A UNIFIED BOUNDARY INTEGRAL FORMULATION FOR ACOUSTOAEROELASTIC ANALYSIS OF SHELLS

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ABSTRACT

In the present work we address the problem of the *acoustoaeroelastic* analysis of structures, *i.e.*, the study of the dynamic response of a fluid-filled elastic structure in motion within a compressible fluid in presence of a sound source. The analysis is accomplished by an integrated approach, taking into account the feedbacks between the structure, the cavity, and the exterior flow, leading to a single *acoustoaeroelastic* (matrix) equation. This is formulated in terms of the amplitudes of the structural modes, considered *in vacuo*. The pressure in the cavity is expressed in terms of acoustic natural modes of vibration, while for the pressure in the exterior field we rely on a direct boundary integral formulation, numerically solved using BEM. The application of BEM for the solution of the exterior problem introduce the so-called Fictitious Eigenvalues Difficulty (FED). This involves the arising of fictitious (not physical) resonances for frequency values coincident with the eigenvalues of the associated interior problem, *i.e.*, the problem which is governed by an operator equation adjoint to that of the case under consideration. Here we introduce a regularization technique in order to overcome this problem, based on the linear combination of the Kirchhoff-Helmoltz equation with its normal derivative. These equations are written in terms of velocity potential function. Preliminary results obtained for radiation and scattering problems are presented, and validated through comparison with analytical solution.

1. INTRODUCTION

This paper presents a methodology for the analysis of *acoustoaeroelastic* systems. We define an *aeroacoustoelastic* system as the complex system composed by an elastic structure (tipically a thin shell) which encloses a compressible fluid and is surrounded by a (possibly different) compressible fluid; the structure may be in motion with respect to the undisturbed exterior fluid. In such a situation, the internal and external pressure fields interact through the elastic boundary, thus imposing the consideration of the underlying feedback mechanism. Such a problem is of great interest in aeronautics, where the structures involved are tipically shells surrounded by a pressure field highly perturbed by the engine noise and the aerodynamic noise of propellers or jets. The acoustoaeroelastic effects due to the airborne noise (in this analysis, the noise propagating through the structure is not taken into account) may dramatically affect the pressure field in the interior of the cabin of the aircraft, with consequences on the passengers comfort.

The dynamics of the structure and the pressure field in the interior of the shell can be effectively described in terms of natural modes of vibration. The latter can be obtained in analytical form for simple configurations, or, for more complex geometries, using numerical methods, such as the finite-element method for the structural modes, and the boundary element method for the acoustic ones (this particular issue is addressed in Iemma, Morino, and Trainelli [8]). For the solution of the external pressure field, the Boundary Element Method (BEM) is again the best candidate. The drawback of such a formulation arises from the so-called Fictitious Eigenvalues Difficulty (FED): the Kirchhoff-Helmholtz Boundary Integral Equation (BIE) used for this approach is affected by spurious (not physical) frequencies values corresponding to the adjoint interior problem eigenvalues. This means that, for example, the resonant frequencies of the Dirichlet interior problem appear in the spectra of a Neumann exterior problem, and vice-versa. This Fictitious Eigenvalues Difficulty is a primary issue in the present research on BIE applied to acoustics. In fact, this non-physical resonances can completely destroy the solution of the numerical method used to solve the Kirchhoff-Helmholtz BIE, as we will see later.

