ANALYSIS OF AIRCRAFT MAXIMUM PERFORMANCES ON OPTIMAL TRAJECTORIES

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Abstract

In this paper a model aircraft is evaluated from the point of view of the agility metrics based on the intrinsic co-ordinates. The question is solved as the inverse problem of determining the proper control actions which maximize the three agility components following assigned flight procedures.

Nomenclature

$\boldsymbol{A}$	agility vector
A_t, A_n, A_b	agility components in $\mathcal{F}_F$
a	acceleration vector
h	altitude
k_1, k_2	curvature and torsion parameters
M_T	test Mach number
P_S	specific excess power
p, q, r	angular velocity components
$\boldsymbol{R}$	position vector of the aircraft c.g.
S	cost function
s	curvilinear abscissa
t	time
$\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}$	tangential, normal and binormal unit vectors
$\boldsymbol{u}$	control vector
V	velocity modulus
V_c	corner velocity
$\boldsymbol{v}$	velocity vector
$\boldsymbol{x}$	state vector
$\boldsymbol{y}$	output vector
α	angle of attack
β	sideslip angle
γ	flight path angle
δ_A	aileron angle
δ_E	elevator angle
δ_R	rudder angle
δ_T	throttle position
ϕ, θ, ψ	Euler angles
$\boldsymbol{\omega}$	angular velocity vector

Introduction

In very recent times more and more research activity has been carried out in order to evaluate the *agility* of high performance aircraft[1-6]. However, as the investigations are rapidly expanding, even a commonly accepted definition of the appropriate metrics of this important flight quality parameter is still matter of debate. The reason for this can be essentially found in the very different motivations and goals of the studies on this subject.

Among the several classifications, one of the more convenient is based on grouping the metrics in two basic sets[2]. The first set groups those metrics which can be addressed as transient agility whereas the second set groups the functional agility metrics. Each particular group corresponds to a different time scale. In particular, the transient agility is related to short time rotational motions and transition between extreme specific–power levels. The time scale is, in this case, of about $2 \div 3$ s. The functional, long time–scale agility quantifies how well the aircraft executes rapid rotations of the velocity vector[6]. The time scale is now of about $20 \div 30$ s.

Here we take advantage of the rigorous definition of the agility as the time derivative of the acceleration vector[3]. This peculiar metrics, based on the instantaneous characteristics of the c.g. motion, is best expressed in terms of the evolution of the natural coordinates of the c.g. along the trajectory and appears very suitable when dealing with the appreciation of the short time maneuvering performances of an aircraft.

Our purpose here is to show how this particular metrics and the use of a solution method of flight inverse problems can lead to the determination of the maximum attainable agility components of an airplane and, at the same time, to the calculation of the related time histories of the state variables and the corresponding control actions. For an assigned short time task our approach takes into account the full dynamics of the aircraft and evaluates the control actions by a local optimization method[7] in such a way that either the variations of the agility components can follow a desired path or the maximum value of one of the components can be reached. Furthermore, all the limitations of the control can be implemented.

After a short presentation of the analytical and numerical method which will be applied, a number of applications to significant situations follows. Some of these applications concern problems already discussed in the pertinent literature. It is the author's opinion that the procedure can be extended to the long time maneuvers where a sequence of specific tasks is performed in order to appreciate the overall agility characteristics of a high performance aircraft.

Analysis

We begin this section by recalling the principal aspects of the agility parameters. In particular, following the differential geometry approach[8], we refer to the Frenet or intrinsic frame $\mathcal{F}_F$ which has tangent $\boldsymbol{t} = d\boldsymbol{R}/ds$, curvature $\boldsymbol{n} = (d\boldsymbol{t}/ds)/|d\boldsymbol{t}/ds|$ and binormal $\boldsymbol{b} = \boldsymbol{t} \wedge \boldsymbol{n}$ as unit vectors, where $\boldsymbol{R}$ is the position vector of the aircraft c.g. The agility vector is defined as the rate of change of the maneuver state and its components are conveniently expressed in the intrinsic frame as[3]

$$\boldsymbol{A} = (A_t, A_n, A_b)^T = \begin{bmatrix} \ddot{V} - V^3 k_1^2 \\ 3V\dot{V}k_1 + V^2\dot{k}_1 \\ V^3 k_1 k_2 \end{bmatrix} \qquad (1)$$

where A_t, A_n and A_b are, respectively the axial, curvature and torsion agility, and $V = |\boldsymbol{v}|$ is the flight speed. Also, in Eq. (1), k_1 and k_2 stay for the curvature and the torsion of the aircraft trajectory. As an observation, the latter parameter is zero for a planar path and, for a constant value of k_1 and k_2, the flight trajectory is a cylindrical helix.

