AUTO-PILOT COMMAND-GENERATOR TRACKER DESIGN BY LEQG/LTR METHOD

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Abstract. The objective of this paper is to propose an autopilot Command-Generator Tracker (CGT) design by the Linear Exponential Quadratic Gaussian and Loop Transfer Recovery (LEQG/LTR) methodology. Since not only the optimal feedback but also the feedforward gains of the resulting controllers can take the covariances of both system and measurement noises into consideration, whereas the traditional LQG method cannot, the proposed method is more robust. This paper also derives the algorithms which can take all the time domain, frequency domain, and robustness design techniques into a unified method. An example of F-16 lateral autopilot design is given, which shows that the proposed method is more robust to disturbance, sensor noise, and parameter variations. In addition, the time domain responses are also better.

I . Introduction. In general, the performance of an aircraft is determined by the lateral mode of maneuvering command responses. The autopilot is always used to control the lateral modes and provide better response to the command inputs. The optimal Command-Generator Tracker (CGT)[1-3], consisting of both adaptive feedforward and robust feedback controllers, is a powerful design technique that can not only produce better time-domain performance but guarantee a zero steady-state error in response to a large class of command inputs. Recently, a lot of attention has been focused on the Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) techniques[1-8], which is characterized by integrating time domain optimization (LQG) with frequency domain approaches (LTR). The LQG/LTR method with state-feedback technique can provide some important guaranteed robustness properties, e.g., at least 60° of Phase Margin (P.M.) and −6dB of Gain Margin (G.M.) for each channel. In addition, the adoption of LTR process can preserve robustness of the system with state observer[4]. On the other hand, some reports[9-14] concluded that the optimal control systems obtained by the Linear Exponential Quadratic Gaussian (LEQG) and Loop Transfer Recovery (LEQG/LTR) methods were insensitive to the load disturbances and sensor noises. The reason is that the optimal feedback controller obtained by the proposed method can take the covariances of both system and measurement noises into consideration, whereas those controllers obtained by the LQG/LTR method cannot. However, the attention is only focused on the feedback control system design. In addition, the applications for an F-16 lateral autopilot [1-2] designed by the above methods suffer from the problem of high gain. This motivate us to develop the

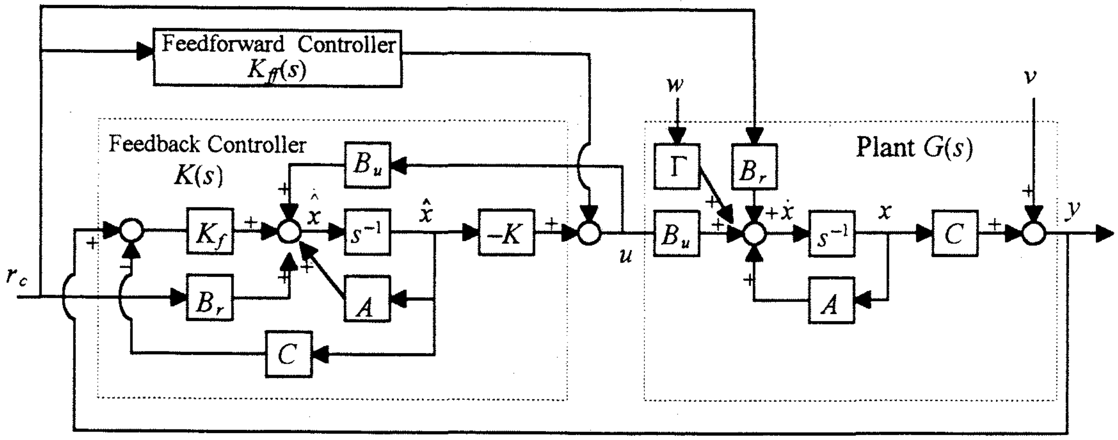

Fig. 1. LEQG/LTR compensator structure.

feedforward and feedback controllers by the proposed method. Therefore, the main contributions in this paper are that both adaptive feedforward and robust state-feedback controllers designed by LEQG and LEQG/LTR methods are originally derived in detail, and comparisons with the traditional LQG and LQG/LTR ones are also made.