Many analytical and numerical regularization have been proposed, but no complete satisfactory schemes have been developed yet. In Iemma, Trainelli, and Morino [9] this problem has been by-passed by means of a finite-state reduction of the aerodynamic operator, as follows. The BIE leads to the expression of the aeroacoustic pressure on the structure as a aeroacoustic matrix acting on the structural displacement vector. This matrix then undergoes a finite-state reduction, obtained through a sampling procedure followed by a least-square approximate reconstruction. We use a preventive BEM analysis of the exterior acoustic field, so that spurious frequencies affecting the adjoint problem are identified, and the sampling procedure is implemented with care in order not to include values in the immediate vicinity of fictitious eigenvalues. This allows for a profitable smoothing of the numerical solution, in which no non-physical resonances appears, thus representing, in the framework of the finite-state approximation, the desired regularization tool. In this way, a matrix-polinomial-rational dependence of the exterior pressure on the frequency is established, thus simplifying the set of governing equations in terms of structural and (internal) acoustic modal amplitudes. The results obtained with this approach reveal that the approximation of the aerodinamic matrix reproduces with remarkable accuracy the spectrum of the aerodynamic operator. Figures 1 and 2 shows the spectra of the normal displacement of the structure w for the case of a fluid-filled spherical shell subject to a spherically-symmetric impinging wave. For this particular configuration the coupled acoustoaeroelastic system can be solved analytically. The exact expression for the amplitude of the radial displacement $\tilde{w}(\omega)$ is

$$\left[\varrho_S\left(\Omega^2-\omega^2\right)-\frac{\varrho_I\omega^2\sin\kappa}{\kappa\cos\kappa-\sin\kappa}-\frac{\varrho_E\omega^2}{1+ik}\right]\tilde{w} = -2\,\frac{\varrho_E}{c_E}\frac{\omega^2}{1+ik}\,A \qquad (1)$$

where c_I and c_E represent the speed of sound in, respectively, the interior and exterior acoustic media, $\kappa=\omega/c_I$ and $k=\omega/c_E$, ϱ_I, ϱ_E, and ϱ_S are the densities of the fluids and of the solid, and A is the magnitude of the impinging wave $A\,e^{ikr}/r$. In Fig. 1 the numerical solution obtained with a zeroth-order BEM formulation for the Kirchhoff-Helmoltz equation is compared with the finite-state approximation of the aerodynamic matrix. In both cases, 10 modes have been used to describe the dynamics of the shell, and 28 for the decomposition of the internal acoustic field. The BEM solution (dashed line) is clearly affected by peaks corresponding to non-physical resonances; these correspond to natural frequencies of the interior Dirichlet problem (see, *e.g.*, Iemma, Morino, and Trainelli[8]). These peaks are completely absent in the finite-state solution, which, on the other hand, perfectly reproduces the natural frequencies of the problem predicted by the BEM. The comparison with the analytical solution of the finite-state approximation (Fig. 2) reveals a good overall behaviour.

As mentioned above, this methodology is nothing more than a convenient way to by-pass the spurious-frequency problem. As mentionad above, the non-physical resonances are *cut-off* by a careful choice of the samples of the aeroacoustic matrix used for the least-square approximation. Two drawbacks make this approach not particularly appealing for design applications on real configurations: the sampling procedure requires the *a priori* knowledge of the natural frequencies of the adjoint problem (for geometries of practical interest, this knowledge can be achieved only by numerical experimentation); small modifications on the geometry of the cavity results in the modification of its resonances, and, as a consequence, in the re-evaluation of the whole finite-state approximation procedure.

From this point of view, the most effective approach is the regularization of the Kirchhoff-Helmoltz operator. In the present work, a regularization technique for the integral representation of the velocity potential in the external field is presented, similar to that suggested by Burton and Miller [3]. This is based on a linear combination of the Kirchhoff-Helmoltz equation for the potential with that obtained by taking the normal derivative. In fact, although the Kirchhoff-Helmoltz operator and the integral equation for the normal derivative are both affected by the spurious resonances at the charachteristic frequencies of the cavity, it was demonstrated that the combination of the two equations circumvents this problem (see, *e.g.*, Colton and Kress [5]). The linear combination of the integral equations is numerically solved using a third order BEM, based on Hermite interpolation of the variables. A special treatment of the hypersingular integral present in the integral equation for the normal derivative is introduced.

A review of the most significant work in the field is beyond the scope of the present paper. Two works closely related to the argument of the present paper are those of Chien, Rajiyah, and Atluri[4], and Amini and Wilton[2]. Extensive reviews on the subject are available, for instance, in Gaunaurd[7], and Amini and Harris[1].