The curvature and torsion can be expressed as functions of the vehicle kinematic variables in body axes $\mathcal{F}_B$ as follows

$$k_1 = \frac{1}{V^2}\left(|\boldsymbol{a}| - \dot{V}^2\right)^{\frac{1}{2}} \qquad (2)$$

where $\boldsymbol{a} = \dot{\boldsymbol{v}} + \tilde{\boldsymbol{\omega}}\boldsymbol{v}$, and

$$k_2 = \frac{\det\{\boldsymbol{v}, \boldsymbol{a}, \dot{\boldsymbol{a}}\}}{V^2\left(|\boldsymbol{a}|^2 - \dot{V}^2\right)} \qquad (3)$$

with $\dot{\boldsymbol{a}} = \ddot{\boldsymbol{v}} + \dot{\tilde{\boldsymbol{\omega}}}\boldsymbol{v} + 2\tilde{\boldsymbol{\omega}}\dot{\boldsymbol{v}} + \tilde{\boldsymbol{\omega}}\tilde{\boldsymbol{\omega}}\boldsymbol{v}$.

The nonlinear set of the governing equations for the aircraft motion in $\mathcal{F}_B$, is written in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \ , \quad \boldsymbol{x}(t=0) = \boldsymbol{x}_0 \qquad (4)$$

where the state vector $\boldsymbol{x} \in R^{11}$ contains the linear and angular velocities $\boldsymbol{v}$ and $\boldsymbol{\omega}$, the Euler angles (ϕ, θ, ψ), the altitude h, and a further state δ_{T_c} accounting for the first-order lag engine dynamics. The control vector $\boldsymbol{u} \in R^4$ is $\boldsymbol{u} = (\delta_E, \delta_A, \delta_R, \delta_T)^T$ for elevator, aileron, rudder angles and commanded thrust level, respectively. Also, for the output variables we write

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \qquad (5)$$

where $\boldsymbol{y} \in R^m$ is the output vector.

The inverse simulation problem is solved by a local optimization technique[7] that is just recalled here. Once a vector $\boldsymbol{y}^D$ of constrained outputs is given, the difference between the number n of control variables and the assigned outputs, being in any case $n \geq m$, is termed degree of redundancy, whereas the situation $n = m$ is referred to as a nominal problem. A nonzero degree of redundancy allows for a suitable cost function to be introduced in order to tailor the resulting maneuver according to certain prerequisites.

We assume $\boldsymbol{u}$ to be a step–constant function, i.e. $\boldsymbol{u}(t) = \boldsymbol{u}_j, t_{j-1} < t \leq t_j$, so that the inverse problem is solved when the discretized input $\boldsymbol{u}_j^*$ is determined at each time step, for an assigned output $\boldsymbol{y}_j^D$, as the inverse of the implicit function

$$\boldsymbol{y}_j^D = \boldsymbol{h}\left(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j^*\right) \qquad (6)$$

where use has been made of Eq. (1) in order to express $\boldsymbol{x}_j$ in the form $\boldsymbol{x}_j = \boldsymbol{F}\left(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j^*\right)$. In the redundant case, the constrained optimization problem represented by the minimization of the performance index $S(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j)$, subjected to Eq. (6), is solved by a Sequential Quadratic Programming (SQP) algorithm, being S a scalar, positive–semidefinite function. Of course, it is $S = 0$ when the problem is nominal and the resulting solution simply satisfies the constraints given by Eq. (6). The SQP procedure is implemented by using the appropriate modules of the NAG Fortran Library[9]. Further details on the above method are reported in Ref. 7.

