AILERON TO RUDDER INTERCONNECTION WITH ANALYTICAL REDUNDANCY

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ABSTRACT

This work illustrates how analytical redundancy can be included in an aileron to rudder interconnection flight control system. The work starts with an ARI system already designed and suggests how to include redundancy in the system without extra sensors. For the inclusion of redundancy a robust observer is designed based on the linear lateral directional aircraft mathematical model. The robust observers designed to be included in the system maintains both, the stability and the flying qualities of the sensor based system. Three observer based control laws are designed and evaluated against the sensor based control law with respect to stability and flying qualities. With this design the system is able to support the failure of two sensors and still giving the same performance as the sensor based system.

Introduction

The ARI, aileron to rudder interconnection, is used in order to minimize sideslip in manoeuvres where aileron is used. So in lateral-directional flight control systems an ARI is included and used together with a roll damper and yaw damper. In this way the lateral-directional flight control systems needs in general three sensors, for roll-rate, bank angle and yaw-rate, and if there is a loss of a feedback path, then the aircraft flying qualities will be seriously degraded and in certain cases even the aircraft safety is implied. In view of this it is desirable to include some degree of redundancy in the lateral-directional flight control system. This work shows how such redundancy can be included without adding extra sensors, that is, including analytical redundancy by means of robust-observer-based control laws. These observer-based control laws have been designed by using the Doyle-Stein[1] observer and the idea of Rynaski[2].

So in the event of a feedback path loss the system has alternative reversionary control laws that maintain the same stability and flying qualities as given by the sensor based control law.

Control Law Structure

The control law structure used in this work to obtain the results has been obtained from McLean[3] and is showed in figure (1). As can be seen is a simple roll-damper and yaw damper with ARI included. So this is the sensor based control law, the design of such control law is discussed in the appropriate references, as McLean[3] . McRuer[4] and many others related to ARI design. As can be seen the control law uses three sensors, and if some of then are lost the aircraft performance will be seriously degraded.

Observers Design

Three reduced order observers can be designed, one using roll-rate as input, the second using yaw-rate as input and the third using bank-angle as input. The lateral directional aircraft mathematical model used is given by,

$$\dot{x} = A x + B_1 \delta_a + B_2 \delta_r \quad (1)$$

with the state vector given by $x^T = [\ \beta \ \ p \ \ r \ \ \phi\ ]$ (2)

It is known that the Doyle Stein observer has its poles located at the open loop transmission zeros of the aircraft. In this particular aspect this study is interesting because in the longitudinal case, the open loop transmission zeros are located on the left half of the s-plane, and in this case there are some transmission zeros located on the right half of the s-plane. However as mentioned by Maciejowski[5] the design works quite well if these transmission zeros are located beyond the operating bandwidth of the system as finally designed. The aircraft example used is the same used by MaLean and the appropriate data is,

$$A = \begin{bmatrix} -0.056 & 0.0 & -1.0 & 0.042 \\ -1.050 & -0.465 & 0.39 & 0.000 \\ 0.600 & -0.032 & -0.115 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 \end{bmatrix}$$

$$B_1^T = [\ 0 \quad 0.14 \quad 0.008 \quad 0\ ]$$

$$B_2^T = [\ 0.0022 \quad 0.153 \quad -0.475 \quad 0\ ]$$

The observer dynamics is given by,

$$\dot{z} = F z + G y + H_1 \delta_a + H_2 \delta_r \quad (3)$$

and the estimated state vector is simply,

$$\hat{x}_2 = My + Nz \quad (4)$$

The design method is described in Chen[6], and

$$y = C\ x \quad (5)$$

To design the observer the aircraft dynamics is partitioned in the form,

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + B_{11} \delta_a + B_{12} \delta_r \quad (6)$$

$$\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_{21} \delta_a + B_{22} \delta_r \quad (7)$$

where $y = x_1$ is the sensed state and x_2 is the state vector to be estimated. In table (1) the open loop transmission zeros are listed in order to be used in the design. The design was performed by choosing the poles of the F matrix as the open loop transmission zeros of the aircraft as a first attempt. Then choosing a G in order that the pair (F, G) be controllable. After these steps have been performed a transformation matrix T is obtained by solving a Lyapunov equation as,

$$F T + T(-A) = -G C \quad (8)$$

Then H_1 and H_2 are obtained from H, that is obtained from, $H = T B$ (9)

