The Robust Fault Diagnosis Based on the singular Value Decompositon

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Abstrct: The key of the fault diagnosis based on observer is the robustness to unknown inputs such as modelling error and noise, which usually cause the false alarm,thus decrease the performance of diagnosis, wunnenberg and frank apply unknown input observer to solving this problem, but their method are very complicate for application. In this study, based on matrix singular value decomposition, the authors suggest a design scheme of robust fault diagnosis observer,, and it is believed that this scheme is much easier to implement.Firstly, from the frequency domain perspective, the authors analyse the errors of the observer, considering the requirement of the fault diagnosis observer's robustness to unknown inputs, get a robust conditional equation of the observer to unknown inputs , then employ the matrix singular value decomposition theory to solving this equation, present a simple robust fault diagnosis observer design procedure, which can be iteratively computed in the digital micro-computer, finally, the numeral simulation is given and the simulation results verify the suggestion.

1. Introduction.

Growing attention has being given to the fault diagnosis of complex large-scale system. Beard (1971) suggested a fault diagnosis method based on observer, and many proceeding papers also studied this method. Whether this method is success or failure deponds on the robustness to unknown inputs,therefore, many researches have been done to this problem recently and many methods have been suggested such as UIO and KCF by Wunnenberg and Frank, but most of thoese methods are very complicate to implement, in this paper, based on the singular value decomposition, the author suggests a simple computer iterative algorithm.

2. Problem Descripition.

Considering a linear system containing unmodelling error and fault, the mathematic description is expressed as equation (1)

$$\begin{cases} \dot{x} = (A+\Delta A)x + (B+\Delta B)u + +N_x \upsilon_x + f_s \\ y = Cx + f_o \end{cases} \tag{1}$$

Where x is $R^{r\times 1}$ state vector, u is $R^{m\times 1}$ control inputs, y $R^{m\times 1}$ observer vector, υ_x disturbance vector, A, B, C are proper demision matrix, $\Delta A, \Delta B$ are modelling error. f_s represent the fault vector or matrix of executeror controller component, f_o the fault vector or matrix of sensor.

Combining the unknown factors into one yields equation (2)

$$\begin{cases} \dot{x} = Ax + Bu + Ed + f_s \\ y = Cx + f_o \end{cases} \tag{2}$$

$$E = [\Delta A, \Delta B, N_x], d^T = [x^T, u^T, \upsilon_x^T].$$

The observer of equation (2) is

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + bu + k(\hat{y} - y) \\ y = c\hat{x} \end{cases} \tag{3}$$

let

$$\varepsilon = \hat{x} - x, e = w(\hat{y} - y)$$

then
$$\begin{cases} \dot{\varepsilon} = (A+KC)\varepsilon - Ed - f_s - Kf_o \\ e = WC\varepsilon - Wf_o \end{cases} \tag{4}$$

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let $H = WC$, Then

$$\dot{\varepsilon} = (A + KC)\varepsilon - Ed - f_s - Kf_o$$
$$e = H\varepsilon - Wf_o \quad (5)$$

When there arenot modelling error, noise and fault, if k the gain of the observer's feedback k is properly selected to make the matrix $(A+KC)$ stable, then

$$\lim_{t\to\infty} \varepsilon(t) = 0, \lim_{t\to\infty} e(t) = 0 \quad (6)$$

When fault occures to the system, equation (6) doesnot held, that is

$$\lim_{t\to\infty} \varepsilon(t) \to 0, \lim_{t\to\infty} e(t) \to 0$$

therefore, employing simple decision-making (such as threshold logic), whether faults occure or not can be determined., when $e(t) > \delta$, $r^*(t) = 1$, system is in fault. when $e(t) < \delta$, $r^*(t) = 0$, system is in normal. The problem is that the unknown inputs such as the modelling errors and disturbances also influence $\varepsilon(t)$ and e(t). and make the error $\varepsilon(t)$ and e(t) away from the nominal status, thus the unknown inputs are the key factors that cause the false alarm, mis-detection and degrade the detection performanc. The design task of robust observer is to design an observer, which is robust to unknown inputs and sensitive to fault.

