (17) and (18) we have direct corelations betwen vertical surface area (Sv) , postion of aerodynamic center of vertical tail in function of dynamic parameters of Dutch roll mode (ω_{nd}, ζ_d) and aerodynamics, inertial and mass parametars :

- wing planform area (**S$_w$**)
- wing span **(b)**
- moment inertia about Z axe (**I$_z$**)

This parametars for an given conditions are constant. Only variable parametars are:

- free stream velocity **(U)**
- air density (ρ)

Parametar $\frac{d\sigma}{d\beta}$ are given from [2] :

$$(1-\frac{d\sigma}{d\beta})*\eta_v = 0.724 + 3.06*\frac{\frac{S_v}{S_w}}{(1+\cos\lambda_{\frac{c}{4}})} + 4*\frac{Z_w}{X_v} +$$

$$0.009 * A_w$$

(23)

Before investigating efect of stability derivative variation, the requirement for Dutch roll will be studied .Critical conditions for**MIL-F-8785C**chapter **3.3.1.1 (Lateral-directional oscilations- Dutch roll**) cheking process are minimum dynamic presure an maximum altitude of operational anvelope. For an given flight condition, dynamic presure are constant parametar. Only variable parametars are:

- vertical surface area **(S_v)**
- position of aerodynamic center of vertical stabilizer due to center of gravity(c.g.)., **(X_v)**

This facts indicates final aproach in evaluation **(S_v)** and **(X_v)**.

Numerical results

In numerical example for an light aeroplane preliminary
defined wing-body configuration with following parametars :

- wing planform area Sw = 13.3 m2
- wing span b_w = 9.649 m
- wing aspect ratio A_w = 7.
- body surface S_b = 5.826 m2
- stability derivativ:
  C_{n_β} (wing+body) = -0.0311 (1/rad)
- stability derivativ :
  C_{n_r} (wing+body) = -0.2287 (1/rad)
  flight conditions are :

- altitude H = 2000. (m)
- Mach number of free stream, M = 0.15

For this flight conditions are defined main prametars in evaluation lateral stability. Next step in evaluation is create function $S_V * X_V = f(\omega_{nd})$ for given flight conditions. This function is clearly shown in Figure 2. Olso are shown in Figure 2 boundary for statistical range of vertical tail volumetric parametar $\overline{V_v} = \frac{S_v * X_v}{S_w * b}$ (statistical range for light aeroplane categories 0.035 < V_v < 0.075) . For numerical example range products Sv*Xv are :

(4.49 < Sv*Xv< 9.62). Boundaries of this range is olso shown in Figure 3. Military specifications MIL-C-18244A chapter (3.1.1..3.1.2 Lateral control) define angle of roll frequencis range for human pilot characteristics :

$$0.6 < \zeta_\phi < 1.2$$

$$\omega_\phi > 0.46 + 1.46 * \zeta_\phi \text{, and :}$$

$$1.336 < \omega_\phi < 2.212$$

In Figure 1 are defined pilot iso-opinion curve for lateral-directional control [2] (chapter 12.9, page 525 ,Figure 12.12).In Figure 1 is shown general trend of pilot rating. From requirements for Level 1 dutch roll mode flying qualities $\zeta_d > 0.19$. The ratio $\left(\frac{\omega_\phi}{\omega_{nd}}\right)$ is a significant parametar in studying lateral-directional flying qualities. The major consequnece is that the ϕ response to aileron becomes non-oscillatory.The general preference for $\left(\frac{\omega_\phi}{\omega_{nd}}\right)^2 = 1$. is apparent.

Depending upon the value of ζ_d the optimum value of $\left(\frac{\omega_\phi}{\omega_{nd}}\right)$ may be different.

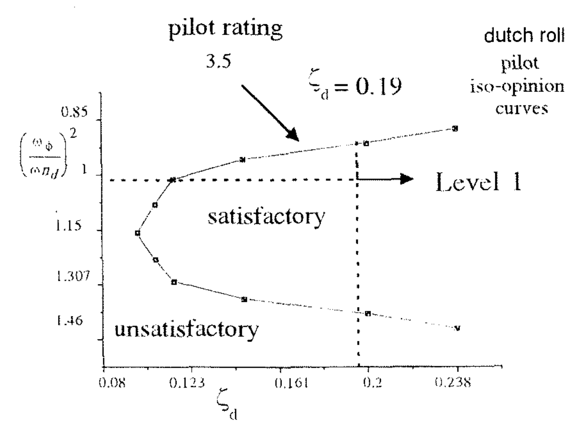

Figure 1

In Table 1 are defined minimum value of Dutch roll frequnecis ω_{nd} =0.1 rad/s. In this case optimal range Dutch roll frequnecis for human pilot chracteristics are (for numerical example):

$$1.336 < \omega_{nd} < 2.212$$

Some important fact in evaluation process is definition of optimum values for the handling qualities parameters and the definition of acceptable ranges for these parameters.

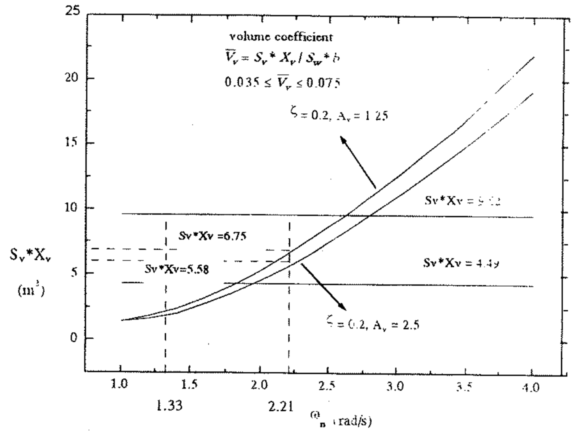

Figure 2

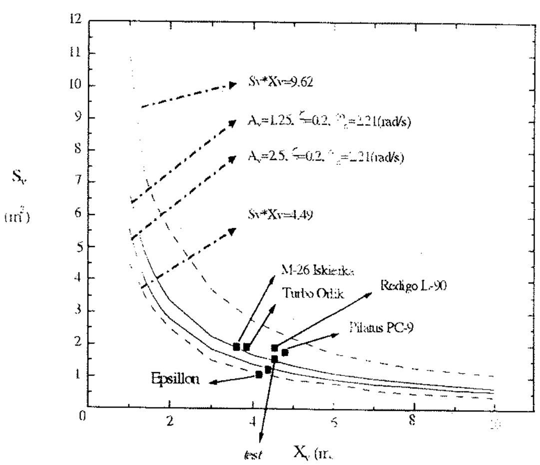

Figure 3

Conlusion

Some important fact in design process of piloted aircraft is configuration optimization procedure. due to pilot abilities and define task. Without this atributes we can quaranted project succes.Configuration optimisation defined by several number of atributes (Military specification requirements, human characteristics. ecsperimental data) which present posible solution anvelope limits.On that way project engineer have oportunity to look whole operational anvelope trough aerodynamical, mass and inertia parametars. Using this project algoritm project engineer can avoided large number of iteration and saving more time and project costs. In this paper present one example of defined procedure in evaluation process. In first step can make aproach to optimal solution area.
Olso , presented method able and compare analysis of realased projects of light aeroplanes in this category. Final result of this evaluation process is create system pilot-machine for performing the assigned task . That system must have positive subject opinion concering the suitability of the aircraft handling qualities.

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