In Section 2 a brief description of the coupling terms in the equation of the structural dynamics is given, in order to put in the proper perspective the results presented in the following Sections. The approach used for the external flow is presented in Section 3, including the regularization technique. Preliminary results obtained with the methodology presented are commented in Section 4. The theoretical aspects of the present work have been developed jointly by Iemma, Gennaretti, and Morino; Iemma and Gennaretti are also responsible for the derivation of the numerical algorithm, which has been implemented and validated with Trainelli, for the zeroth order formulation, and with Giordani, for the third order one.

2. AEROACOUSTOELASTIC COUPLING

Consider a solid body $\mathcal{B}$ which divides the fluid domain in two regions: the interior volume $\mathcal{V}_I$, and the unbounded exterior region $\mathcal{V}_E$. Introducing the natural modes of vibration of the free structure *in vacuo*, $\mathbf{\Phi}_n$, the generic displacement $\mathbf{u}$ of the structure can be decomposed as

$$\mathbf{u}(\mathbf{x},t) = \sum_m a_m(t)\, \mathbf{\Phi}_m(\mathbf{x}) \tag{2}$$

while $\langle \mathbf{\Phi}_k, \mathbf{\Phi}_m \rangle = \delta_{km}$. The corresponding Lagrange equations of motion are

$$\ddot{a}_m + \Omega_m^2\, a_m = f_m \tag{3}$$

where $f_m := \langle \mathbf{f}, \mathbf{\Phi}_m \rangle$ denotes the mth generalized (modal) force on the shell. Next, we assume that the body is a thin shell, and that the outer and inner flows are inviscid. Denoting with $\mathcal{S}$ the surface of the shell (note the difference between $\partial\mathcal{B}$ and $\mathcal{S}$), we have $\mathbf{f} = -\Delta p\, \mathbf{n}$, where $\Delta p = p_E - p_I$ and $\mathbf{n} = \mathbf{n}_E$, and hence

$$f_m = -\int_{\mathcal{S}} \Delta p\, \mathbf{\Phi}_m \cdot \mathbf{n}\, d\mathcal{S} := -\langle \Delta p, \Phi_m \rangle_{\mathcal{S}} \tag{4}$$

The issue of the influence of the interior pressure field in the forcing terms f_m is addressed in Iemma, Trainelli, and Morino[8]. The approach used in that work is closely related with that of Dowell *et al.*[6]. Note that in Iemma *et al.*[8] the analysis includes the coupling with both interior and exterior flows (*aeroacoustoelastic* problem), whereas in Dowell *et al.*[6] only the interior fluid is considered (simple *acoustoelasticity*). In the following of the present work the contribution of the internal pressure field will be neglected, since the emphasis is here on the regularization of the external integral operator used to evaluate the velocity potential. Thus, in the following,

$$f_m = f_{E_m} = -\langle p_E, \Phi_m \rangle_{\mathcal{S}} \tag{5}$$

3. EXTERNAL ACOUSTICS

In this section we turn our attention on the evaluation of the external pressure load. Although the following derivation could be easily extended to aeroacoustic cases (*i.e.*, when the body is in motion with respect to the undisturbed exterior fluid field), we limit ourselves to simple acoustic (*i.e.*, no-flow) cases (see, *e.g.*, Morino[10] for the aerodynamic general theory). We make use of the superposed $\tilde{}$ to indicate Laplace-transformation throughout. The external pressure field can be expressed in terms of potential using the Bernoulli's theorem in the frequency-domain

$$\tilde{p}_E = -s\varrho_\infty \left(\tilde{\varphi}^{sc} + \tilde{\varphi}^{inc}\right) \tag{6}$$

where we distinguish the total scattered field φ^{sc}, and the incident field φ^{inc} due to the external source. The boundary integral equation for the scattered velocity potential φ^{sc} in the frequency-domain has the form

$$\tilde{\varphi}_*^{sc} = \int_{\mathcal{S}} \left[\frac{\partial \tilde{\varphi}^{sc}}{\partial n} G - \tilde{\varphi}^{sc} \frac{\partial G}{\partial n} \right] d\mathcal{S} \tag{7}$$