3.Sloution to the Problem.

Perform laplacian transformation to equation (5) and put it in order,get equation (7)

$$e(s) = -H(SI - A - KC)^{-1}Ed$$
$$-H(SI - A - KC)^{-1}Kf_o - Wf_o$$
$$-h(SI - a - KC)^{-1}Kf_s \quad (7)$$

from equation (7), it is clear that the robust condition to unknown inputs is

$$H(SI - A - KC)^{-1}E = 0, \text{ i.e. } HE = 0. \quad (8)$$

so the problem focus on getting H that satisfy equation (8), if H has p rows and rank$E > n-p$, then H can be got by directly solving equation (8), but when rank $E > n-p$, H cannot be got by directly solving equation (8), the authors make use of the singular value decomposition, get a simple iterative algorithm to get H that satisfy eq.(8)

Lema 1. The p that has the best orthogonality with $z_1, z_2, \cdots, z_q$ can be got by the following optimization [2][3].

$$J = \min \sum_{k=1}^{q} \left\| p^T z_q \right\|_F^2 \quad (9)$$

Lema 2. The p, which make equation (9) minimal, can be got by the following singular value decomposition [2][3].

let $z = [z_1, z_2, \cdots, z_q]$, suppose the singular value decomposition of z be

$$z = U\Sigma V \quad (10)$$

$(\Sigma = dig(\sigma_1, \sigma_2, \cdots \sigma_n, 0, \cdots, 0))$

then $p = [u_1, u_2, \cdots, u_p]^T$,where u_i is the i-th column of u and

$$pz = 0 \quad (11)$$

According to Lema.1 and Lema.2, considering equation (8), it is easy to get Therory.1.

Therory.1. For the system described by equation (2),take equation (3) as observer, the residue of the observer is described as equation (5).The H whose $e(t)$ is robustness to the unknown inputs such as modelling error and noise can be got by the singular value decomposition of E

if

$$E = U\Sigma V$$

then

$$H = [u_1, u_2, \cdots, u_p]^T$$

The entire procedure of Therory.1 can be implemented by the following algorithm in the micro-computer.

Algorithm. Step1. Transform the matrix E into double diagonal matrix by a series of Householder transformation.

$$U_n \cdots U_2 U_1 E V_1 V_2 \cdots V_m = \mu E v = D_1$$

Step 2. Transform the double diagnoal matrix D_1 into diagnoal matrix D by a series of Given transformation.

$$U_l' \cdots U_2' U_1' E V_1' V_2' \cdots V_k' = \mu' E v' = D$$

Step.3.
$U=\mu'\mu, V=vv', \Sigma=D, E=U\Sigma V$

4. Application Example.

Suppose that certain control system be described by state equation

$$\begin{cases}\dot{x}=\begin{bmatrix}0 & 1.0\\ 8.04915 & -0.09255\end{bmatrix}x+\begin{bmatrix}0\\ -8.92329\end{bmatrix}u\\ y=\begin{bmatrix}10\\ 01\end{bmatrix}x\end{cases}$$

place the close-loop polars at -2 and -5 by output feedback, the feedback control matrix is

$$L=[2.0227 \quad 0.7741]$$

the close-loop system is described by the state equation as

$$\dot{x}=\begin{bmatrix}0 & 1\\ -10 & 7\end{bmatrix}x+\begin{bmatrix}0 & 0\\ 18.04915 & 6.90745\end{bmatrix}\begin{bmatrix}r\\ 0\end{bmatrix}$$

where r is the reference input signal. Take equation (3) as observer, the gain of the observer K is properly selected to place the two characteristic value of the close-loop observer system at -10.

$$K=\begin{bmatrix}-10 & -1\\ 10 & -3\end{bmatrix}$$

Considering the unmodelling error

$$\Delta A=\begin{bmatrix}0 & 0\\ 1.48068 & -0.00318\end{bmatrix}, \Delta B=\begin{bmatrix}0\\ -0.04781\end{bmatrix},$$

that is

$$E=\begin{bmatrix}0.00000 & 0.000000 & 0.0000\\ 1.48068 & -0.00318 & -0.04781\end{bmatrix}$$

employing the Therory.1 described in 3 to solve H, gets $H=[1,0]$.