By the derivation of this paper in Section II, we would demonstrate that the proposed method can take all the time domain, frequency domain, and robustness design techniques into a unified process. In addition, not only feedforward but feedback controllers obtained by the LEQG/LTR method can take account of the covariances of both system and measurement noises. Therefore, the proposed method would be more robust to disturbance, sensor noise, and parameter variations. In addition, the time domain responses would be also better. In Section III, the proposed method is applied for an F-16 lateral autopilot design[1-2], and comparisons with those results obtained by the traditional LQG/LTR method are also made. Finally, brief conclusions are drawn in Section IV.

II. Problem and Methodlogy Formulation. Let the dynamic equations of a controllable and observable multivariable system shown in Fig. 1 be

$$\dot{x}(t) = A(t)x(t) + B_u(t)u(t) + B_r r_c(t) + \Gamma w(t) \quad (1)$$

and

$$y(t) = C(t)x(t) + v(t) \quad (2)$$

where $x(t)$ is an n-dimensional state vector, $u(t)$ is an m-dimensional control vector, $r_c(t)$ is an m-dimensional deterministic inputs, $y(t)$ is a q-dimensional measurement vector, $A(t), B_u(t), B_r(t)$, Γ, and $C(t)$ are respectively $n \times n$, $n \times m$, $n \times m$, $n \times p$, and $q \times n$ matrices, $w(t)$ and $v(t)$ are p- and q-dimensional uncorrelated Gaussian white noise processes with zero-mean and covariances to be as

$$\begin{aligned} E\{w(t)w^T(\tau)\} &= W(t)\delta(t-\tau) \\ E\{v(t)v^T(\tau)\} &= V(t)\delta(t-\tau) \end{aligned} \quad (3)$$

and

$$E\{v(t)w^T(\tau)\} = 0 \quad (4)$$

respectively, where $E\{\cdot\}$ is an expectation function operator.

Let the command tracking error $e(t)$ be

$$e(t) = y(t) - r_c(t) \quad (5)$$

and then the optimal command-generator tracker can be obtained by minimizing the following LEQG performance index

$$J = \sigma E\{\exp[\frac{\sigma}{2}e^T(t_f)Q_f e(t_f) + \frac{\sigma}{2} \int_{t_0}^{t_f} e^T(t)Q(t)e(t) + u^T(t)R(t)u(t)dt]\} \quad (6)$$

where $\exp\{\cdot\}$ is an exponential function operator, Q_f is an m×m positive semi-definite weighting matrix for the terminal states, $Q(t)$ is an m×m positive semi-definite state weighting matrix, $R(t)$ is an m×m positive definite control weighting matrix, and σ is a positive number which is also a weighting factor of the LEQG method.

A. LEQG Problem Formulation with Perfect Measurement. In this section, the optimal control system design based on the LEQG method would be derived by applying the minimum principle of Pontryagin[15-17]. By taking the covariance of only system noise into consideration, the performance index defined by Eq.(6) can be rewritten in the form of noncooperative differential game[9,14] as follows

$$J = \underset{u(t)}{Min}\ \underset{w(t)}{Max}\{\frac{1}{2}e^T(t_f)Q_f e(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[e^T(t)Q(t)e(t) + u^T(t)R(t)u(t) - \sigma^{-1}w^T(t)W^{-1}(t)w(t)]dt\} \quad (7)$$

thus the Hamilton function can be obtained as (for the sake of easy presentation, the augment parameters are neglected hereafter)

$$H = \frac{1}{2}(r_c^T Q r_c - 2r_c^T QCx + x^T C^T QCx + u^T Ru - \sigma^{-1}w^T W^{-1}w) + P^T(Ax + B_u u + B_r r_c + \Gamma w) \quad (8)$$

where P is the costate (or Lagrange multiplier). Then the optimal control u^* can be obtained by satisfying the Euler-Lagrange equations as

$$\dot{x}^* = \frac{\partial H}{\partial P} = Ax^* + B_u u^* + B_r r_c + \Gamma w \quad (9)$$

$$\dot{P}^* = -\frac{\partial H}{\partial x} = C^T Q r_c - C^T QCx^* - A^T P^* \quad (10)$$

$$0 = \frac{\partial H}{\partial u} = Ru^* + B_u^T P^* \quad (11)$$

$$0 = \frac{\partial H}{\partial w} = -\sigma^{-1}W^{-1}w + \Gamma^T P^* \quad (12)$$