Then M and N are obtained from,

$$\hat{x} = P^{-1} [\ y \quad z\ ]^T \quad (10)$$

With the help of table (1) it is possible to choose the F matrix of each observer. So in the case of y = p the observer poles are chosen as : -0.0966 ± i 0.8114 and -1.204 as a first design attempt. With this choice the system performance was then assessed with respect to tracking, regulation and frequency response. This choice has showed a good performance with respect to tracking and frequency response, however not with respect to regulation. In view of this fact the complex pole ($-0.0966 \pm i\ 0.8114$) was not used and it was replaced by -0.01 as an approximation for the transmission zero located at 0 and -5 as the third pole of the observer, this third pole was chosen by looking to the system performance. With this choice the system has a better performance in the regulation characteristic maintaining the good tracking performance and the same frequency response as obtained with the sensor based control law.

In the case of $y = r$, the observer poles are the following : -0.7752, -0.5894 and -10, and this choice was good enough with respect to all aspects, again the pole at -10 was obtained by looking to the system performance.

Finally in the case of $y = \phi$ the poles are chosen as -1, -1.204 and -10. The pole at -10 was chosen in order to approximate the transmission zero located at infinity and the pole at -1 was chosen in order to obtain good regulation characteristic.

It must be noticed that in the case of $y = r$ a pole at $s = -10$ was chosen, this was due to the fact that in this case there are only two transmission zeros in the left half s-plane, and so the third pole was taken in order to approximate the infinity and taking care to not degrade the system performance with respect to the sensor based control law. Then the observer F matrices are identified as F_p for the case of $y = p$, F_r for the case of $y = r$, and finally F_ϕ for the case of $y = \phi$, and they are the following:

F_p = diagonal (-0.01 , -1.204 , -5)

F_r = diagonal (-0.7752 , -0.5894 , -10)

F_ϕ = diagonal (-1 , -1.204 , -10)

The matrix G was chosen as, $G^T = [\,1\ \ 1\ \ 1\,]$, for the three cases. With these F and G the H_1 and H_2 matrices obtained are the following :

In the case of $y = p$;

$H_1^T = [\,-0.1124\ \ 0.2030\ \ 0.0308\,]$

$H_2^T = [\,0.50\ \ 0\ \ 0.0376\,]$,

In the case of $y = r$;

$H_1^T = [\,0.0\ \ -0.1357\ \ 0.0009\,]$

$H_2^T = [\,-0.3123\ \ 0.001\ \ -0.0477\,]$,

In the case of $y = \phi$;

$H_1^T = [\,-0.296\ \ -0.168\ \ -0.0015\,]$

$H_2^T = [\,0.145\ \ 0\ \ -0.0017\,]$.

The M matrices obtained are the following :

In the case of $y = p$; $M_p^T = [\,-3.52\ \ 6.0\ \ 12.65\,]$,

In the case of $y = r$; $M_r^T = [\,18.48\ \ 7.69\ \ 329.87\,]$,

In the case of $y = \phi$; $M_\phi^T = [\,-17.3\ \ 11.6\ \ 6.2\,]$.