where $G = -e^{s\theta}/4\pi r$, with the acoustic delay given by $\theta = r/c_E$, $r = \|\mathbf{x} - \mathbf{x}_*\|$, while c_E represents the unperturbed external sound speed. The boundary condition on $\mathcal{S}$, expressed in the frequency domain, are

$$\begin{aligned} \frac{\partial \tilde{\varphi}^{sc}}{\partial n} &= s\left(\tilde{\mathbf{u}} \cdot \mathbf{n} - \alpha_E \varrho_\infty \tilde{\varphi}^{sc}\right) \\ &\quad - \left(\frac{\partial \tilde{\varphi}_{inc}}{\partial n} + s\alpha_E \varrho_\infty \tilde{\varphi}^{inc} \right) \end{aligned} \tag{8}$$

(α_E is the exterior acoustic admittance). Thus, the total scattered field φ^{sc} may be decomposed into the rigidly scattered field $\varphi^{sc\infty}$, corresponding to a structurally rigid surface (*i.e.*, $\mathbf{u} = 0$), and the radiated field φ^{rad}, produced by the vibration of the boundary. For $\alpha_E = 0$ the boundary condition 8 are of the Neumann type. Thus, Eq. 7 will be affected by fictitious peaks corresponding to the resonances of the interior Dirichlet problem. The regularization is obtained by combining Eq. 7 with its normal derivative with respect to the normal n_* in the observation point

$$\frac{\partial \tilde{\varphi}_*^{sc}}{\partial n_*} = \int_{\mathcal{S}} \left[\frac{\partial \tilde{\varphi}^{sc}}{\partial n} \frac{\partial G}{\partial n_*} - \tilde{\varphi}^{sc} \frac{\partial^2 G}{\partial n_* \partial n} \right] d\mathcal{S} \tag{9}$$

The linear combination of Eqs. 7 and 9 yields

$$\tilde{\varphi}_*^{sc} \quad + \quad \zeta(s) \frac{\partial \tilde{\varphi}_*^{sc}}{\partial n_*} =$$

$$\int_S \left[\frac{\partial \tilde{\varphi}^{sc}}{\partial n} G - \tilde{\varphi}^{sc} \frac{\partial G}{\partial n} \right] dS \qquad (10)$$

$$+ \quad \zeta(s) \int_S \left[\frac{\partial \tilde{\varphi}^{sc}}{\partial n} \frac{\partial G}{\partial n_*} - \tilde{\varphi}^{sc} \frac{\partial^2 G}{\partial n_* \partial n} \right] dS$$

where $\zeta(s)$ is a linear function of the Laplace variable. In Eq. 10 there is an integral term with an highly singular kernel, the evaluation of which represent the most challenging issue in the numerical implementation of the method. In the present work, the hypersingular integral is integrated by parts, and regularized taking advantage to the equivalence of doublet and vortex layers.

4. NUMERICAL RESULTS

In this section preliminary resuts obtained with the formulation presented are validated through comparisons with analytical solutions. Eqs. 7 and 10 are discretized using a third order BEM based on the formulation introduced in Morino, Gennaretti, and Calcagno[11]. In all the results presented the attention is posed on the capability of the present formulation to overcome the occurence of the spurious resonances in the solution. In the following, we will address as *method A* the third-order BEM solution of Eq. 7, whereas the solution of Eq. 10 will be indicated as *method B*.