Expriment.1. condition: (1). There arenot unmodelling error and fault. (2). There are fault but not unmodelling error. Take the standard observer to simulate the two condition, the results is shown in Fig.1, where solid-line curve showes the result when the system is in condition 1, the dash-line curve showes the result when the system is in condition 2, the vertical cooridinate is the norm of the residue, the horizontal coordinate is the time.

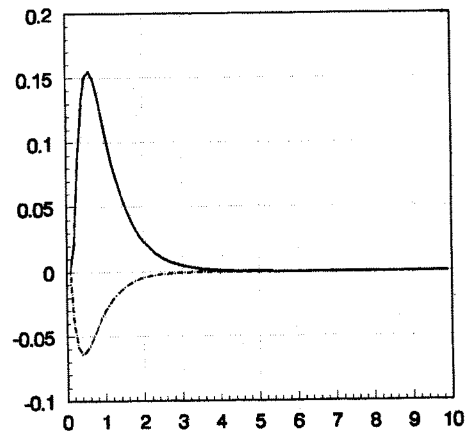

Fig.1

From Fig.1, we can see that the unmodelling error make the norm of the residue away from the nominal state greatly.

Expriment.2. Under the same condition as experiment 1, but the robust observer is used instead of normal observer, the result is shown in Fig.2, where the solid-line curve represent the residue when the system is in nominal state and the dash-line curve represent the residue when the system has unmodelling errors E.

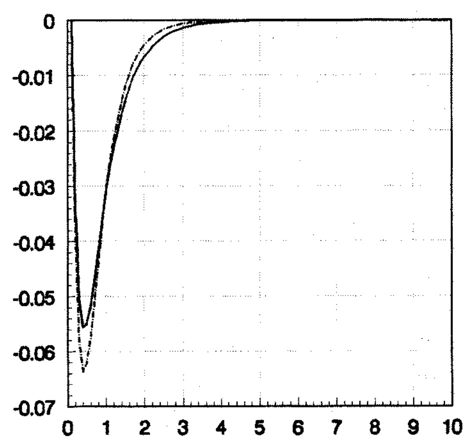

Fig.2

From Fig.2 we can see that the residue got by the robust observer is robust to unmodelling error E.

Experiment.3. suppose that there exist a open-circuit fault in the rate feedback inputs to the executor inputs, the residue got by the robust observer is shown as the solid-line curve of Fig.3,

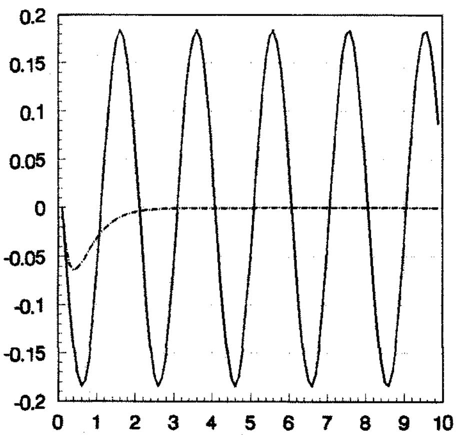

Fig.3

From Fig.3 we can see that there are big positive bias and period vibration when fault ouccur to the system.

From experiment 1 to 3, the following conclusion can be achieved.

(1). If take the standard observer as observer, there exist false alarm when the system contains unmodelling error.

(2). the robust observer suggested by this paper is robust to unmodelling errors and random disturbances and sensitive to faults.

5. Conclusion

In this paper, the authors make use of the matrix singular value decomposition therory, suggest a design procedure for observer, the observer designed by this method is robust to unknown inputs such as unmodelling errors and random disturbances and sensitive to fault, furthermore, the entire design procedure can be implemented by the micro-computer, it is very simple and convinent to put it into application, the simulation result verify the conclusion.

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