The N matrices obtained with this design are the following :

$$N_p = \begin{bmatrix} 0.0275 & 0.314 & 14.00 \\ 0.0776 & 1.730 & -38.1 \\ 1.76 & 3.63 & -74.9 \end{bmatrix}$$

$$N_r = \begin{bmatrix} 0.426 & -0.0822 & -186.80 \\ 4.340 & -1.2570 & -108.25 \\ 44.15 & -2.960 & -3572.6 \end{bmatrix}$$

$$N_\phi = \begin{bmatrix} 2.39 & -5.90 & 198.20 \\ -0.37 & 0.87 & -119.1 \\ -4.3 & 8.3 & -87.3 \end{bmatrix}$$

In this way the observers design are completed and they can be used together with the control law showed in figure (1).

Observer-Based Control Laws

With the three reduced order observers designed it is now possible to design three observer based control laws and then the required redundancy, with respect to sensor failures, can be introduced in the flight control system. The observer-based control law when the aircraft output is considered as roll-rate will be called CLP, the observer based control law when the aircraft output is yaw-rate will be called CLR and finally the observer based control law when the aircraft output is bank angle will be called CLF. In figure (2) the CLP control law is showed as an example.

Similar figures can be drawn for the cases of CLR and CLF. In this way with only three sensors, one for roll-rate, one for yaw-rate and one for bank-angle there are three alternative observer based control laws. This allows the system to support two sensor failures and still working with the same level of stability and flying qualities. The closed loop system can be represented by :

$$\dot{x} = A\,x + B_1\,\phi_d + B_2\,r_d \qquad (11)$$

with the state vector x given by :

$x^T = [\ x_1\ \ x_2\ \ \delta_a\ \ \delta_r\ \ \delta_{ryd}\ \ \delta_{rcf}\ \ z\ ]$

Performance Comparison

In order to assess the performance of the three observer based control laws with respect to the sensor based-control law a comparison of the sideslip (β) time response to a step in the commanded ϕ_d , of the sideslip time response for an initial sideslip perturbation, that is, a beta-release response, and of the bode plot of the frequency response obtained with the transfer function of β / ϕ_d is showed. So in figure (3) these responses are showed for the sensor based control law, in figure (4) they are showed for the observer based control law CLP, in figure (5) they are showed for the observer based control law CLR and finally in figure (6) they are showed for the observer based control law CLF. From these figures it can be noticed that the same flying qualities are maintained with any one of the observer based control laws and so not only stability robustness is assured but also performance robustness.

These figures show that the frequency response is the same for any of the control laws, the step response is also the same for any of the control laws and finally the beta release is almost the same, however the regulation performance is not affected, and can be made the same by redesigning the observers by means of changing only the observer poles that have been choose based on the system performance. So the observer based control laws are able to maintain the same flying qualities as given by the sensor based control law. A comparison of the M matrices of each observer shows that the observer with $y = r$ has some disadvantage with respect to the others, since it has terms of higher magnitude and so it is more dangerous in the event of a sensor failure. The observer that gives M with elements of lower magnitude is when $y = p$, followed by the M of the observer with $y = \phi$. A comparison of H_1 and H_2 matrices of each observer shows a similar magnitude for the three cases, and so very close to zero as required by the Doyle-Stein observer.

Redundancy Aspect

As noticed it is then possible to include a certain degree of redundancy in the system by means of these three observer-based control laws. So in this way it is allowed to the flight control system to support a double sensor failure and still maintaining the original designed flying qualities and stability characteristics.

In the event of a roll rate sensor failure the system can switches to CLR or CLF, if then it happens a yaw rate-sensor failure the system can switches to CLF, and similar cases are obvious. Using the notation SBCL for the sensor based control law, the following situations are then possible :

SBCL → CLR → CLF

SBCL → CLF → CLR

SBCL → CLP → CLF

SBCL → CLF → CLP

SBCL → CLP → CLR

SBCL → CLR → CLP

Conclusions and Observations

The research has shown that redundancy can be introduced analytically in the lateral directional flight control system maintaining the original flying qualities and stability by means of the guidelines given to design a Doyle-Stein observer, that is, a robust observer. Obviously the observer parameters must be scheduled with flight conditions in order to maintain the robustness obtained. The main aspect of the research is that some of the open loop transmission zeros are located in the right half s-plane and so here it has been showed that the Doyle-Stein observer also works in these cases. Similar time responses for the other state variables have been obtained and it has been assessed against the sensor based control law responses resulting in a very good agreement. It must also be mentioned that an observer based control law with sideslip input to the observer could also be designed, however it was not used here due to the fact that sideslip sensors are not used in general in flight control systems.