Results

In what follows a number of situations were considered to show the practicality of adopting the inverse simulation technique in aircraft agility evaluation. The

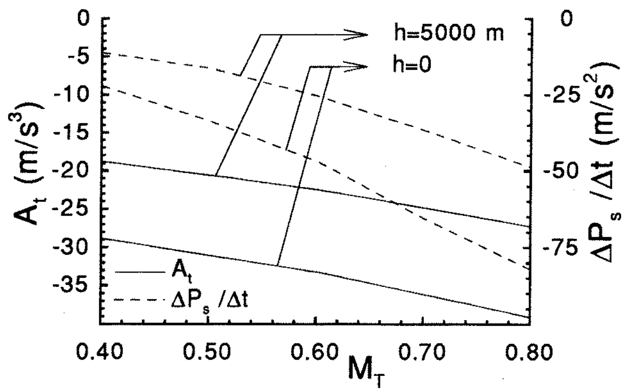

Figure 1: $A_{t_{max}}$ and $\Delta P_S/\Delta t$ vs. M_T at two altitudes; $k_1 = 0$. Speed brake not deflected.

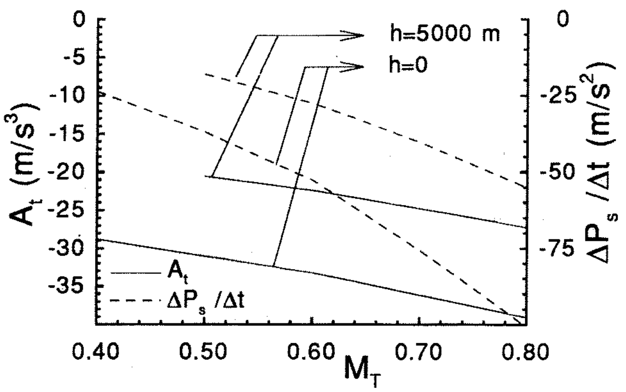

Figure 2: $A_{t_{max}}$ and $\Delta P_S/\Delta t$ vs. M_T at two altitudes; $k_1 = 0$. Speed brake deflected.

three components of $\boldsymbol{A}$, in the order, are considered and calculated, with reference to a simplified F–16 model[10] where the Stability Augmentation System is excluded and the c.g. position is conveniently located so that the longitudinal stability is assured. Furthermore, only subsonic flight conditions are taken into account and no consideration is given to control via thrust vectoring.

The tangential agility, at decreasing speed, is first dealt with according to three different approaches. Two of them refer to the calculation of $A_{t_{max}}$ for A_n and A_b both exactly zero, i.e. for curvature and torsion of the trajectory both vanishing. The third case corresponds to the evaluation of $A_{t_{max}}$ for $A_n \neq 0$.

Firstly a case is carried out which is thought to be particularly significant since it is devoted to show how our results compare to those in Ref. 2, where flight simulations were performed and a different agility metrics was considered. In particular, in the symmetry plane, $k_2 = 0$, the altitude h is constant, the speed is increased from an initial trimmed state by reaching the maximum throttle value, an assigned test speed is attained and then the thrust is abruptly decreased to zero, and the speed brakes are possibly fully extended. This inverse problem is nominal since $k_1 = 0$, and the

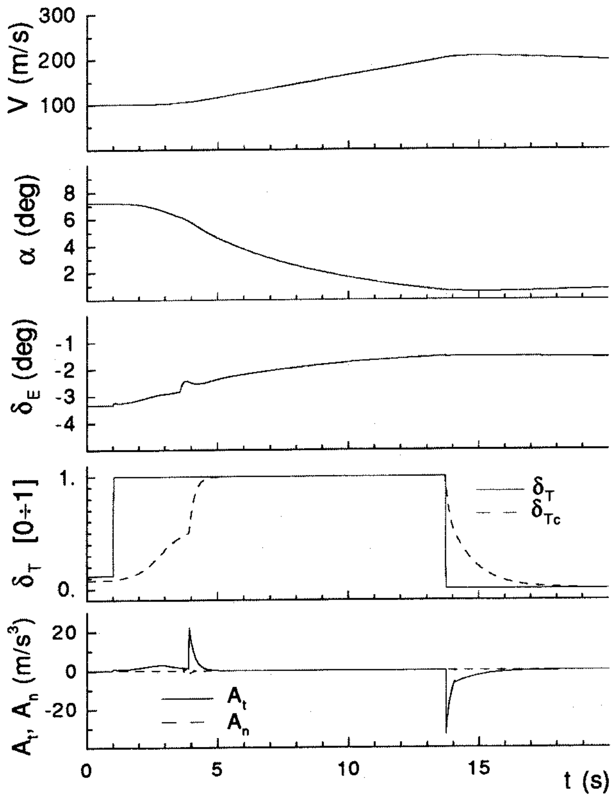

Figure 3: Time histories for for assigned $\delta_T(t)$, $k_1 = 0$. Speed brake not deflected, $M_T = 0.6$, $h = 0$.