and with the transversality condition at terminal time

$$P^*(t_f) = Q_f x^*(t_f) \quad (13)$$

Thus, the relationships among the optimal costate P^*, optimal control u^*, and system noise w can be obtained as

$$u^* = -R^{-1}B_u^T P^* \quad (14)$$

and

$$w = \sigma W \Gamma^T P^* \quad (15)$$

and a set of 2n linear homogeneous differential equations can be obtained in matrix form as

$$\begin{bmatrix} \dot{x}^* \\ \dot{P}^* \end{bmatrix} = H_m \begin{bmatrix} x^* \\ P^* \end{bmatrix} + \begin{bmatrix} B_r \\ C^T Q \end{bmatrix} r_c \quad (16)$$

where the Hamiltonian matrix H_m of dimensions 2n×2n is defined as

$$H_m = \begin{bmatrix} A & -B_u R^{-1} B_u^T + \sigma \Gamma W \Gamma^T \\ -C^T QC & -A^T \end{bmatrix} \quad (17)$$

Let the solution of the costate P^* be as

$$P^* = P_c x^* + g \quad (18)$$

where P_c and g are some differentiable matrices to be determined as follows. Substituting Eq.(18) into (16), and after some manipulations, one has

$$[\dot{P}_c + P_c A + A^T P_c - P_c(B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)P_c + C^T QC]x^* + \dot{g} + [A^T - P_c(B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)]g + (P_c B_r - C^T Q)r_c = 0 \quad (19)$$

Since Eq.(19) is true for any x^* and r_c, it can be deduced that P_c is defined as

$$-\dot{P}_c = P_c A + A^T P_c - P_c(B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)P_c + C^T QC \quad (20)$$

and g is an adaptive feedforward term defined as

$$-\dot{g} = [A^T - P_c(B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)]g + (P_c B_r - C^T Q)r_c \quad (21)$$

Therefore, the optimal control input, consisting of both robust state-feedback and adaptive feedforward controllers, is obtained as

$$u^* = -R^{-1}B_u^T P_c x^* - R^{-1}B_u^T g \quad (22)$$

From practical point of view, we use suboptimal control strategy. The P_c is then defined by the following Controller Algebraic Riccati Equation (CARE)

$$P_c A + A^T P_c - P_c (B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T) P_c + C^T Q C = 0 \quad (23)$$

and g admits a steady-state solution to be as

$$g = -[A^T - P_c (B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)]^{-1} (P_c B_r - C^T Q) r_c \quad (24)$$

Therefore, the optimal controller can be obtained as

$$u^* = -K_c x^* + K_g r_c \quad (25)$$

where K_c is the optimal feedback gain matrix defined

$$K_c = R^{-1} B_u^T P_c \quad (26)$$

and K_g is the feedforward gain matrix defined as

$$K_g = R^{-1} B_u^T [A^T - P_c (B_u R^{-1} B_u^T - \sigma \Gamma W \Gamma^T)]^{-1} (P_c B_r - C^T Q) \quad (27)$$

From above derivations, we have demonstrated that the optimal control gain obtained by the LEQG method can take the covariance of system noise into account.

B. Formulation of LEQG with LTR Procedure. By applying the separation theorem, a Kalman filter can be used to provide the estimated state $\hat{x}(t)$, which is defined by the following state estimation equation

$$\dot{\hat{x}} = A\hat{x} + B_u u + B_r r_c + K_f (y - C\hat{x}) \quad (28)$$

where K_f is the Kalman filter gain matrix defined as

$$K_f = P_f C^T V^{-1} \quad (29)$$

and where P_f is the covariance of $x - \hat{x}$ propagated forward in time, defined as

$$P_f = E[(x-\hat{x})(x-\hat{x})^T] \quad (30)$$

which can be obtained by the following Filter Algebraic Riccati Equation (FARE)

$$\dot{P}_f = P_f A^T + A P_f + \Gamma W \Gamma^T - P_f C^T V^{-1} C P_f \quad (31)$$

with the boundary condition $P_f(t_0) = P_{f0}$. Therefore, by specifying the non-frequency-sensitive and/or frequency-sensitive weighting parameters W and V, the Kalman filter would be chosen to obtain suitable step response of the target feedback loop. In addition, the principal gains (singular values) of the return ratio $G_{kf}(s)$, sensitivity function $S_{kf}(s)$, and complementary sensitivity function $T_{kf}(s)$ at the plant output would meet the frequency-domain requirements, where