Control Law Data

The following data are with respect to figure (1). The aileron actuator transfer function is given by : 10 / (s + 10). The rudder actuator transfer function is given by : 4 / (s + 4) . The yaw-damper wash-out transfer function is given by : s / (s + 1) . The cross-feed wash-out transfer function is given by : s / (s + 20) And finally the gains are the following :
$G_p = 9.5156$; $G_r = 10$; $K_{cf} = 0.035$; $G_\phi = 1$ and $K_{fa} = 10$.

References

[1] Doyle, J. C. and Stein, G. *Robustness with observers.* - IEEE Transactions on Automatic Control. - vol. AC-24, no.4, aug 1979, pp 607-611.

[2] Rynaski, E.G. *Flight control synthesis using robust output observers.* Proc. AIAA, Guidance and Control Conference. - San Diego - CA - aug 1982 pp 825-831.

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[5] Maciejowski, J.M. *Multivariable Feedback Design.* - Addison-Wesley Publishing Company - 1989 .

[6] Chi-Tsong Chen *Linear System Theory and Design.* CBC College Publishing, Holt. - Rinehart and Winston, 1984.

Tables and Figures

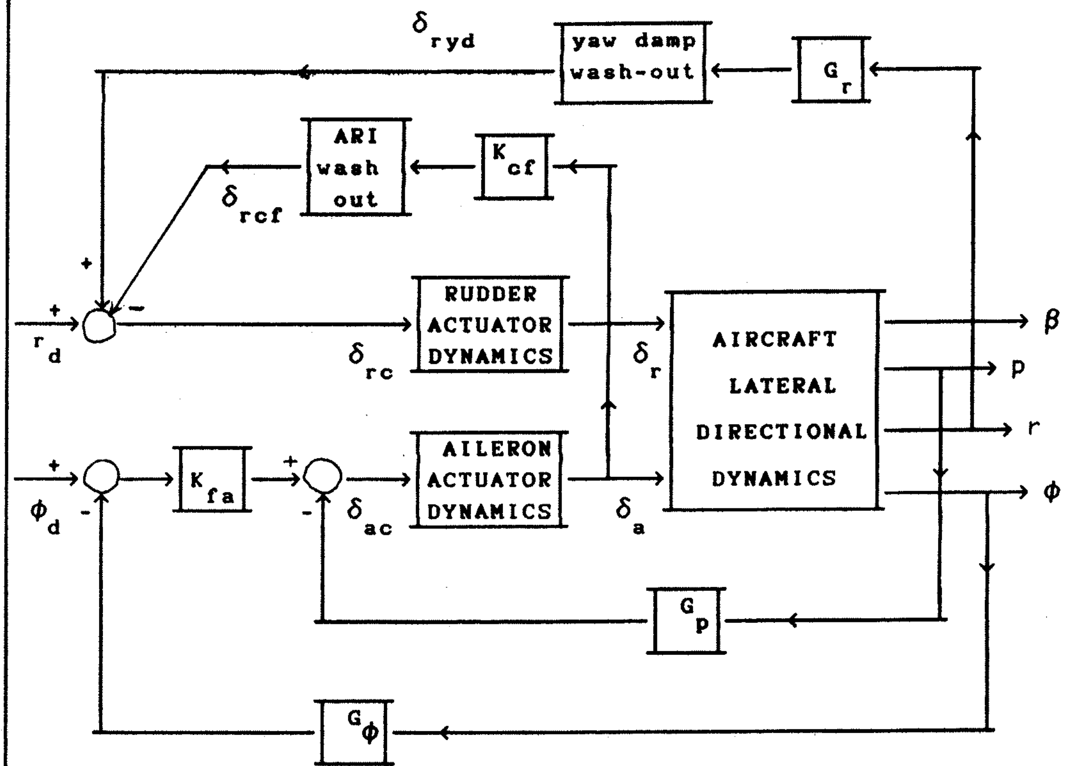

figure 1 - ARI included in the lateral-directional flight control system with sensors of roll-rate (p), yaw-rate (r) and bank angle (ϕ).