only unknown is δ_E. For this case, Figs. 1 and 2 show the values of $A_{t_{max}}$ which are obtained following the indicated flight procedure, as function of the assigned test Mach number M_T, of the altitude, and of the brake action. For this case, Fig. 3 reports the time histories of some significant quantities, namely the agilities A_t, A_n, the states V and α, the control actions δ_E and δ_T, and the actual thrust level δ_{T_c}. Note the time lag and the effect of the afterburner ignition on A_t taking place at $t = 4$ s.

At sea level and for an altitude $h = 5,000$ m, the obtained tangential agility versus M_T is shown in Fig. 1,

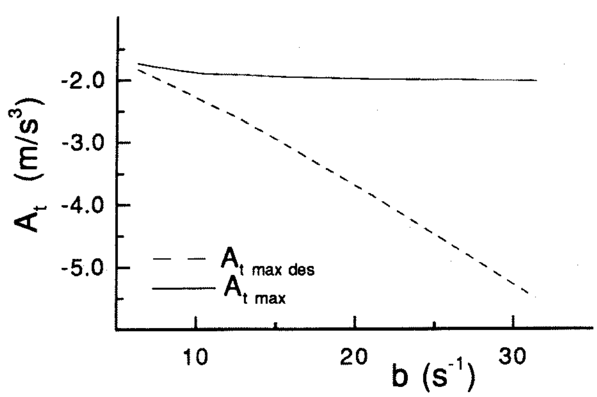

Figure 4: $A_{t_{max}}$ and $A_{t_{max_{des}}}$ vs. b; $M_0 = 0.6$, h=0.

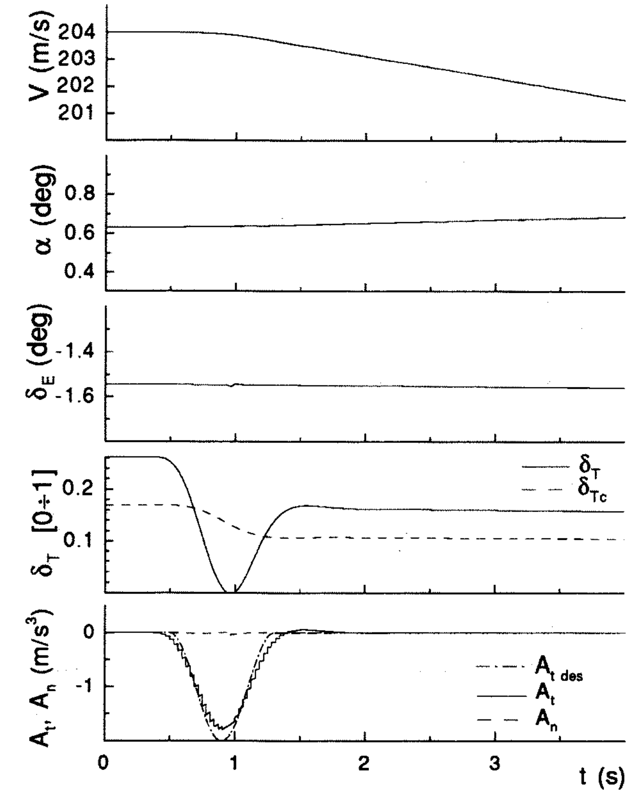

Figure 5: Time histories for assigned $A_{t_{des}}$; $k_1 = 0$, $V = V(t)$, $b = 7.85$ s^{-1}.