$$G_{kf}(s) = C(sI - A)^{-1} K_f \quad (32)$$

$$S_{kf}(s) = [I + G_{kf}(s)]^{-1} \quad (33)$$

and

$$T_{kf}(s) = [I + G_{kf}(s)]^{-1} G_{kf}(s) \quad (34)$$

Since the Kalman filter is an unbiased estimator, the statistics of the innovation term $K_f(y - C\hat{x})$ in Eq.(28) can be obtained respectively as

$$E[K_f(y - C\hat{x})] = 0 \quad (35)$$

and

$$Cov[K_f(y - C\hat{x})] = K_f V K_f^T \quad (36)$$

Therefore, by taking the covariances of both system and measurement noises into consideration, the original optimal control problem can be solved by replacing the performance index to be

$$J = \underset{u(t)}{Min}\, \underset{v(t)}{Max}\{\frac{1}{2}\hat{e}^T(t_f) Q_f \hat{e}(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [\hat{e}^T Q \hat{e} + u^T R u - \sigma^{-1}(y - C\hat{x})^T V^{-1} (y - C\hat{x})] dt\} \quad (37)$$

where $\hat{e}$ is the estimated tracking errors defined as

$$\hat{e} = C\hat{x} - r_c \quad (38)$$

Therefore, the optimal control input, consisting of both robust state feedback and adaptive feedforward controllers, can be derived by the same procedure from Eqs.(9) to (22), and the result is

$$u^* = -R^{-1} B_u^T P_c \hat{x}^* - R^{-1} B_u^T g \quad (39)$$

where P_c is defined as

$$-\dot{P}_c = P_c A + A^T P_c - P_c (B_u R^{-1} B_u^T - \sigma K_f V K_f^T) P_c + C^T Q C \quad (40)$$

and g is the adaptive feedforward term defined as

$$-\dot{g} = [A^T - P_c (B_u R^{-1} B_u^T - \sigma K_f V K_f^T)] g + (P_c B_r - C^T Q) r_c \quad (41)$$

For suboptimal control strategy, the P_c is defined by the following Controller Algebraic Riccati Equation (CARE)

$$P_c A + A^T P_c - P_c (B_u R^{-1} B_u^T - \sigma K_f V K_f^T) P_c + C^T Q C = 0 \quad (42)$$

and g admits a steady-state solution to be as

$$g = -[A^T - P_c(B_u R^{-1} B_u^T - \sigma K_f V K_f^T)]^{-1} (P_c B_r - C^T Q) r_c \quad (43)$$

therefore, the optimal controller can be derived as

$$u^* = -K_c \hat{x}^* + K_{ff} r_c \quad (44)$$

where K_c is the optimal feedback gain matrix defined in Eq.(25) and K_{ff} is the feedforward gain matrix defined as

$$K_{ff} = R^{-1} B_u^T [A^T - P_c(B_u R^{-1} B_u^T - \sigma K_f V K_f^T)]^{-1} (P_c B_r - C^T Q) \quad (45)$$

Therefore, the closed-loop system can be obtained as

$$\begin{bmatrix} \dot{x}^* \\ \dot{\hat{x}}^* \end{bmatrix} = \begin{bmatrix} A & -B_u K_c \\ K_f C & A - B_u K_c - K_f C \end{bmatrix} \begin{bmatrix} x^* \\ \hat{x}^* \end{bmatrix} + \begin{bmatrix} B_u K_{ff} + B_r & \Gamma & 0 \\ B_u K_{ff} + B_r & 0 & K_f \end{bmatrix} \begin{bmatrix} r_c \\ w \\ v \end{bmatrix} \quad (46)$$

and

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \hat{x}^* \end{bmatrix} + \begin{bmatrix} 0 & 0 & I \end{bmatrix} \begin{bmatrix} r_c \\ w \\ v \end{bmatrix} \quad (47)$$

From above derivations, we have also demonstrated that the gains of feedforward and feedback controllers obtained by the LEQG/LTR method can take the covariences of both system and measurement noises into consideration.