The first test case deals with an elastic spherical shell subject to a spherically-symmetric impinging wave. This problem is similar to that of Fig. 1 and 2, with the difference that here the interior acoustic field is not taken into account. The analytical solution of the problem can be obtained from Eq. 1, setting $\varrho_I = 0$. In Figure 3, the frequency spectrum for the amplitude of the normal displacement w is presented. The analytical solution (continuous line) presents only one peak, corresponding to the frequency of the spherically symmetric mode of vibration of the shell (dot-dashed vertical line).† The solution of Eq. 7 (method A) presents non-physical peaks at the resonances of the corresponding interior Dirichlet problem, indicated with small squares. On the contrary, the solution of Eq. 10 (method B) is not affected by the FED; the numerical result is smooth and in excellent agreement with the exact solution. Note that in all the results presented, the function ζ is $\zeta(k) = i/k$ This form is suggested in Chien *et al.*[4] as the optimal value for ζ in radiation and scattering problems. A second test confirms the capability of the regularized operator to smooth out the non-physical resonances. The spherical shell is subject to the perturbation due to an impinging plane wave. This kind of disturbance excites modes of the shell which are axially-symmetric with respect to the axis of propagation of the wave front. In Fig. 4 the spectrum of w obtained with an external pressure field evaluated using Eq. 10 is presented. All the resonances in the numerical solution correspond to the frequencies of the axially-symmetric modes of vibration of the shell. Note that no spurious effects are present.

The formulation presented for the evaluation of the exterior pressure field can be successfully used also to evaluate scattering pattern due to rigid or elastic bodies. In Fig. 5 preliminary results obtained for the scattering signature for the same problem of Fig. 4. The angular distribution of the potential function at a distance of $5r$ from the sphere is presented. The solution is obtained for $k = 3.14$, which corresponds to the first spurious frequency. Note that the solution of Eq. 7 is completely different from the analytical solution. This result demonstrates as the FED completely destroy the numerical solution of the Kirchhoff-Helmoltz equation. Conversely, the regularized operator yields to a numerical result (method B) which is in very good agreement with the exact solution.

0† Note that the analytical solution of Eq. 1 in Fig. 2 presents several peaks, corresponding to the natural modes of vibration of the acoustic cavity. Here, the interior pressure field is not taken into account, so only the natural mode of vibration of the structure are present in the solution.