TABLE 1 - OPEN LOOP TRANSMISSION ZEROS

T.F.	ZEROS		
p/δ_a	$-0.0966 \pm i\ 0.8114$	0	
p/δ_r	2.2588	-1.204	0
r/δ_a	$0.407 \pm i\ 0.678$	-0.7752	
r/δ_r	$0.0304 \pm i\ 0.2452$	-0.5894	
ϕ/δ_a	$-0.0966 \pm i\ 0.8114$		
ϕ/δ_r	-1.204	2.2588	

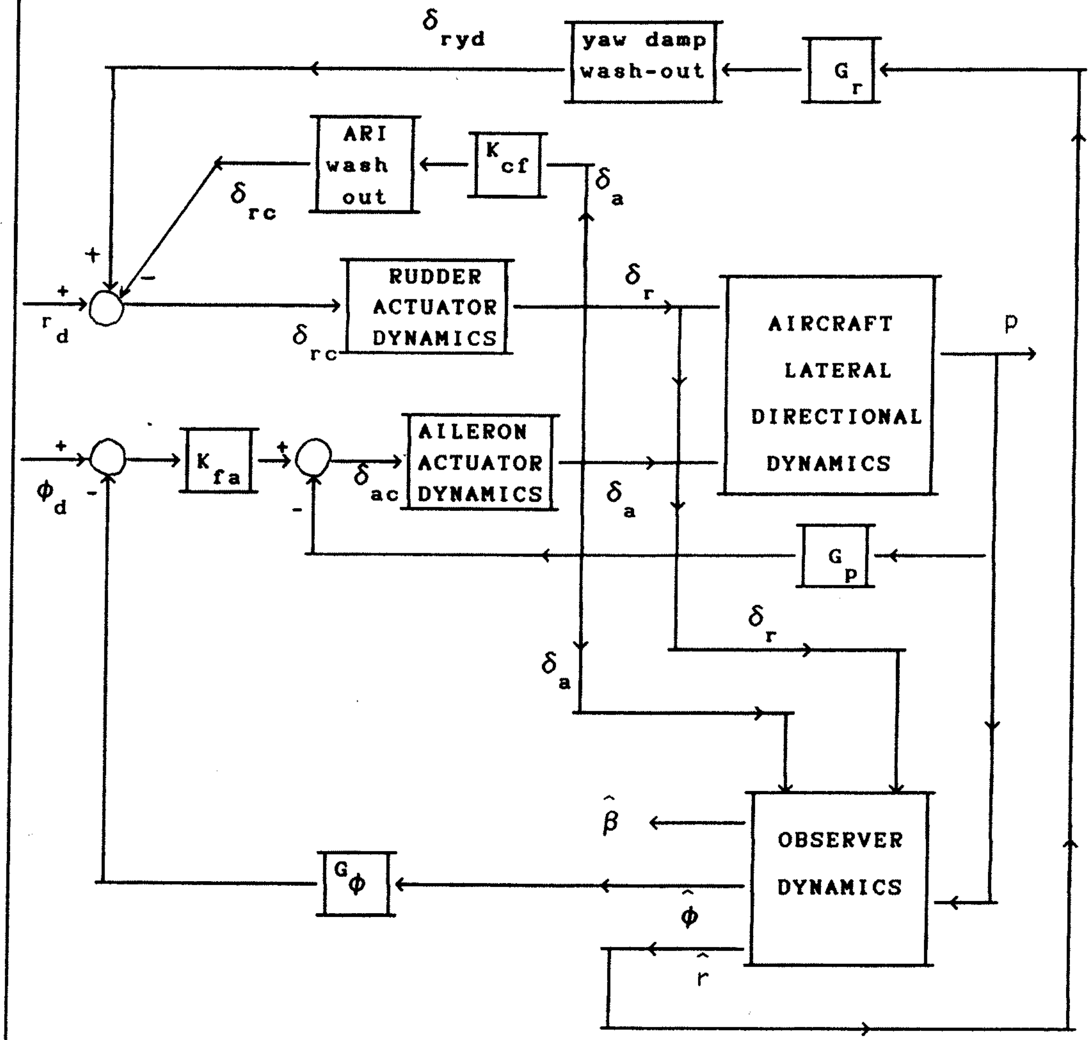