Since the stability and robustness of compensated system are not influenced by the feedforward controller, therefore, our focus is aimed on the state feedback control law of the linear exponential quadratic regulator. Let the transfer function of the observer-based compensator be as

$$K(s) = K_c (sI - A + B_u K_c + K_f C)^{-1} K_f \quad (48)$$

then the return ratio, sensitivity function, and complementary sensitive function at the output of the plant would be as

$$G_{GK}(s) = C(sI - A)^{-1} B_u K_c (sI - A + B_u K_c + K_f C)^{-1} K_f \quad (49)$$

$$S_{GK}(s) = [I + G_{GK}(s)]^{-1} \quad (50)$$

and

$$T_{GK}(s) = [I + G_{GK}(s)]^{-1} G_{GK}(s) \quad (51)$$

By specifying the weighting matrices Q and R, the dynamic compensator would recover the guaranteed robustness properties. In addition, the weighting factor σ can be manipulated to get better performance, which provides another degree-of-freedom for the designer. It should be noted that for either of the proposed method to design the return ratio at the input or output of the plant, in addition to the parameter σ can be tuned, all the weighting parameters W, V, Q, and R can also be modified to get better performance.

<u>C. Guaranteed Robustness Properties</u>. From those derived in Section II, it can be seen that if $\sigma > 0$ and $(A, Q^{1/2})$ is observable, both optimal feedforward and feedback gains obtained by the proposed method is more robust. The reason is that there is an additional positive weighting term $\sigma K_f V K_f^T$ which takes the covariances of both system and measurement noises into consideration, and is different from those controllers obtained by the LQG method.

In addition, if σ_{max} is the upper limit of σ to make the effective control weighting matrix R_{eff} to be positive-definite, where R_{eff} and σ_{max} are defined as

$$B_u R_{eff}^{-1} B_u^T = B_u R^{-1} B_u^T - \sigma K_f V K_f^T \quad (52)$$

and

$$R_{eff} > 0 \text{ for } \sigma < \sigma_{max} \quad (53)$$

After some manipulations with Eq.(42), the return difference identity can be obtained as

$$[I + H_o^T(s)] R_{eff} [I + H_o(s)] = R_{eff} + B_u^T(-sI - A^T)^{-1} C^T Q C (sI - A)^{-1} B_u \quad (54)$$

where H_0 is the open-loop transfer function, i.e.,

$$H_o(s) = -K_c (sI - A)^{-1} B_u \quad (55)$$

Since the state weighting matrix Q is positive semidefinite, one has

$$[I+H_o^T(s)]R_{eff}[I+H_o(s)] \geq R_{eff} \tag{56}$$

Therefore, it can be concluded that if $\sigma > 0$ and $(A, Q^{1/2})$ is observable, the compensated system obtained by the proposed method as shown in Fig.1 is always stable, i.e., the gain margin and phase margin are -6 dB to ∞ dB and 0° to $\pm 60^\circ$, respectively. The reason is that there is an additional positive weighting term $\sigma P_c K_f V K_f^T P_c$ in R_{eff} which takes the covariances of both system and measurement noises into consideration; therefore, the performance robustness of CGT design based on the LEQG/LTR method would be better than those obtained by the LQG/LTR method, which will also be shown later by computer simulation.

III. Numerical Example and Simulation Results.

Considering an F-16 lateral autopilot[1-2], the ill-conditioned system is augmented by a pre-compensator to balance the principal gains at zero frequency. The specification of the time domain requirement is to provide coordinated turns by causing the bank angle $\phi(t)$ to follow a desired command while maintaining the sideslip angle $\beta(t)$ at zero. The dynamic equations are as follows

$$\dot{x} = Ax + B_u u$$

and

$$y = Cx$$

where

$$x = [\beta \ \phi \ p \ r \ \delta_a \ \delta_r \ \varepsilon_\phi \ \varepsilon_\beta]^T$$

$$u = [u_\phi \ u_\beta]^T$$

and

$$y = [\phi \ \beta]^T$$

The numerical data for the system are also given in Ref. 1-2. The principal gains of return ratio $C(jwI - A)^{-1} B_u$ for the nominal model is shown in Fig. 2. It can be seen that the principal gain plots of F-16 model including the integrators and precompensator are balanced at zero frequency and where the slope is -20dB/decade, so the steady-state error of the closed-loop system would be zero. The proposed method can be applied as follows:

A. Kalman Filter Design for Target Feedback Loop. Let Γ, W, and V be assigned as[1-2]

$$\Gamma = I \tag{57}$$

$$W = diag[0.01 \ 0.01 \ 0.01 \ 0.01 \ 0 \ 0 \ 1 \ 1] \tag{58}$$

and

$$V = \rho_1 I \tag{59}$$

Suppose that the disturbances would couple into the system through directly on the states rather than the inputs, thus Γ is chosen by Eq.(57) instead of being as input matrix. In order to obtain time domain responses as well as good robustness properties, one can let ρ_1 be as

$$\rho_1 = 1 \tag{60}$$

Thus the Kalman filter gain matrix can be obtained and the result is the same as Ref. 1-2. Therefore, the unit-step responses of target feedback loop are shown in Fig. 3. In addition, the principal gain plots of return ratio, sensitivity function, and complementary sensitivity function for the target feedback loop are shown in Figs. 4 and 5. It can be seen that they can meet the time-domain and frequency-domain requirements.

B. LEQG Optimal Controller Design. Specifying the state and control weighting matrices as follows

$$Q = I \tag{61}$$

and

$$R = \rho_2 I \tag{62}$$

one then manipulates σ and ρ_2 to meet the time-domain requirement and recover the principal gains of the return ratio $G_{GK}(s)$ at the plant output. In general, the smaller ρ_2, the better the loop transfer recovery. Therefore, one can let that

$$\rho_2 = 10^{-8} \tag{63}$$

and

$$\sigma = 0.3 \tag{64}$$

Thus, the optimal feedback and feedforward control gain matrices can be obtained as

$$K_c = \begin{bmatrix} -3.6336 & 2.0844 & 0.3036 & 1.0446 & -0.0109 & -0.0012 & 0.0000 & 0.0000 \\ 0.5443 & 0.3480 & -0.0022 & -0.1048 & 0.0006 & 0.0002 & 0.0000 & 0.0001 \end{bmatrix} \times 10^6 \quad (65)$$

and

$$K_{ff} = \begin{bmatrix} 3.7600 & -6.7274 \\ 0.5951 & 1.0307 \end{bmatrix} \times 10^4 \quad (66)$$

By the adoption of feedforward controller, the problem of high gain can be well improved for both methods.

The principal gain plots of the return ratio $G_{GK}(s)$, the sensitivity function $S_{GK}(s)$, and complementary sensitivity function $T_{GK}(s)$ at the output of the compensated system are shown in Figs. 6 and 7. The unit-step responses of the bank angle $\phi(t)$ as well as the actuator inputs while maintaining the sideslip angle $\beta(t)$ at zero are shown in Figs. 8 and 9. For comparison purpose, the results obtained by the traditional LQG/LTR method (by letting $\sigma = 0$) are also shown in Figs. 6 to 9. It can be seen that the smallest principal gains of the return ratio for the proposed method at lower frequencies are increased, i.e., the capability to eliminate steady-state errors is increased. In addition, the condition numbers $(\bar{\sigma}/\underline{\sigma})$ or the separations between σ_i's for the proposed method are also decreased, i.e., the proposed method is more robust. The largest principal gains of sensitivity function obtained by the proposed method at lower frequencies are decreased, i.e., the disturbance rejection capability is increased. In addition, the maximal overshoot of the largest principal gains of sensitivity function at the crossover frequency w_c is also decreased, i.e., the proposed method can provide better noise rejection, dynamic decoupling, and reference command transmission. Moreover, the separations of the principal gains of the complementary sensitivity function at higher frequencies for the proposed method are also decreased, i.e., the ability to reject measurement noises, dynamic cross-coupling as well as parameter variations of the proposed method are better. On the other hand, in the time domain all the command tracking, overshoots of the system unit-step responses, cross-coupling effect as well as amplitudes of the actuators inputs are improved and reduced by the proposed method.

IV. Conclusions. From the previous derivation, we have shown that the proposed method can take all the time domain, frequency domain, and robust decoupling design techniques into a unified process. It should be also noted that in the design procedure, in addition to the tunable weighting factors W, V, Q, and R, the parameter σ of the LEQG method can also be manipulated to get better performance response. This provides another degree of freedom for the designer. By the results of numerical simulation, we also have demonstrated that the proposed method can make the system to be more robust to disturbance, sensor noise, and parameter variations. In addition, the time domain responses are also better.