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FIGURES

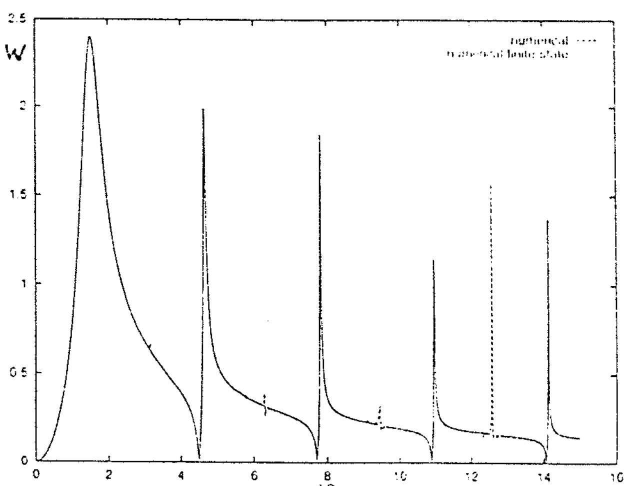

Figure 1

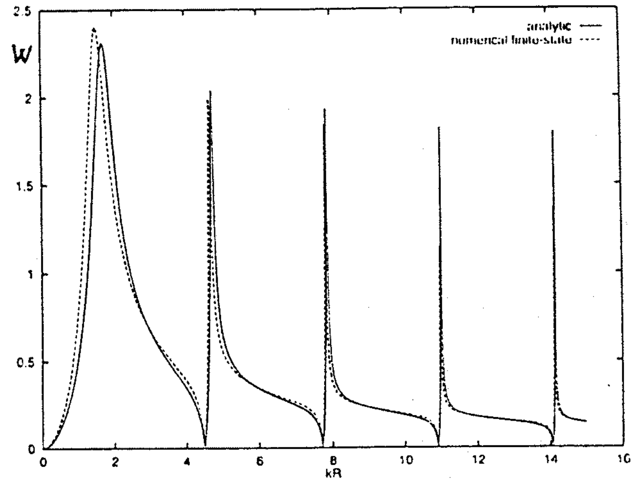

Figure 2

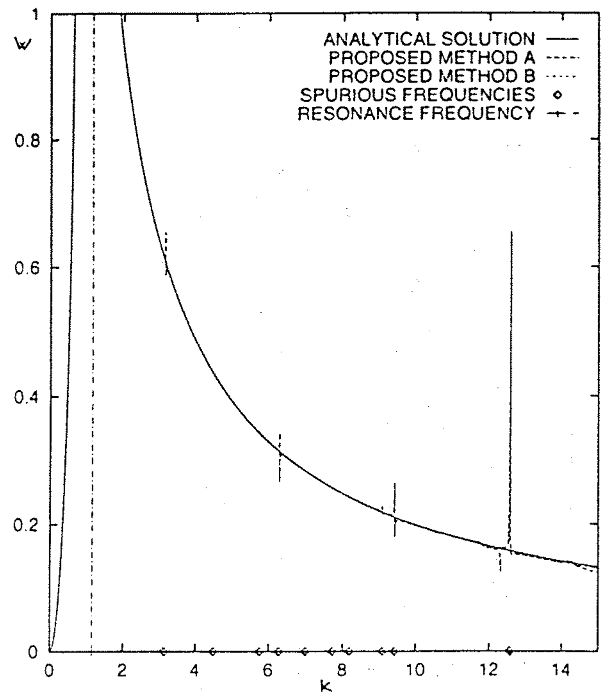

Figure 3

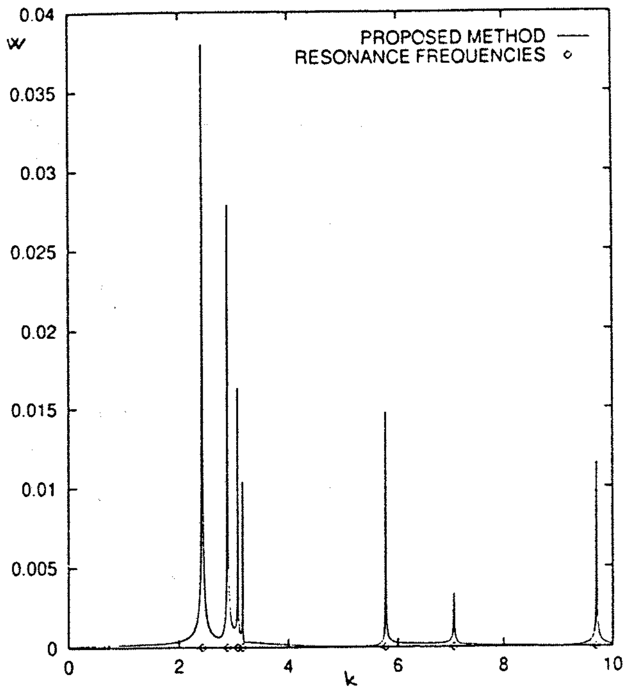

Figure 4

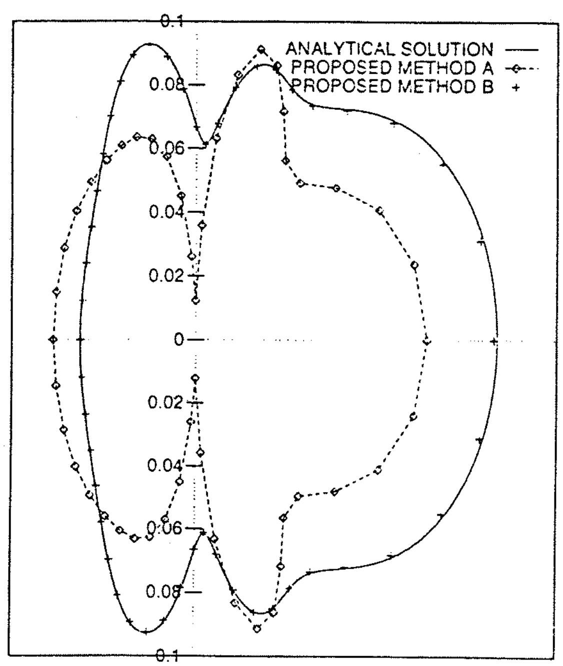

Figure 5