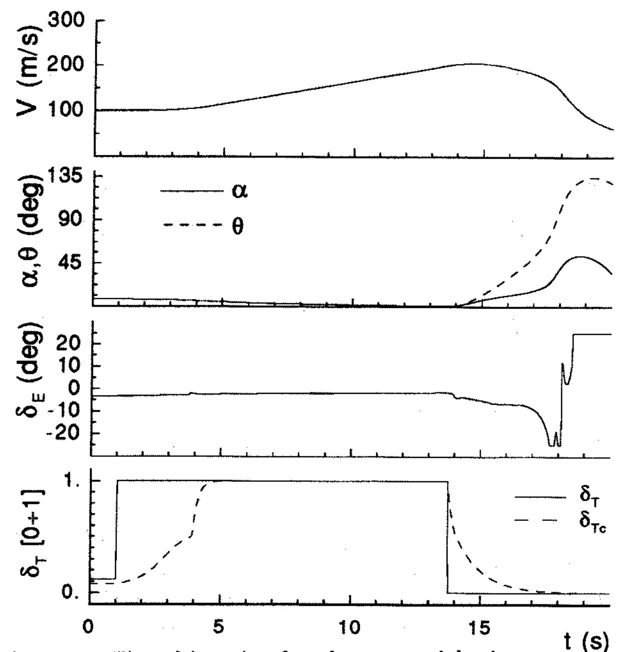

Figure 7: Time histories for the case with A_n not constrained

when the speed brake is retracted. According to Ref. 2 the power loss parameter, i.e. $\Delta P_S/\Delta t$, defined as the increment of specific excess power in going from a maximum power/minimum drag condition to a minimum power/maximum drag condition divided by the time necessary to complete the transition, can be chosen as a different and practical agility metrics. The time difference Δt is between the instant at which the minimum deceleration occurs and the time when the value of M_T is obtained.

According to our computations, at sea level, and for M_T ranging from 0.4 and 0.8, Δt changes between 6.78 and 4.40 s. For $h = 5,000$ m , the time increments Δt are of the order of ten percent greater with respect to the values at $h = 0$ and at the same M_T. Fig. 1 also shows the power loss parameter and one can immediately verify that the two sets of curves give comparable results at higher M_T's only. This can be realized since $\Delta P_S/\Delta t$ evaluates the rate of change of the maneuver state over the entire time length to performe the prescribed task. On the other hand, the definition of A_t leads to the calculation of the maximum instantaneously achieved value.

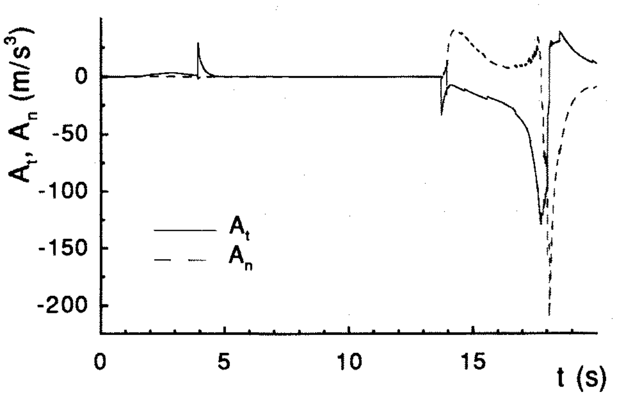

Figure 6: Agitily components vs. t; A_n not constrained.

Figure 2 shows the corresponding results when the action of the speed brake is present. Apparently the braking force has no influence at all on $A_{t_{max}}$ whereas a little effect is felt on $\Delta P_S/\Delta t$. In the first case this can be understood since, as the thrust is abruptly brought to zero, the speed brake takes its time to become effective. Furthermore, to keep the altitude constant and to compensate for the longitudinal moment, the angle of attack is decreased and the induced drag, which is significant at low speed, is reduced. On the short time all this determines very little changes of the tangential agility with respect to the brakeless case. Substantially different results might be envisaged by properly timing the combined actions of thrust and speed brake. The little more sensitivity of the power loss parameter to the action of the brake can, once again, be explained after the meaning of $\Delta P_S/\Delta t$ as a global definition of agility over the entire maneuver is taken into account.