figure 2 - ARI included in the lateral-directional flight control system with observer-based control law CLP using only roll-rate as input

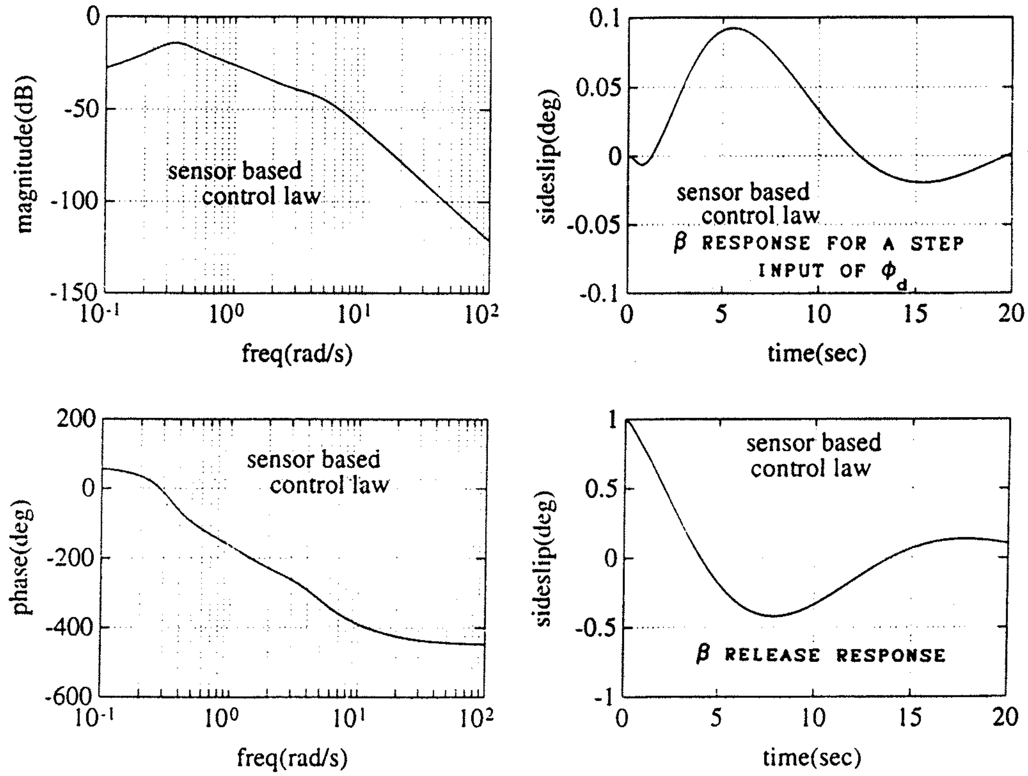

figure 3 - flight control system performance obtained with the sensor based control law