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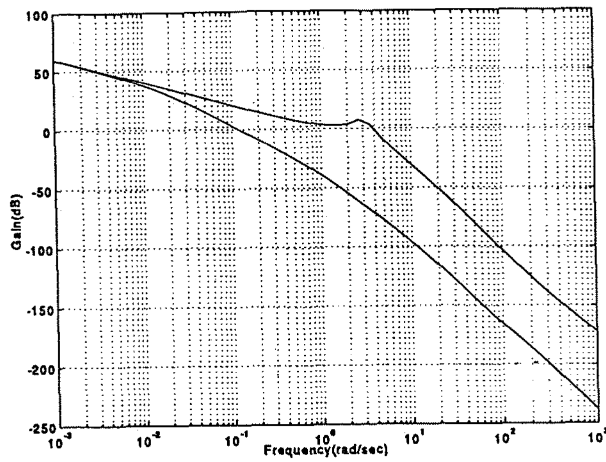

Fig. 2. Principal gains plots of $C(jwI - A)^{-1}B$ for nominal system.

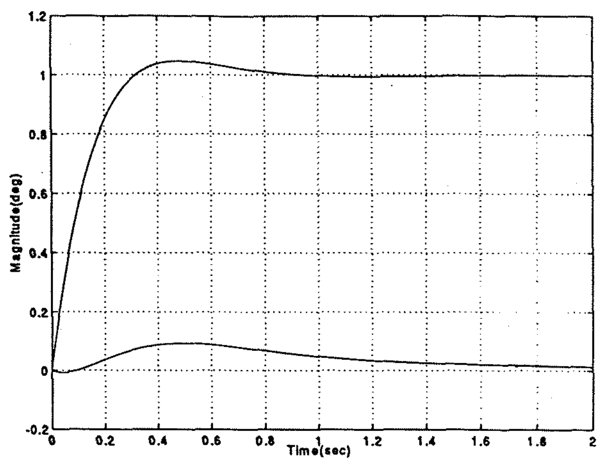

Fig. 3. Unit-step responses of ϕ for target feedback loop design.

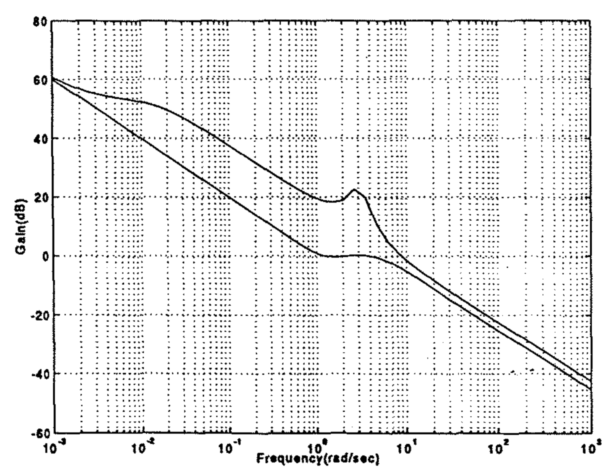

Fig. 4. Principal gains plots of $G_{kf}(s)$ for target feedback loop design.

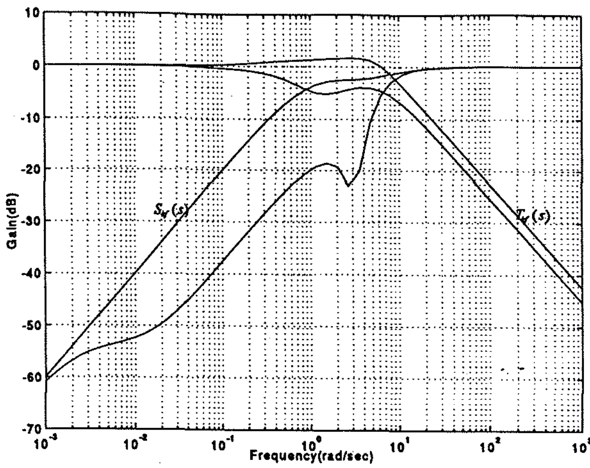

Fig. 5. Principal gains plots of $S_{kf}(s)$ and $T_{kf}(s)$ for target feedback loop design.