In the second application of the inverse simulation, a functional law is assigned to $A_t(t)$ in the form $A_{t_{des}} =$

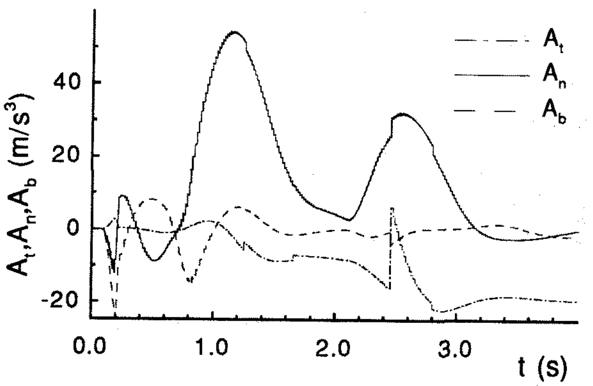

Figure 8: Agility components vs. t in a maximum performance turn; $V = V_c$, $h = 0$, $\dot{\psi}$ maximized.

$A_{t_{max_{des}}} \left[1 - \cos b\left(t - t_0\right)\right]/2$, $t_0 \leq t \leq t_0 + 2\pi/b$, and the maximum value of A_t effectively realized by the aircraft model is determined by varying b in the range of obtainable aircraft maneuvers. In other words, this euristic approach consists in forcing the aircraft, at constant altitude and in the symmetry plane, to execute limit maneuvers in order to verify its possibilities in terms of A_t, when A_n is zero. In this case the unknown quantities are the control actions δ_E and δ_T, the constraints are $k_1 = 0$ and $V = V(t)$ as obtained through Eq. (1) and the assumed A_t law. The inverse problem is nominal and the results depend on h. For $h = 0$, the main results are shown in Figs. 4 and 5. In particular, Fig. 4 shows how the values of b have been varied and the corresponding changes of $A_{t_{max_{des}}}$ and $A_{t_{max}}$, for $h = 0$ and an initial Mach number M_0 equal to 0.6. For $b = 7.85$ s^{-1}, Fig. 5 reports the final optimal solutions as far as the state variables, the relative control actions and the tangential and normal agility components are concerned. Note that A_n is everywhere vanishing. Note also that the maximum values of A_t in this case are much lesser than the ones obtained in the preceding situation. The problem however, presents some interest since it

shows the capability of the method of determining the control actions when the agility law itself is imposed.

After relaxing the conditions for a rectilinear flight path, the same maneuver of the first case was dealt with as a redundant problem. In particular, after attaining M_T, the condition of vanishing curvature is not imposed any more and the cost function $S = 10^3/A_t^2 + 10^{-3}A_n^2 + 10e^{-10\sin\gamma}$ is assigned, where γ is the flight path angle, with the objective of maximizing the local value of A_t realized along the trajectory, not letting A_n and γ assume very high values. The results of Figs. 6 and 7 show that the value of the tangential agility is increased compared to the data reported in Fig. 3. Note also the high value of A_n at the end of the simulation, due to the abrupt variation of the ele-

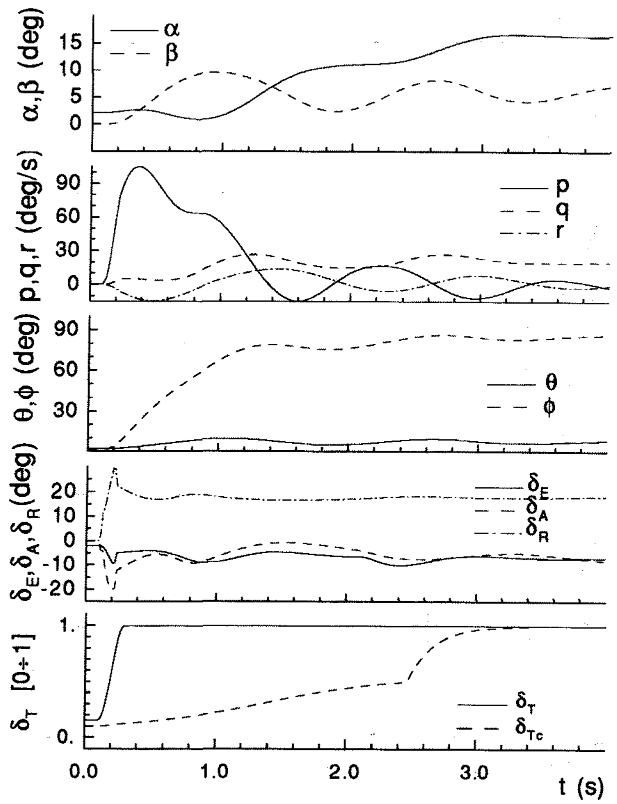

Figure 9: Time histories for a maximum performance turn; $V = V_c$, $h = 0$, $\dot{\psi}$ maximized.