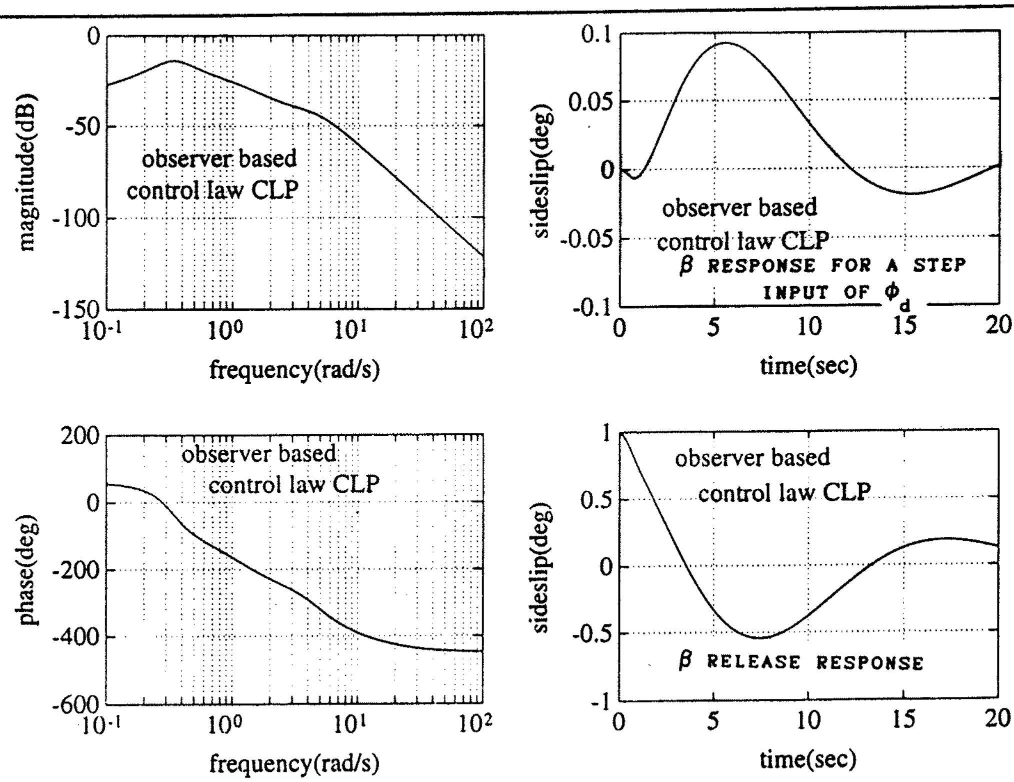

figure 4 - flight control system performance obtained with the observer based control law CLP