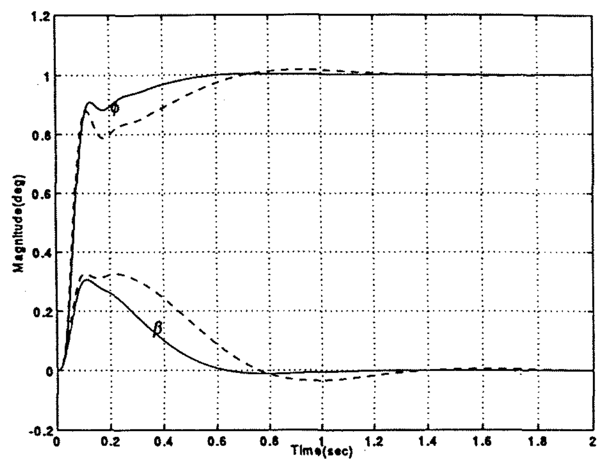

Fig. 8. Unit-step responses of ϕ by LEQG (solid) and LQG (dashed) methods for nominal system.

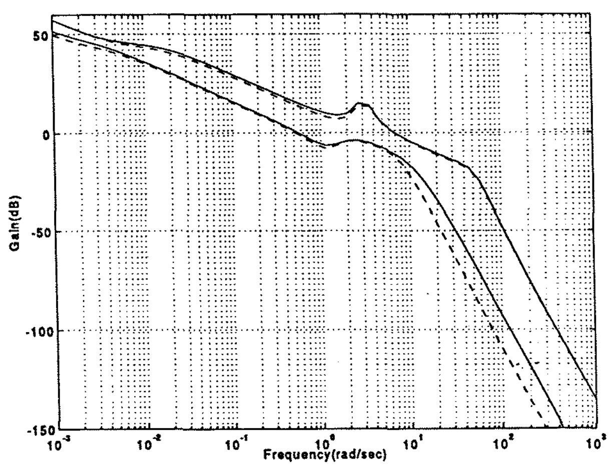

Fig. 6. Principal gains plots of $G(s)K(s)$ by LEQG (solid) and LQG (dashed) methods for nominal system.

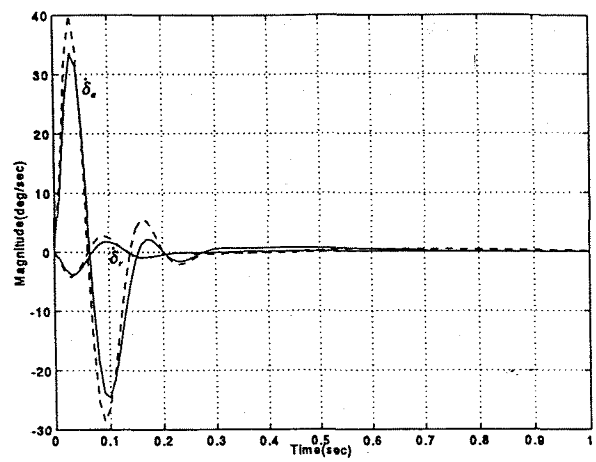

Fig. 9. Actuator rate inputs for unit-step responses of ϕ by LEQG (solid) and LQG (dashed) methods for nominal system.

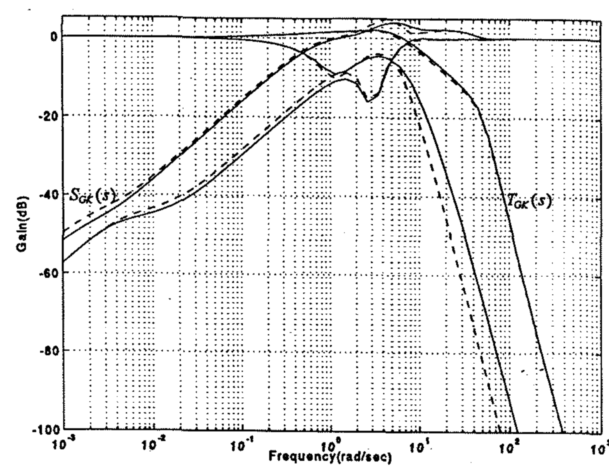

Fig. 7. Principal gains plots of $S_{GK}(s)$ and $T_{GK}(s)$ by LEQG (solid) and LQG (dashed) methods for nominal system.