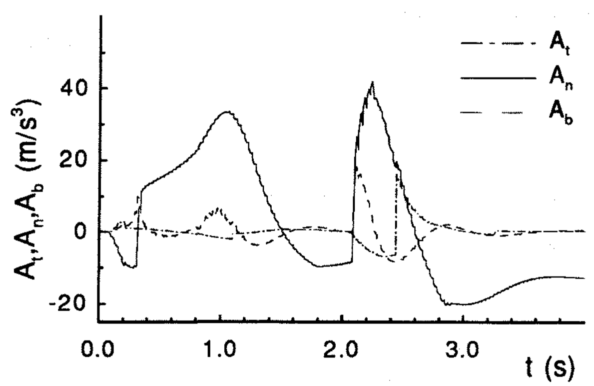

Figure 10: Agility components vs. t in a maximum performance turn; $A_t = 0$, $h = 0$, $\dot{\psi}$ maximized.

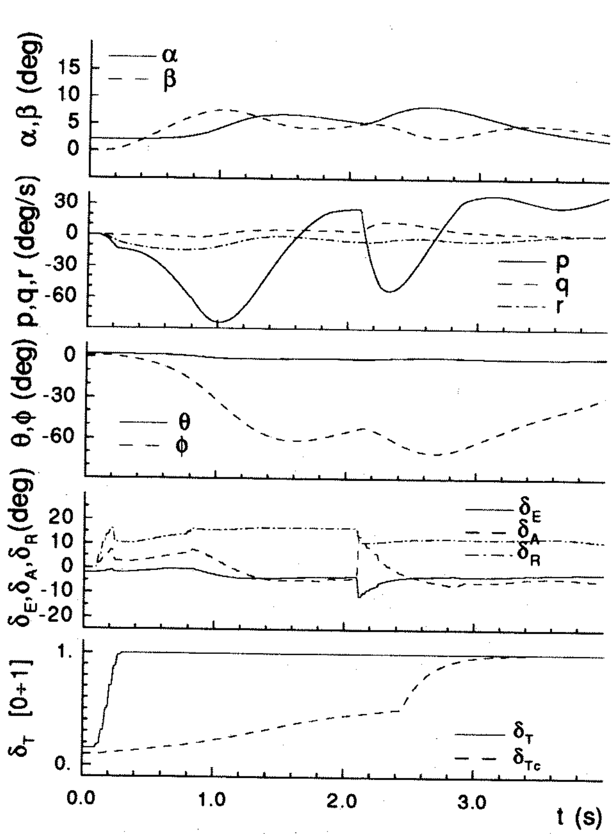

Figure 11: Time histories for a maximum performance turn; $A_t = 0$, $\dot{h} = 0$, $\dot{\psi}$ maximized.

vator angle for trimming the aircraft at high angle of attack.

We pass now to the case of looking for the maximum of A_n according to the commonly accepted maneuver of Ref. 3. This maneuver corresponds to a maximum performance turn at constant altitude so that $A_b = 0$. We start from trimmed rectilinear flight conditions at the corner velocity V_c, then the thrust is increased to its maximum, the speed and the altitude are kept constant and A_t is calculated along the turn. The unknowns are δ_A, δ_E and δ_R and the inverse problem is one degree redundant. Following the procedure of local optimization to solve this kind of problems, a penalty function is assumed in order to maximize $\dot{\psi}$ as the aircraft turns at an increasing load factor. The results concerning the agility components, and the time histories of the relevant state and control variables are reported in Figs. 8 and 9. These results can then be compared with those in Figs. 10 and 11, which were obtained by imposing that, the speed is not constant and the turn is executed at $A_t = 0$, $A_b = 0$, while still keeping $\dot{\psi}$ locally maximum. Here one should observe that the maximum A_n values are greater when the value of A_t is not constrained to be zero. Also, observe in Fig. 10 that the constraint $A_t = 0$ could not be satisfied at $t \simeq 2.1$ s due to the delay in the ignition of the afterburner.