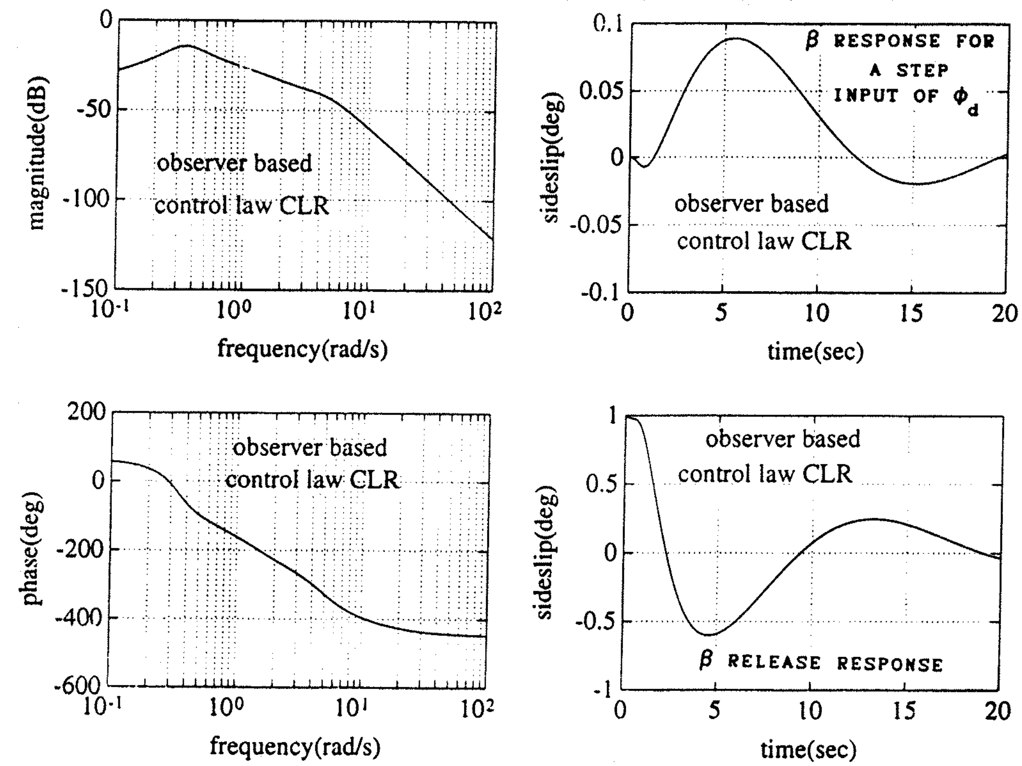

figure 5 - flight control system performance obtained with the observer-based control law CLR

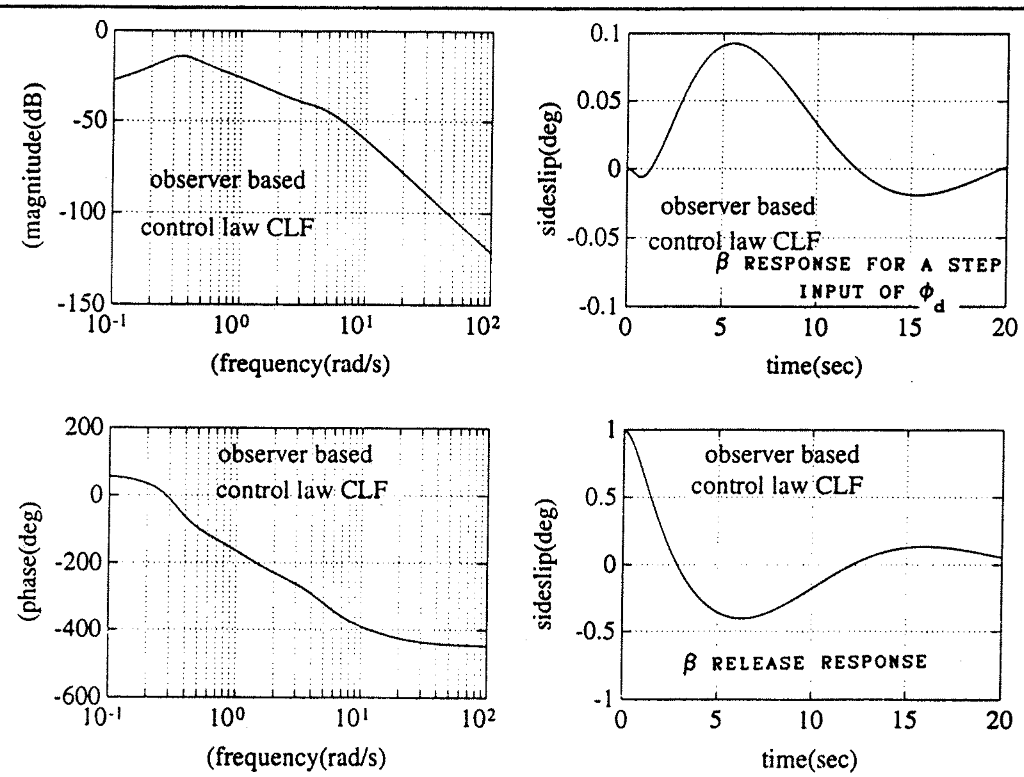

figure 6 - flight control system performance obtained with the observer based control law CLF