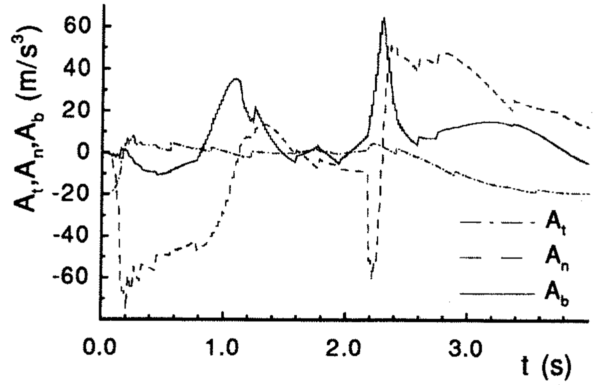

Figure 12: Agility components vs. t in a reversed turn; $\phi = \phi(t)$.

A reversed turn maneuver was carried out as a final application to the evaluation of the torsional agility characteristics of our model aircraft. The initial state $\boldsymbol{x}_0$ was a sustained turn at the maximum installed thrust with afterburning and at the corner speed. The initial angular velocity was $\boldsymbol{\omega}_0$. A law for the roll angle was assigned, $\phi = \phi_0 \cos[\pi(t - t_0)/T]$, with $\phi_0 = 80.3$ deg and $T = 2.25$ s. The problem is twice redundant since the unknown control actions were δ_E, δ_A and δ_R, and was solved by imposing that $|(\boldsymbol{\omega} - \boldsymbol{\omega}_0)|^2$ should be kept minimum along the trajectory. The results are shown in Figs. 12 and 13. One can observe that the aircraft keeps an almost constant altitude during this maneuver which presents extremely high values of the torsional and normal agilities.

Conclusion

In the preceding sections we adopted the agility metrics based on the definition of $\boldsymbol{A}$ as the derivative of the acceleration vector. This may be considered a choice which is more or less convenient depending on the particular purposes the evaluation of this peculiar parameter is aimed at. Other proposals were put forward in order to tailor the agility metrics definition according to the practical situations where the flight qualities of an aircraft are to be evaluated. However, it is the authors' opinion that the present work follows a rigorous approach which is founded on the differential geometry of the aircraft flight trajectories.

Furthermore the problem of determining the agility characteristics of an airplane is properly dealt with as an inverse flight mechanics problem either from the point of view of the numerical simulation and for envisaging pertinent flight tests. In most of the reported applications our principal concern was to consider maneuvers where the component of the agility vector either greatly prevails on the other two or can be possibly determined alone.

As we already noted the constraints of keeping two components of $\boldsymbol{A}$ equal to zero lead to the evaluation of

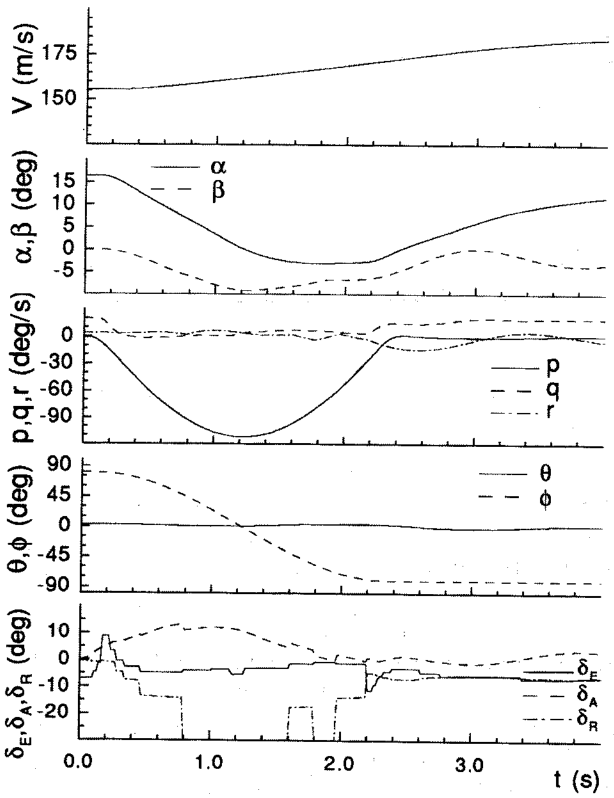

Figure 13: Time histories for a reversed turn; $\phi = \phi(t)$.

smaller maximum values of the third component then in the case where these constraints are relaxed. However this should be interpreted as a way to provide reference conditions for comparing aircraft performances.

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